SUPPLEMENT TO THE LECTURE ON GAUGES

REVIEW OF THE PRESENT STATE OF CONFUSION IN STRONG-FIELD PHYSICS

These lectures have implicitly associated the length gauge with tunneling and the velocity gauge with the SFA. Nevertheless, there is complete chaos in the literature, with the widespread assumption that there is no difference between tunneling and the SFA.

Keldysh: Keldysh begins with a Volkov solution gauge-transformed to the LG. He stays with this concept only long enough to point out that there can be an inference of an integer *n* that can be associated with the number of photons absorbed. He never uses that concept, and the paper continues as a pure tunneling treatment.

PPT: They never introduce the Volkov solution, but they make a gauge transformation to the VG in order to show (as did Keldysh) that one can introduce the concept of a number *n* of photons absorbed. Beyond that observation, everything else is tunneling.

HRR 1990: The confusion between tunneling and the Volkov-based VG paper of 1980 [HRR, PRA **22**, 1786 (1980)] had become damaging and required clarification. The SFA terminology was introduced [HRR, PRA **42**, 1476 (1990)] in an unsuccessful attempt to clarify matters.

Lewenstein HHG paper 1994: Lewenstein et al. took note of the SFA suggestion, but interpreted it to mean: *SFA ≡ use of Volkov solutions.* They then used the LG with gauge-transformed Volkov solutions, and called it the SFA.

RESOLUTION OF THE TUNNELING – SFA CONFUSION

Within the *"blue triangle"*, VG SFA and the so-called "LG SFA" give similar results, so repeated attempts to make the distinction have not achieved much.

From personal conversations, it seems that many people assume that I introduced the SFA terminology to avoid using the "personalized" KFR terminology. (Examples: Peter Knight, Misha Ivanov, Wilhelm Becker, Dejan Milosevic, etc.). That is, there are multiple misunderstandings causing the confusion.

An important clue to the resolution of the problem: The Volkov solution is written in terms of a vector potential ***A***. The gauge transformation factor that generates the gauge transformation from the VG to the LG is $exp(i\boldsymbol{A}\cdot\boldsymbol{r})$. That means that the Volkov solution employed in the LG requires a vector potential ***A*** from a different gauge; there is no vector potential in the LG. This has been noticed many times, so the suggested solution is to replace ***A*** by an integral representation in terms of the electric field ***F***:

$$\boldsymbol{A}(t) = -c\int_{-\infty}^{t} d\tau \boldsymbol{F}(\tau).$$

This means that a Volkov solution can be introduced into the LG only by making it a nonlocal expression – evidence of the inappropriateness of a "LG Volkov solution".

TUNNELING – SFA CONFUSION (continued)

The Volkov solution is “inappropriate” in the LG because the Volkov solution is a plane-wave (PW) solution and the LG can describe only quasistatic electric (QSE) fields.

The LG interaction Hamiltonian is $\boldsymbol{r}\cdot\boldsymbol{F}(t)$. This contains only the electric field, and selects the direction of the electric field explicitly.

If one considers the logical low-frequency limit of the $\boldsymbol{r}\cdot\boldsymbol{F}(t)$ interaction Hamiltonian, it is $\boldsymbol{r}\cdot\boldsymbol{F}_0$, where $\boldsymbol{F}_0$ is a constant electric field. As has been shown (repeatedly) the low-frequency limit of a plane wave is an extreme-relativistic quantity.

The use of a gauge-transformed Volkov solution in the LG serves only to disguise the inescapable fact the $\boldsymbol{r}\cdot\boldsymbol{F}(t)$ interaction Hamiltonian can describe only longitudinal fields.

IMPORTANT ADDENDUM

It has been remarked (frequently) that the VG and LG theories give similar results in the "blue triangle". This is the domain in which the dipole approximation is valid, and where there is a gauge equivalence between the VG and the LG. However…

The upper frequency limit on the dipole approximation comes from simplification of the phase of a traveling wave from ωt - $\boldsymbol{k}\cdot\boldsymbol{r}$ to ωt . **This limit on gauge equivalence is available *only* from the VG. There is no PW phase in the LG.**

The lower frequency limit on the dipole approximation comes from the onset of magnetic-field effects. **This limit on gauge equivalence is available *only* from the VG. There is no magnetic field possible in the LG, which refers strictly to a QSE field.**

ALL INFORMATION ABOUT THE DOMAIN OF APPLICABILITY OF GAUGE EQUIVALENCE COMES FROM THE VG; WHICH IS THE PHYSICAL GAUGE.