STABILIZATION

Strong-field processes in general have a property that is generally a surprise, although it has been known for a long time. That is, as the field intensity increases, certain field-induced processes exhibit a maximum rate, after which a further increase in intensity leads to a decline in rate.

In strong-field ionization of atoms, this has had the name “stabilization” attached to it, although that name is not universally liked.

Processes with this property have been noted in the literature at least since 1970. See: HRR, PRL **25**, 1149 (1970).

It is particularly easy to demonstrate with the SFA for circular polarization, so this example will be presented; but keep in mind that laser-induced ionization is not the only process to have this property.

IONIZATION BY CIRCULARLY POLARIZED LIGHT

Circular polarization leads to a rate expression identical to that for linear polarization except that the generalized Bessel function is replaced by an ordinary Bessel function.

$$\frac{dW}{d\Omega} = \frac{1}{(2\pi)^2} \sum_{n=n_0}^{\infty} p \left(\frac{p^2}{2} + E_B \right)^2 \left| \tilde{\phi}_i(p) \right|^2 J_n^2(\alpha_0^{cir} p \sin\theta).$$

$$p = \sqrt{2(n\omega - U_p - E_B)}; \quad \alpha_0^{cir} = \sqrt{2z/\omega}; \quad define\ \zeta \equiv \alpha_0^{cir} p \sin\theta$$

When the field is very intense, U_p is large, and so the index *n* is also large.
Then the Bessel function has asymptotic forms that are analytically simple, and have the property that

$$\left| J_n(x) \right| \ll 1 \quad unless \left| x \right| \approx n$$

The appropriate constraint here comes from

$$\left| \zeta \right| = \left| \alpha_0^{cir} p \sin\theta \right| \le \left| \alpha_0^{cir} p \right| = \left(\frac{2z}{\omega} \right)^{1/2} \sqrt{2(n\omega - U_p - E_B)} = 2\sqrt{z(n - z - \varepsilon_B)}$$

$$where\ \varepsilon_B \equiv E_B / \omega$$

The quantity $|\zeta|$ has a maximum as a function of *z*, easily found to occur when

$$z = \frac{1}{2}(n - \varepsilon_B) \quad \Rightarrow \quad \zeta_{\max} = n - \varepsilon_B$$

Since

$$n\omega > U_p \gg E_B, \quad or \quad n > z \gg \varepsilon_B, \quad then$$
$$n_{\max} > \zeta_{\max}, \quad but \quad n_{\max} \approx \zeta_{\max}$$

From these constraints, it is clear that the largest result will come from ζ_{max} as large as possible. There is a standard asymptotic form of the Bessel function for this situation:

$$J_n^2(\zeta_{\max}) \approx \frac{\exp\left[2n(\tanh\alpha - \alpha)\right]}{2\pi n \tanh\alpha}, \quad \zeta_{\max} = \frac{n}{\cosh\alpha}$$

This has to be solved for α. With everything substituted,

$$J_n^2(\zeta_{\max}) \approx \frac{1}{2\pi(2\varepsilon_B)^{1/2}} \frac{\exp\left(-B/n^{1/2}\right)}{n^{1/2}}; \quad B \equiv \frac{2^{5/2}\varepsilon_B^{3/2}}{3}$$

$$\frac{dW}{d\Omega} \approx \frac{1}{2\pi^2\omega^{3/2}\varepsilon_B^{1/2}} \frac{1}{(n-z)^{3/2} n^{1/2}} \exp\left(-\frac{B}{n^{1/2}}\right).$$

This, in turn, has a maximum as a function of intensity.

To explore the possibility of stabilization, the expression for $dW/d\Omega$ should be written entirely in terms of intensity. This is most easily done if z is used as the measure of intensity. The connection is

$$z = \frac{I}{4\omega^3} \quad or \quad I = 4\omega^3 z$$

At this point, it is sufficient to use

$$z = \frac{1}{2}\left(n - \varepsilon_B\right) \approx \frac{1}{2}n \quad or \quad n \approx 2z$$

The function whose maximum is sought is then of the form

$$f\left(z\right) = \frac{1}{z^2}\exp\left(-\frac{C}{z^{1/2}}\right), \quad where \quad C \equiv \frac{4}{3}\varepsilon_B^{3/2}$$

The maximum is found to occur at

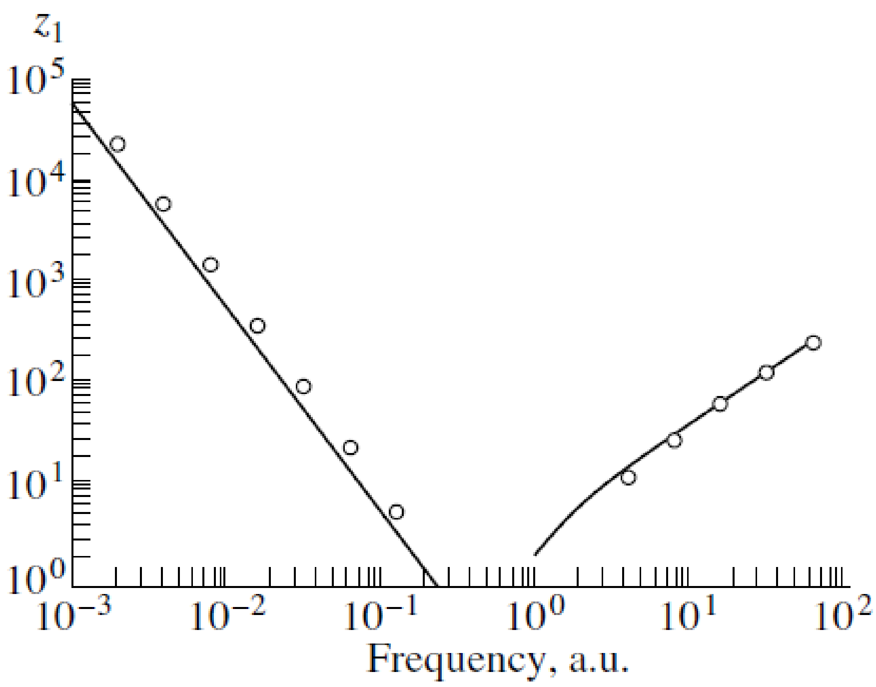
$$z = \left(C/4\right)^2,$$

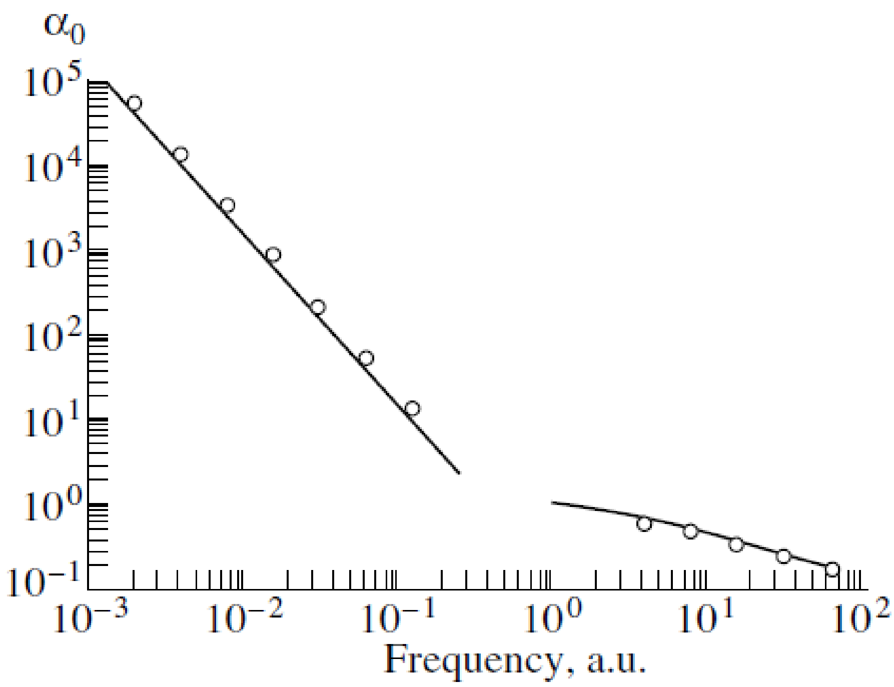
which corresponds to the simple result

$$\boldsymbol{I_{stab} = \frac{4}{9}\left(\omega\varepsilon_B\right)^3 = \frac{4}{9}E_B^3.}$$

COMPARISON BETWEEN ANALYTICAL PREDICTION OF STABILIZATION POINT AND RESULT OF FULL RATE CALCULATION

Two different types of plots are shown. The continuous line is the result of the explicit analytical prediction of the stabilization intensity, and the circles come from complete rate calculations. Vertical axes: $z_1 = 2U_p / E_B$; $\alpha_0 = (2z/\omega)^{1/2}$.





See: HRR, Las. Phys. **7**, 543 (1997).

It is the low-frequency line that was just derived. What is the high-frequency line?

STABILIZATION AT HIGH FREQUENCIES

High frequency is defined as $\omega >> E_B$ (or $1 >> \varepsilon_B$). When this is true, then a single photon is sufficient to cause ionization until the intensity gets so high that U_p is large enough to close the single-photon ionization channel. The transition from single-photon to two-photon minimum order is dramatic, and causes a sharp drop in transition rate. This is interpreted as stabilization.

The maximum intensity at which a single photon suffices for ionization is when

$$\omega = E_B + U_p, \quad or \quad 1 = \varepsilon_B + z \quad \Rightarrow \quad z = 1 - \varepsilon_B$$

In terms of z or z_1 or I , this condition is

$$\boldsymbol{z_{stab} = 1 - \varepsilon_B = \frac{1}{\omega}(\omega - E_B); \quad (z_1)_{stab} = 2\left(\frac{1}{\varepsilon_B} - 1\right); \quad I_{stab} = 4\omega^2(\omega - E_B)}$$

This gives the high-frequency result on the preceding slide.

There is a gap between the low-frequency and the high-frequency stabilization results because the SFA requires $U_p >> E_B$, or, equivalently, $z_1 >> 1$. (See preceding slide.) Also, “high frequency” requires $\omega >> E_B$. That is, the SFA is not valid in the gap.

DIRECTLY CALCULATED RATES FOR CIRCULAR POLARIZATION

Notice how the way in which the peak location changes with frequency is altered as the frequency gets higher.

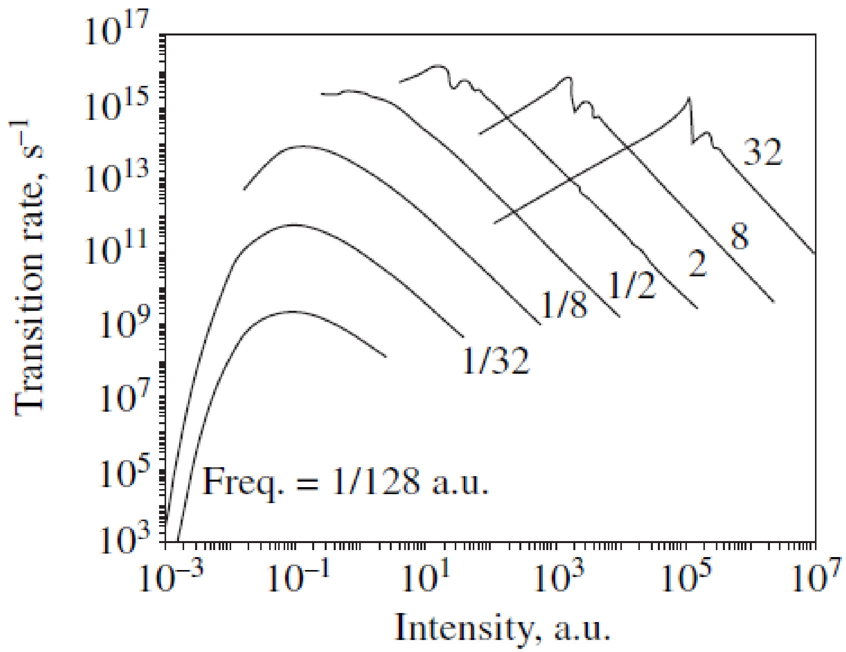


Fig. 2. *Transition rate for photoionization of ground state atomic hydrogen by a circularly polarized laser as a function of laser intensity for a variety of frequencies from low (1/128 a.u.) to very high (32 a.u.). Where the terminus of each curve remains on the figure, the low intensity end is taken at* $z_1 = 1$ *(where SFA validity begins) and the high intensity end is at* $z_f = 1$ *(where relativistic effects occur).*

HRR, Hatzilambrou, and Crawford, Las. Phys. **3**, 285 (1993).

RELATIVISTIC EFFECTS DO NOT ALTER STABILIZATION LOCATION FOR CIRC. POL.
HRR, Opt. Express **8**, 99 (2001)

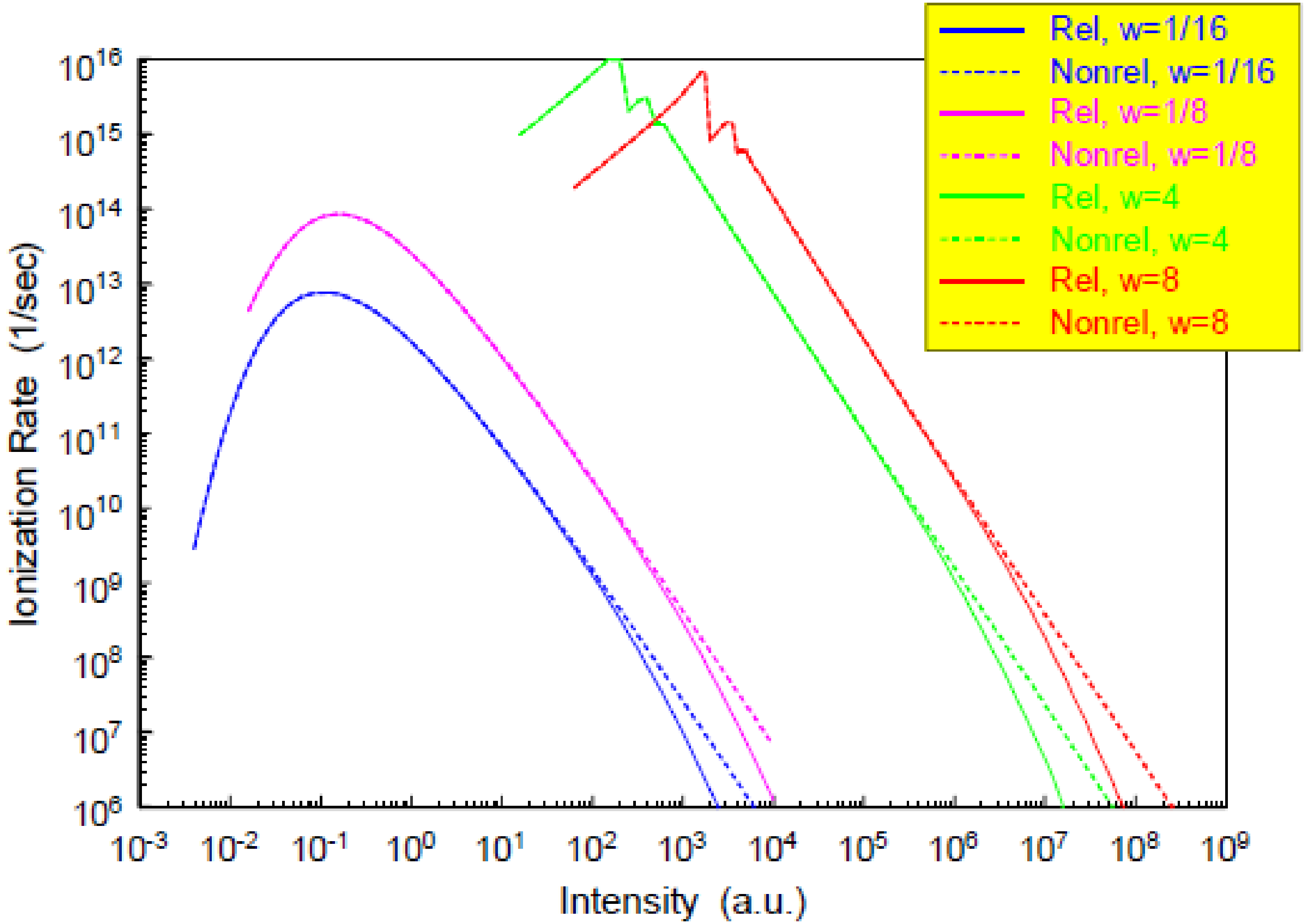


Fig. 1. Atomic hydrogen monochromatic ionization rates in a circularly polarized laser beam.

COMPARISON OF STABILIZATION POINTS FROM TDSE AND FROM THE SFA OVER A WIDE RANGE OF FREQUENCIES FROM LOW TO HIGH

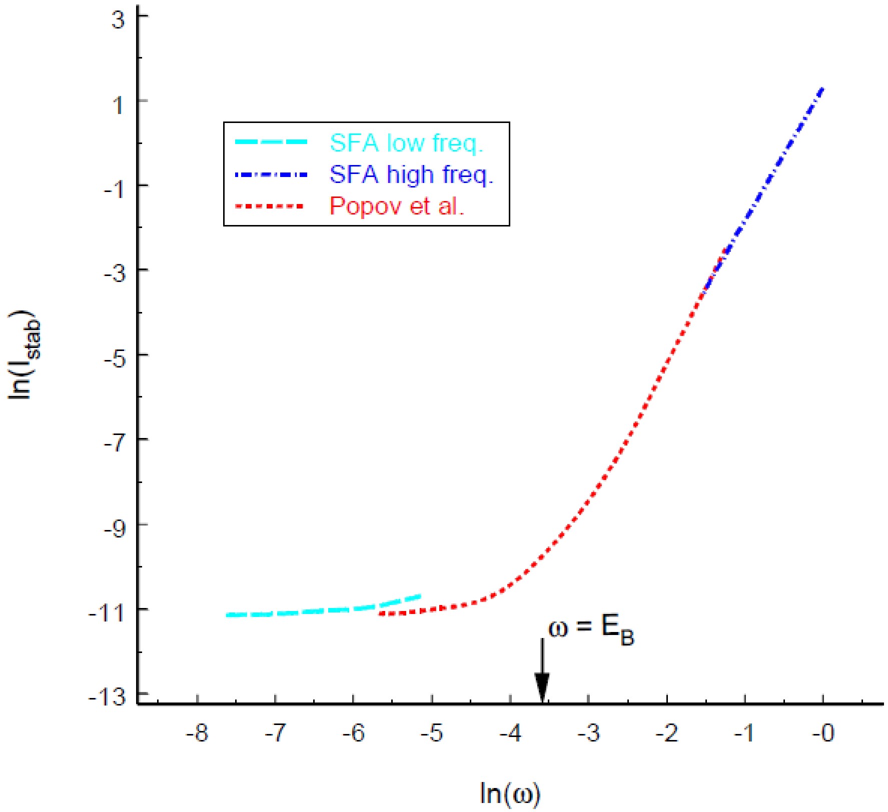


Fig. 2. Value of the stabilization intensity as a function of field frequency for ionization from a hydrogenic atom with binding energy 0.75 *eV*. The Popov et al. data are from Ref. [14], and proportions in this figure mimic those of Fig. 10 of this reference. See text for explanation of the gap in SFA results.

A.M. Popov, O.V. Tikhonova, E.A. Volkova, J. Phys. B **32**, 3331 (1999)

HRR, Opt. Express **8**, 99 (2001)

TUNNELING THEORIES DO NOT (AND CANNOT) EXHIBIT STABILIZATION

The dominant algebraic behavior in a tunneling theory is the *tunneling exponential:*

$$W \sim \exp\left(-C / F\right),$$

where *F* is the electric field amplitude. The tunneling exponential is a function that increases monotonically, and cannot exhibit tunneling behavior:

$$F \to 0 \Rightarrow W \to \exp\left(-\infty\right) \to 0;$$
$$F \to \infty \Rightarrow W \to \exp\left(0\right) = 1.$$

Stabilization is another item in the list of basic laser-field properties that are described in the VG, but are different or absent in the LG.

PHYSICAL ORIGIN OF THE STABILIZATION PROPERTY

"The physics is in the phase." Much of quantum mechanics is governed by quantities in the time-dependent phase factor.

In first-order perturbation theory, the time-dependent phase has the simple form

$$\exp\left[-i\left(E_i - E_f \pm \omega\right)t\right].$$

A quantum process occurs when oscillations of this function are minimal, which happens when

$$E_f = E_i \mp \omega,$$

which is simple energy conservation.
With plane-wave fields (laser fields) present, the time dependent phase takes the more complicated qualitative form

$$\exp\left[i\left(E_i - E_f \mp n\omega + \vec{A}\cdot\vec{p} + \frac{1}{2}\vec{A}^2\right)\right].$$

There is no simple way to achieve a zero in the exponential, and the presence of ***A*** and ***A***^{2} means that increases in field intensity will inevitably lead to such rapid oscillations that transition rates will be damped as field strength increases.

THIS LEADS TO STABILIZATION

ATTEMPTS TO OBSERVE STABILIZATION

Attempts to observe stabilization have been limited by the perceived need to use high frequencies.

This follows from theoretical work by Mihai Gavrila and, with other techniques, by Robin Shakeshaft. They used high frequencies because their methods were too difficult to apply to low frequencies. This is true also of TDSE, which cannot be extended to stabilization intensities except for high frequencies.

This has led to the assumption that stabilization is a high-frequency effect.

The earliest attempt to observe stabilization (at AMOLF in Amsterdam) examined a bound-bound transition in neon based on the presumed need to have $\omega \gg \Delta E$.

The result was ambiguous:
M. P. de Boer et al. (including Harm Muller), PRL **71**, 3263 (1993).

The view presented here is that
LOW FREQUENCIES are the most promising.

For high frequency ionization, saturation will occur before stabilization.

For sufficiently low frequencies, saturation is not a problem.