TUNNELING THEORIES OF IONIZATION

KELDYSH

L. V. Keldysh, JETP **47**, 1945 (1964) [Sov. Phys. JETP **20**, 1307 (1965)]

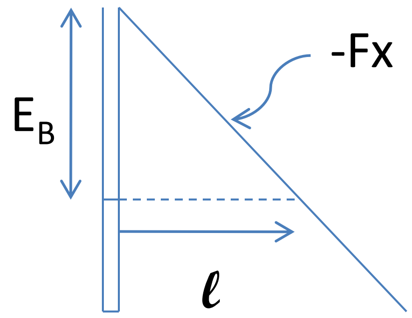
Keldysh is famous in solid-state physics. The introduction in his paper is written from the point of view of solid state physics. It contains almost every misconception that continues to plague strong-field (or high-field) laser physics to the present day.

List of problem concepts from pages 1 and 2 of the Keldysh paper:

- Atomic ionization is basically a tunneling process at low frequencies. *Stated without justification or apparent awareness that lasers do not produce longitudinal fields.*
- There is a transition between tunneling and multiphoton mechanisms that depends only on a specific frequency or an equivalent specific field intensity. *(To be explained further.)*
- There are "direct" transitions to the continuum, or indirect if there is an intervening resonance in the bound system. *This arises from a perturbative point of view.*
- The initial atomic state is Stark-shifted by the applied field. *This is again a perturbative notion, where the structure of quantum transition matrix elements are not inspected from a fundamental measurements point of view (as with S matrices).*
- All physical effects depend on electric field strength. *Laser effects are measured by energies.*

TUNNELING TIME, TUNNELING LENGTH, TUNNELING FREQUENCY

$\ell \approx E_B / F$
$v \approx (E_B)^{1/2}$
$\omega_t \approx F / (E_B)^{1/2}$
$\tau \approx 1 / \omega$



The frequency ω_t is taken to be the dividing line between tunneling ($\omega < \omega_t$) and multiphoton ($\omega > \omega_t$) processes.

There are statements like: "…the tunnel effect…is the most effective mechanism for the absorption of high-power radiation…in the region of frequencies… $\omega < E_B$ …"

Keldysh introduces a gauge-transformed Volkov solution as the wave function for the ionized electron with the statement: *"The difference between our procedure and the usual perturbation theory lies..only..in that we calculate the probability of transition not to a stationary final state, but to a state that takes exact account of the main effect of the electric field – the acceleration of the free electron."*

What is wrong with that statement?

The statement is completely QSE (quasistatic electric field) in nature. The Volkov solution for plane waves has nothing to do with net acceleration; it describes the oscillatory nature of an electron in a transverse PW field.

All of the descriptive statements by Keldysh describe a physical scenario that is unrelated to strong-field laser phenomena.

“LENGTH-GAUGE VOLKOV SOLUTION”

Quotation marks are used around the phrase “length-gauge Volkov solution” because, in principle, there is no such thing. The free-particle Schrödinger equation in the LG is

$$i\partial_t \Psi_{LG}^{free} = \left[\frac{1}{2}\left(-i\nabla\right)^2 + \mathbf{r}\cdot\mathbf{F}\left(t\right)\right]\Psi_{LG}^{free}.$$

There is only the scalar potential $\mathbf{r}\cdot\mathbf{F}\left(t\right)$ and no vector potential at all. The solution for this SE is a solution for a QSE field. The Volkov solution is explicitly for a PW, and the result used here is a nonrelativistic dipole-approximation limit of the Volkov solution for the Dirac equation (spin-1/2 electron) or the Klein-Gordon equation (spin-0 electron). There is no way to find directly a Volkov solution from the above SE.

DIPOLE APPROXIMATION VOLKOV SOLUTION

The concept of "dipole approximation" for a free particle seems strange, because a free particle solution, in principle, should extend over an unbounded range in space, thus negating the usual $\mathbf{k}\cdot\mathbf{r}\ll 1$ justification for the dipole approximation. Nevertheless, for a solution to be used in a matrix element together with a wave function for an atomic bound state, the effective range of $\mathbf{r}$ is limited, and the dipole approximation can be employed for the free-particle part of the matrix element. The free-particle dipole-approximation Schrödinger equation in the VG is

$$i\partial_t\Psi_{VG}^{Volk}=\frac{1}{2}\left(-i\nabla+\frac{1}{c}\mathbf{A}\left(t\right)\right)^2\Psi_{VG}^{Volk},$$

with the normalized solution

$$\Psi_{VG}^{Volk}\left(\mathbf{r},t\right)=\frac{1}{\left(2\pi\right)^{3/2}}\exp\left\{i\mathbf{p}\cdot\mathbf{r}-\frac{i}{2}\int_{-\infty}^{t}d\tau\left[\mathbf{p}+\frac{1}{c}\mathbf{A}\left(\tau\right)\right]^2\right\},$$

as is easily verified. In the LG, all that is necessary is to multiply the above result by the gauge transformation factor

$$\exp\left(\frac{i}{c}\mathbf{r}\cdot\mathbf{A}\left(t\right)\right)$$

to get the "length-gauge Volkov solution"

$$\Psi_{LG}^{Volk}\left(\mathbf{r},t\right)=\frac{1}{\left(2\pi\right)^{3/2}}\exp\left\{i\left[\mathbf{p}+\frac{1}{c}\mathbf{A}\left(t\right)\right]\cdot\mathbf{r}-\frac{i}{2}\int_{-\infty}^{t}d\tau\left[\mathbf{p}+\frac{1}{c}\mathbf{A}\left(\tau\right)\right]^2\right\}.$$

A PARADOX

Definition of a paradox: "a statement or proposition that seems self-contradictory or absurd, but in reality expresses a possible truth"

The length-gauge Schrödinger equation has only a QSE solution, but a PW solution of the length-gauge Schrödinger equation can be found by a gauge transformation from the VG.

This is paradoxical.

Resolution of the paradox: Note that the Volkov solution in the LG is in terms of ***A****(t)*, which does not exist in the LG. To remove it, one can set

$$\mathbf{A}\left(t\right) = -c\int_{-\infty}^{t} d\tau \mathbf{F}\left(\tau\right).$$

F*(t)* does exist in the VG, but using the above device makes the Volkov solution in the LG a **nonlocal** solution; one must refer to the electric field at all previous times, not just at the "local time" *t*.

KELDYSH RESOLUTION OF THE PARADOX

Keldysh uses a different approach to resolution of the paradox. Instead of

$$\mathbf{A}(t) = -c \int_{-\infty}^{t} d\tau \mathbf{F}(\tau) .$$

Keldysh notes that ***A*** and ***F*** will have the same trigonometric behavior, except for a phase shift of π /2:

$$\mathbf{A}(t) = \mathbf{A}_0 \sin \omega t$$

$$\mathbf{F}(t) = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{A}(t) = -\frac{\omega}{c}\mathbf{A}_0 \cos \omega t$$

Keldysh retains the vector potential phase, while replacing the amplitude factor so as to hide the fact that it is a (non-existent in LG) vector potential, and give it the appearance of being an electric field.

$$\mathbf{A}_0 \rightarrow -\frac{c}{\omega}\mathbf{F}_0$$

$$\mathbf{A}_0 \sin \omega t \rightarrow -\frac{c}{\omega}\mathbf{F}_0 \sin \omega t$$

The vector potential actually remains in the solution, but it is made to look like a local electric field.