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Recap: Strong field ionization

⇒ SFA predicts the time and the position + velocity of the electron @ tunnel exit

⇒ everything is obtained by applying a saddle-point method to SFA

$$S_{\vec{p}}(t, t') = \frac{1}{2}\int_{t'}^{t} \frac{1}{2}(\vec{p} + \vec{A}(t''))^2 + I_p \, dt''$$

Saddle point Approx: $\frac{\partial S_{\vec{p}}}{\partial t'} = 0$
(SPA)

⇒ Result in complex time, t'

1 ⇒ $Re(t')$ ⇒ time the electron appears @ tunnel exit*

⇒ $Im(t')$ ⇒ imaginary time under the barrier

for $\gamma << 1$ ⇒ $Im(t') = \tau_{Keldysh} = \frac{\sqrt{2I_p}}{F}$

Has the SFA SPA prediction of when the electron appears @ tunnel exit been tested experimentally?

Yes! ⇒ Shafir et. al. Nature, Vol. 485, 2012

"Resolving the time when an electron exits a tunneling barrier"

⇒ This experiment uses HHG spectra to extract ionization & recombination times

⇒ Nicely combines many topics covered in this class ☺

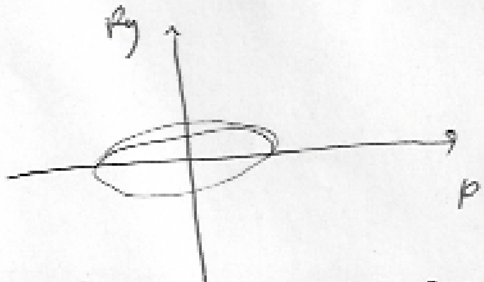
<u>Overview of topics</u> : strong field ionization & HHG (attosecond pulses) ⇒ Lecture 1

I. SFI ⇒ SFA, particularly using saddle-pt. approx. (SPA)

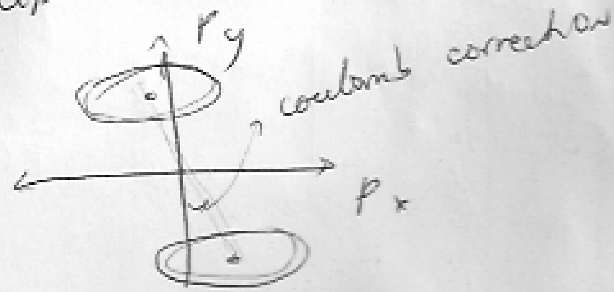
A. Lecture 2: Basic Derivation
- Keldysh theory

B. Lecture 3: SFA predictions for experimentally observed electron momenta distributions

Linear polarization



Elliptical polarization



SFA predicts momenta spreads: σ_x ; $\sigma_\perp$

C. Lectures 2, 8, 10 : SFA in the adiabatic ($\gamma \ll 1$) limit + connection to full solution in a static E-field (lecture 8)

→ Difference btwn SFA & the full solution

→ neglects Coulomb potential, hence triangular barrier or "short-range potential" approximation

D. Lectures 9, 10 : Non-adiabatic SFA (relaxing the $\gamma \ll 1$ assumption)

Best known result: PPT '60s

others: Bondar ; Yadin & Ivanov, etc '2000

Main result: Both velocity @ tunnel exit & the tunnel exit itself change compared to the adiabatic assumption

	Adiabatic	Non-adiabatic
velocity @ tunnel exit	elliptical : $\vec{v}_0 = 0$ linear ; $\vec{v}_0 = 0$	Elliptical: $\vec{v}_\perp \neq 0$; $v_\parallel = 0$ linear: $\vec{v}_0 = 0$
Tunnel exit itself	determined by inst. electric field $\vec{r}_{exit} = \frac{I_p}{F(t)} \hat{r}$	$r_{exit}^{non\text{-}adiab} < \frac{I_p}{F}$

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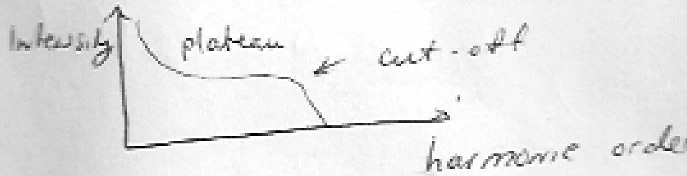
II. HHG

A. Lecture 4: Semiclassical Theory of HHG

- "The Simpleman's model"
  - ⇒ electrons start with $\vec{v} = 0$ @ tunnel exit
  - ⇒ KE of the returning electron determines the frequency of the emitted harmonic: $\hbar\omega = I_p + KE$
  - ⇒ Predicts the approximate cut-off freq. for the harmonic



B. Lectures 5 & 6: The Quantum model of HHG or Lewenstein model

⇒ gets harmonic orders from the Fourier transform of the dipole $\langle \Psi | x | \Psi \rangle$, where $\Psi(t)$ is obtained from SFA (see I) & using saddle-pt. approx.

C. Lecture 7: Propagation of as pulse (created by HHG), pulse dispersion & time delays (phase shifts)

⇒ Wigner time delay: $\tau_W = \hbar \frac{\partial \Phi}{\partial E}$

(or Eisenbud-Wigner-Smith)

Shafir et. al, Nature reconstructs when electron first appears @ tunnel exit by observing HHG spectra

- Tests predictions of saddle-point SFA
- Finds they're correct, while the "simpleman's" model is off.
- Going back to $S_{\vec{p}}(t, t') = \frac{1}{2}\int_{t'}^{t} [\frac{1}{2}(\vec{p}+\vec{A}(t''))^2 + I_p]\, dt''$

this experiment concludes that $Re(t')$ is the time the electron appears @ tunnel exit where t' satisfies $\frac{\partial \vec{S}_p}{\partial t'} = 0$

Experimental details: 2 color-field (almost linear polarization) $\vec{F}(t) = F_0 \cos(\omega t)\,\hat{x} + F_1 \cos(2\omega t + \varphi)\,\hat{y}$ where $F_1 \ll F_0$

Experimental observable: intensity of different harmonics as a function of a 2-color delay.

Reconstruction of t_i & t_R (ionization & recombination times) assumes the dynamics along the $\hat{y}$ axis are completely known

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⇒ dynamics along $\hat{y}$ have adiabatic probability distribution, completely classical, feel no Coulomb field ⇒ determined by: $\ddot{y} = F_1 \cos(2\omega t + \phi)$

⇒ Goal: reconstruct dynamics along the $\hat{x}$-axis (major-axis), in particular t_i & t_R

⇒ motion along the y-axis acts merely as a probe (not suppose to affect t_i & t_R)

⇒ A lot of assumptions!

⇒ Zhao & Lein recently showed the "correct" answer was only obtained because of a fitting parameter (i.e. intensity) ⇒ see PRL 111, 2013 ⇒ however they got the same answer (that SFA saddle point or as they call it "the Quantum orbit model" is correct using Schrodinger simulations

<u>Back to Shafir et al experiment</u>: classical reconstruction procedure comes up with 2 Eqns & 2 unknowns: t_i & t_R

Eqn. 1 <u>Displacement gate</u> ⇒ drift imparted by 2nd harmonic has to cancel out the initial velocity.

$V_d = -V_{0y}$ where V_{0y} is initial velocity @ exit: $P(V_{0y}) \propto e^{-V_{0y}^2/2\sigma_\perp^2}$

$$\int_{t_i}^{t_R} v_y(t)\,dt = 0 \Rightarrow \int_{t_i}^{t_R} \left(v_{0y} - A_{2\omega}(t_i) + A_{2\omega}(t)\right)dt = 0$$

$$\Rightarrow v_{0y} = -\varepsilon \frac{F_0}{2\omega}\left[\sin(2\omega t_i + \phi) + \frac{\cos(2\omega t_R + \phi) - \cos(2\omega t_i + \phi)}{2\omega(t_R - t_i)}\right]$$

different for each harmonic

Displacement gate: $G_y^n(t_R^n, t_i^n, \phi) = e^{-v_{0y}^2/2\sigma_\perp}$

Velocity gate $G_v^n(t_R^n, t_i^n, \phi) = \dfrac{v_y(t_R, t_i, \phi)}{v_x(t_R, t_i, \phi)}$

velocities @ time $t = t_R$

$\Rightarrow$ equivalent to the measured intensity ratio of even to odd harmonics

$$v_y(t_R, t_i, \phi) = v_{0y} - A_{2\omega}(t_i) + A_{2\omega}(t_R)$$

$$v_x(t_R, t_i, \phi) \approx \sqrt{2\,KE_{t_R}}$$

$\Rightarrow$ most of velocity directed along the x-axis

2 Eqns, 2 unknowns:

$$\left.\frac{\partial G_y^n}{\partial \phi}\right|_{\phi_y^{max}} = 0 \qquad \left.\frac{\partial G_v^n}{\partial \phi}\right|_{\phi_v^{max}} = 0$$

solve for t_R, t_i for each harmonic