STRUCTURE OF THE SFA

THERE IS A SIMPLE CONNECTION BETWEEN EACH ANALYTICAL ELEMENT OF THE SFA AND ITS PHYSICAL SIGNIFICANCE

Spherical coordinates, linear polarization:

$$\frac{dW}{d\Omega} = \frac{1}{(2\pi)^2}\sum_{n_0}^{\infty} p\left(\frac{p^2}{2} + E_B\right)^2 \left|\hat{\phi}_i(\vec{p})\right|^2 \left[J_n(\alpha_0 p\cos\theta, -\beta_0 c)\right]^2$$

From phase space integration.

Net energy transfer to ionized electron.

From interaction Hamiltonian.

Momentum distribution in initial atomic state.

Generalized Bessel function; α_0 is the amplitude of the figure-8 in the ***E*** direction; β_0 is the amplitude of the figure-8 in the ***k*** direction.

From the Volkov solution.

$$\frac{p^2}{2} = n\omega - E_B - U_p$$

1

MOMENTUM-SPACE WAVE FUNCTIONS

For hydrogenic atoms:

$$\tilde{\phi}(p)=\left[\frac{2}{\pi}\frac{(n-l-1)!}{(n+l)!}\right]^{1/2} n^2 2^{2l+2} l!\frac{(np)^l}{\left(n^2p^2+1\right)^{l+2}} C_{n-l-1}^{l+1}\left(\frac{n^2p^2-1}{n^2p^2+1}\right),$$

$C_N^{\nu}(x)$ *are Gegenbauer functions*

n = principal quantum number,
l = angular momentum quantum number
For small momenta p:

$$\left|\tilde{\phi}(p)\right|^2 \sim p^{2l}$$

With the additional p from the phase-space factor, the behavior of SFA spectra can be strongly dependent on p at low energies, and certainly do not have maxima at the low-energy end as tunneling theory predicts.

2

HOW THE SFA IS APPLIED

For single-intensity results:

$$W = \int d\Omega \frac{dW}{d\Omega} = 2\pi \int_{1}^{-1} d(\cos\theta) \frac{dW}{d\Omega}$$

This gives angular distributions and total rates directly.

The expression for $dW/d\Omega$ contains a sum over photon order n, and energy conservation gives $p^2 / 2 = n\omega - E_B - U_p$,
so spectra can be obtained by "binning" contributions to the total rate in accordance with the energy associated with each contribution.

3

APPLICATION OF THE SFA TO PRACTICAL CONDITIONS

For single ionization (for example), apply the rate equation:

$$\frac{dN_1(t,\vec{r})}{dt} = \left[N_0 - N_1(t,\vec{r})\right] W_1(t,\vec{r})$$

$$N_1(t) = N_0\left\{1 - \exp\left[-\int_{t_1}^{t} d\tau W_1(\tau)\right]\right\},$$

$$N_1(t_1) = 0$$

where N_0 = number density of neutrals and N_1 = number density of singly ionized atoms. With an intensity distribution:

$$\mathcal{N}_1 = 2\pi N_0 \int_0^\infty r dr \int_{-\infty}^{\infty} dz \left(1 - exp\left\{-\int_{-\infty}^{\infty} d\tau W\left[I(\tau,\vec{r})\right]\right\}\right)$$

where cylindrical symmetry of the focal volume is assumed, and where $W[I(\tau,\mathbf{r})]$ signifies the transition rate as a function of the intensity that varies as a function of time and position within the focal region.

4

Using the rate equation approach fully accounts for depletion effects.

All results using the so-called “length-gauge SFA” have been without any depletion effects. The onset of depletion considerations is rather abrupt. For most problems it matters very little, but when it does begin to matter, a slight increase in intensity can make a major qualitative difference.

For more details and for application to multiple ionization see:
HRR, *Prog. Quant. Electron.* **16**, 1 (1992).

See also HRR, *Phys Rev A* **54**, R1765 (1996).

5

SFA IN CYLINDRICAL COORDINATES
Suited to momentum distributions

$$d^3p = 2\pi\, p_\perp dp_\perp dp_\parallel$$

$$\frac{dW}{dp_\parallel} = \frac{1}{2\pi}\sum_{n=n_0}^{\infty}\left(\frac{p^2}{2}+E_B\right)^2\left|\tilde{\phi}_i(p)\right|^2\left[J_n\left(\alpha_0 p_\parallel,-\beta_0 c\right)\right]^2$$

$$\frac{dW}{p_\perp dp_\perp} = \frac{1}{2\pi}\sum_{n=n_0}^{\infty}\frac{1}{p_\parallel}\left(\frac{p^2}{2}+E_B\right)^2\left|\tilde{\phi}_i(p)\right|^2\left[J_n\left(\alpha_0 p_\parallel,-\beta_0 c\right)\right]^2$$

$$with \quad \frac{p^2}{2} = n\omega - E_B - U_p = \frac{1}{2}\left(p_\parallel^2 + p_\perp^2\right)$$

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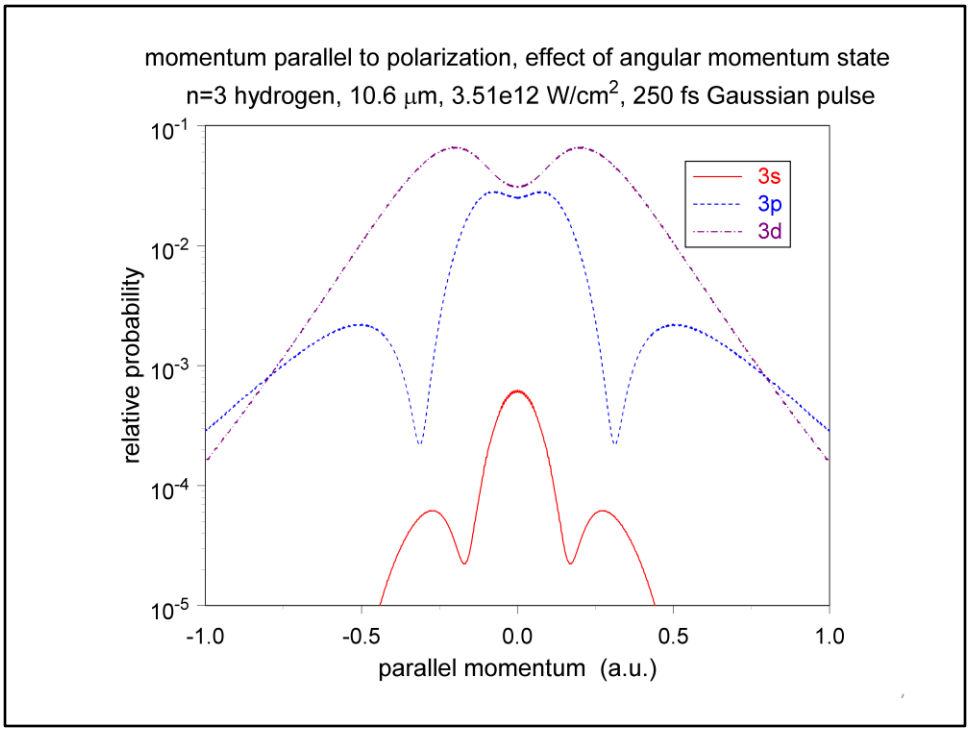
momentum parallel to polarization, effect of angular momentum state
n=3 hydrogen, 10.6 μm, 3.51e12 W/cm2, 250 fs Gaussian pulse
relative probability
10-1
10-2
10-3
10-4
10-5
-1.0
-0.5
0.0
0.5
1.0
parallel momentum (a.u.)
3s
3p
3d

AVERAGING OVER INITIAL m SUBSTATES

So far, only the radial wave function of the initial state Φ_i has been mentioned.
If the initial state is other than an *s* state, and no pre-pulse alignment of the atoms is assumed, then an average over initial *m* substates should be done.

For an incoherent average over *m* substates for an initial p-state atom:

$$\left|Y_1^0\right|^2 = \left(\frac{3}{8\pi}\right) 2\cos^2\theta$$

$$\left|Y_1^{\pm 1}\right|^2 = \left(\frac{3}{8\pi}\right) \sin^2\theta$$

$$\frac{1}{3}\left(\left|Y_1^0\right|^2 + \left|Y_1^1\right|^2 + \left|Y_1^{-1}\right|^2\right) = \frac{1}{4\pi}$$

$$Note: \quad \left|Y_0^0\right|^2 = \frac{1}{4\pi}$$

8

COMPARISON WITH EXPERIMENTS

No systematic study has been done comparing SFA momentum distributions with experiments has been done, but some general remarks are possible.

- Tunneling distributions are always too narrow.
- SFA distributions have about the same width as experimental results, but the behavior for small momenta is not reliable.
- LG SFA calculations have not been done.
- An attempt to extract a “tunneling limit” from the VG SFA had an interesting result, as shown on the next slide.

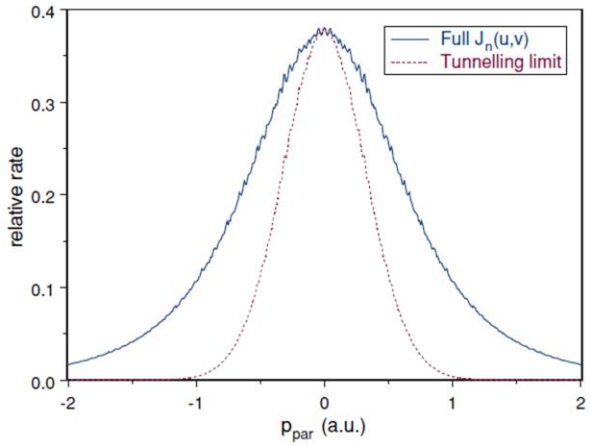

Figure 1. Momentum distribution of photoelectrons from ionization of 1s hydrogen by light of 800 nm, as in an example given by Bauer [2]. Field intensity is such that $z_1 = 40$, where $z_1 \equiv 2U_p/E_B$, U_p is ponderomotive energy, and E_B is atomic binding energy. The distribution is with respect to the momentum component parallel to the polarization direction of the laser. The tunnelling limit and the full $J_n(u, v)$ coincide only for small momenta. Relative rates are calculated by the SFA assuming a Gaussian time distribution in a laser pulse of 25 fs.

HRR and Krainov, J. Phys. A **38**, 527 (2005).

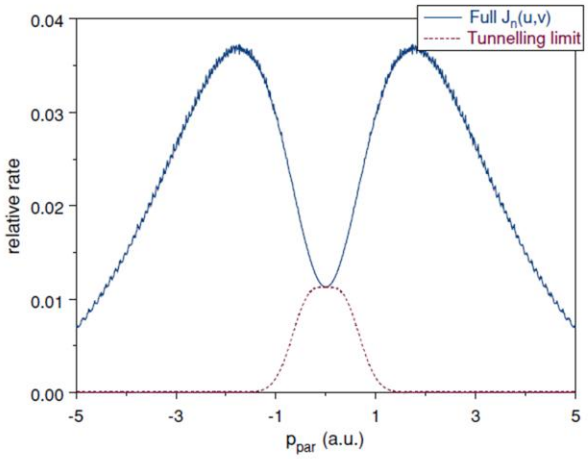

Figure 2. This momentum distribution is comparable to figure 1 except that it is for the case of neon, which has a dip in the momentum distribution near $p_{\parallel} = 0$. Wavelength and z_1 values are the same as for figure 1. The result of the dip is that most of the transition rate comes from values of the momenta that are neglected in the tunnelling approximation to $J_n(u, v)$.

MOMENTUM DISTRIBUTIONS AT HIGH FREQUENCIES

Recent attempts to find theoretical Coulomb corrections (Ivanov & Smirnova) to the LG SFA were evaluated by Bondar *et al.* by comparing with TDSE for high-frequencies.

Included in the comparisons with the VG SFA with no corrections.

PHYSICAL REVIEW A **79**, 063404 (2009)

Photoelectron spectra in strong-field ionization by a high-frequency field

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We analyze atomic photoelectron momentum distributions induced by bichromatic and monochromatic laser fields within the strong-field approximation (SFA), separable Coulomb-Volkov approximation (SCVA), and *ab initio* treatment. We focus on the high frequency regime—the smallest frequency used is larger than the ionization potential of the atom. We observe a remarkable agreement between the *ab initio* and velocity gauge SFA results while the velocity gauge SCVA fails to agree. Reasons of such a failure are discussed.

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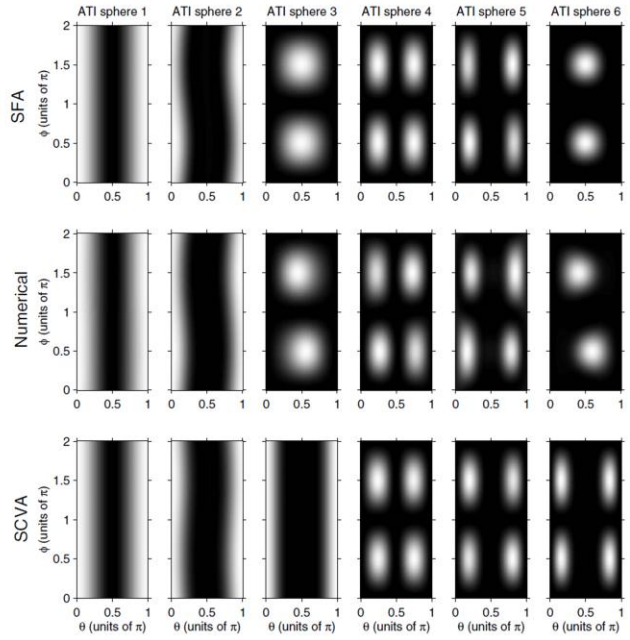

FIG. 1. Normalized ATI spheres (within the SFA, *ab initio*, and SCVA results) for the ground state of a hydrogen atom with field parameters given by $\mathbf{F}_1^{(1)}=(0,0,0.1)$ (a.u.), $\mathbf{F}_2^{(1)}=\mathbf{0}$, $\Omega_1=1$ (a.u.), $\varphi_1=0$ and $\mathbf{F}_1^{(2)}=(0,0.1,0)$ (a.u.), $\mathbf{F}_2^{(2)}=\mathbf{0}$, $\Omega_2=3$ (a.u.), $\varphi_2=0$; θ and ϕ are spherical angles (zenith and azimuth). Linear color scale goes from zero (black) to maximum (white).

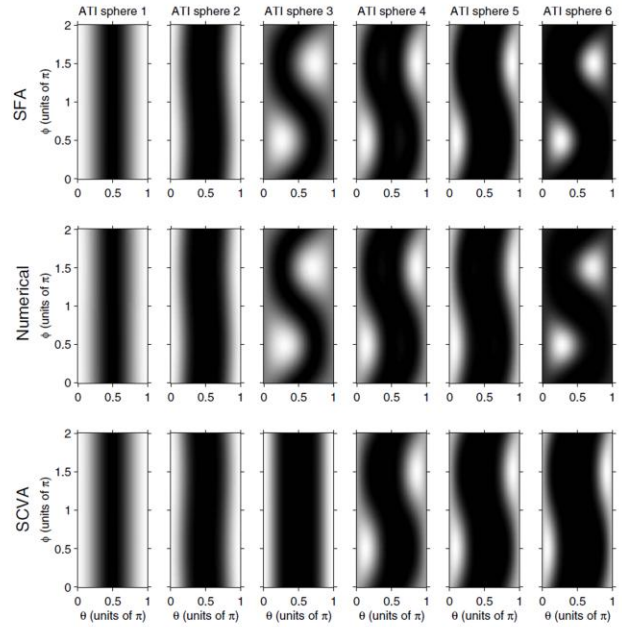

FIG. 2. Normalized ATI spheres (within the SFA, *ab initio*, and SCVA results) for the ground state of a hydrogen atom with field parameters given by $\mathbf{E}_1^{(1)}=(0,0,0.1)$ (a.u.), $\mathbf{E}_2^{(1)}=\mathbf{0}$, $\Omega_1=1$ (a.u), $\varphi_1=0$ and $\mathbf{E}_1^{(2)}=0.1(0,\sin[\pi/4],\cos[\pi/4])$ (a.u.), $\mathbf{E}_2^{(2)}=\mathbf{0}$, $\Omega_2=3$ (a.u.), $\varphi_2=0$. Linear color scale goes from zero (black) to maximum (white).