Ionisation of atomic systems lecture 3

Keldysh 1965

(Ammosov)
Delone 1986
Krainov 1991

Last week

Keldysh ADK

$$P(t_0, V_\perp) \propto \exp\left(-\frac{2\,(2I_p)^{3/2}}{3F(t_0)}\right)\exp\left(-\frac{V_\perp^2}{2\sigma_\perp^2}\right) \quad (1)$$

(to exponential accuracy) $\sigma_\perp = \sqrt{\frac{\omega}{2\gamma}}$ $\gamma = \frac{\sqrt{2I_p}}{F_{max}}\,\omega$

critical assumptions:
- Strong field (neglect Coulomb after ionisation)
- quasistatic (adiabatic limit)
- $V_\parallel = 0$

now: investigate these issues further

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compare theoretical predictions & experiment

- Analytical calculation of trajectories
- Monte Carlo Simulation
  - ↳ initial conditions from (1)
  - ↳ propagation in Coulomb & laser field
- COLTRIMS / VMIS
  - → final momentum Distro's

$\boxed{\text{Ellipticity scan of final momenta}}$ COLTRIMS SIMULATIONS

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a) two peaks wander apart with increasing ellipticity

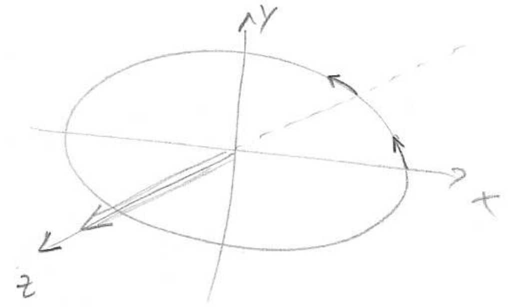
$$p_{y_{10}} = \pm\frac{\varepsilon F_{max}}{\omega} \quad \text{as centres} \quad \rightarrow \text{propagation in laser field}$$

b) slight tilt of the two blobs → Coulomb correction

c) longitudinal momentum spread → propagation
→ $V_\parallel$ question

propagation in the laser field

(atomic units)



$$\vec{F}(t) = \frac{F_0}{\sqrt{1+\varepsilon^2}} \begin{pmatrix} \cos(\omega t) \\ \varepsilon \cdot \sin(\omega t) \\ 0 \end{pmatrix}$$

neglecting Coulomb field from ion $\Rightarrow$ canonical momentum $\vec{p}$ is conserved

because $\dot{\vec{p}} = \frac{\partial H}{\partial \vec{x}} = 0$

so: $\vec{p} = \vec{v}_{final} = \vec{v}_{initial} + q \cdot \vec{A}(t_0)$

laser pulse has passed, $m_e = 1$

t_0 = exit time from tunnel

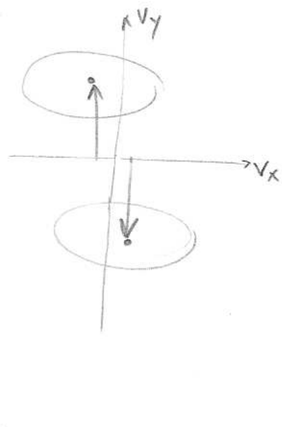
$q = -1$

most probable $\vec{v}_{initial}$ from SFA : $\vec{v}_{initial} = 0$

so: $$\vec{v}_{final} = -\vec{A}(t_0) = \frac{F_0}{\sqrt{1+\varepsilon^2}\,\omega} \begin{pmatrix} \sin(\omega t_0) \\ -\varepsilon\cos(\omega t_0) \\ 0 \end{pmatrix} \quad (2)$$

most probable time t_0 @ maximum of the field

so: $$\vec{v}_{final,0} = \pm \frac{\varepsilon F_0}{\sqrt{1+\varepsilon^2}\,\omega}\,\hat{y} \quad (3)$$



$\boxed{\text{a) explained}}$

note $\frac{F_0}{\sqrt{1+\varepsilon^2}} = F_{max}$

$F_0^2 = I$ cycle averaged quantity

same intensity $\leftrightarrow$ different $F_{max}(\varepsilon)$

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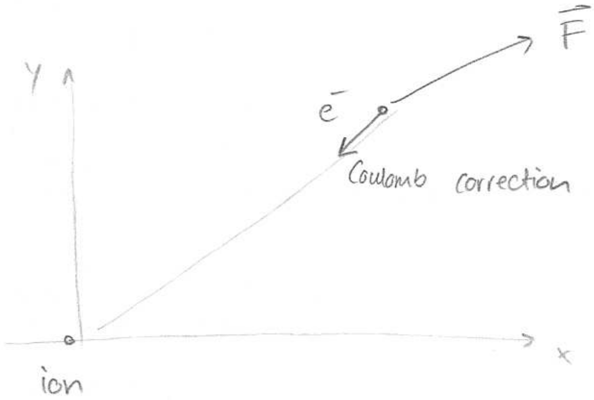
Coulomb Correction



- approximation I:

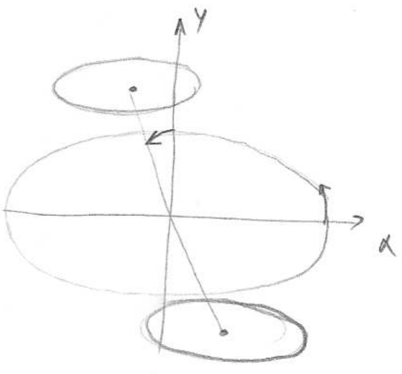
  calculate correction along the unperturbed trajectory $\vec{r}\,(t)$

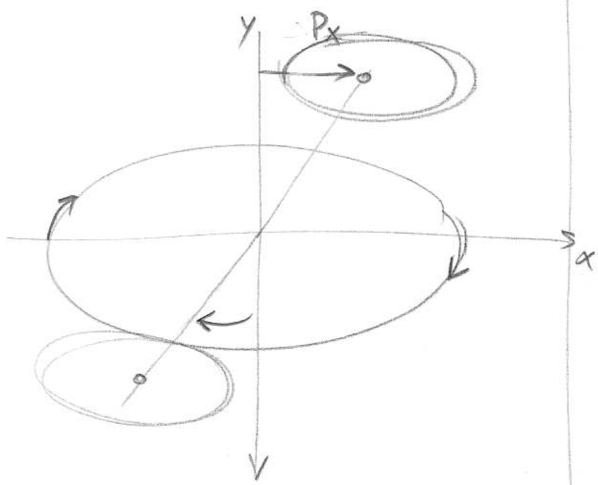
$$\vec{p} \approx - \int_0^\infty \frac{\vec{r}(t)}{r^3(t)} \, dt \qquad (4)$$

- approximation II: only consider dynamics immediately @ exit point

then:
$$r(t) = \tilde{r}_e + \begin{pmatrix} \frac{1}{2} F_{max} t^2 \\ 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad P_x \approx - \frac{\pi}{2\sqrt{2 \cdot F_{max}}} \, r_e^{-3/2} \qquad (5)$$





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$\boxed{\text{b) explained}}$

Longitudinal momentum spread

In eq. (1), only the transverse distribution $\sigma_\perp = \sqrt{\frac{\omega}{2\gamma}}$ is given.

Spread along major axis of polarisation is a lot wider! Beamer

again, canonical momentum conservation

$$\vec{v}_{final} = - \vec{A}\,(t_0) = \frac{F_0}{\sqrt{1+\varepsilon^2}\,\omega} \begin{pmatrix} \sin(\omega t_0) \\ -\varepsilon \cos(\omega t_0) \end{pmatrix} \qquad (2)$$

(4)

(2) - assumes $\vec{V}_{initial} = 0$!

- describes momentum acquired during propagation

note: V_z distribution at exit is the same as @ detector!

we already found tunnelling probability distribution $\sigma_\perp$ for V_z and V_y

now ionisation t_0 not exactly at peak of the field ($t_n = n \cdot \pi/\omega$)

then electrons acquire longitudinal momentum

$$V_{x,final} = \frac{F_0}{\sqrt{1+\varepsilon^2}\,\omega} \sin(\omega t_0) \approx \frac{F_0 \cdot t_0 \omega}{\sqrt{1+\varepsilon^2}\,\omega}$$

first order approximation

$$\Rightarrow \omega \cdot t_0 = \frac{V_{x,final} \cdot \omega \sqrt{1+\varepsilon^2}}{F_0}$$

(1): expand $\frac{1}{F(t_0)}$ around $t_0 = 0$ up to second order

$$\frac{1}{F_{max}\sqrt{\cos^2(\omega t) + \varepsilon^2 \sin^2(\omega t)}} = 1 + \frac{\omega^2 t^2}{2}(1-\varepsilon^2) + \mathcal{O}(t^3)$$

insert in (1):

$$\exp\left(-\frac{2(2I_p)^{3/2}}{3F_{max}}\right) \exp\left(-\frac{2(2I_p)^{3/2}}{3F_{max}} \frac{V_x^{\,2}}{2F_{max}^2} \omega^2(1-\varepsilon^2)\right)$$

or: longitudinal momentum distribution due to propagation:

$$P(V_x) \propto \exp\left(-\frac{V^{\,2}}{2\sigma_\parallel^2}\right)$$

with $$\sigma_\parallel = \sqrt{\frac{3\omega}{2\gamma^3(1-\varepsilon^2)}} \qquad (6)$$

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figure: analytical form (6) quite good, but not enough.

$\Rightarrow$ current topic of research: what about $V_{x,\text{initial}}$?

Non-adiabatic effects

not so directly visible in data shown so far

BUT: often experiments are in regime $\gamma \sim 1$ and mathematical approximations with $\gamma \ll 1$ are not valid anymore.

$\sim$ take dynamics of the field into account

famous treatment: PPT

Perelomov
Popov 1966
Terent'ev

$\Rightarrow$ similar formula

$$P(t_0, \vec{v}_{final}) \propto \exp\left(-\frac{2\,(2I_p)^{3/2}}{3F_{max}} \cdot g(\gamma, \varepsilon)\right) \qquad (7)$$

$$\cdot \exp\left(\frac{V_x^2}{2\sigma_x^2}\right) \exp\left(\frac{(V_y \pm v_0)^2}{2\sigma_y^2}\right) \exp\left(\frac{V_z^2}{2\sigma_z^2}\right)$$

in adiabatic case: $\sigma_y = \sigma_z = \sigma_\perp$, here not!

"nicer" writeup:
Mur et al
JETP
2001

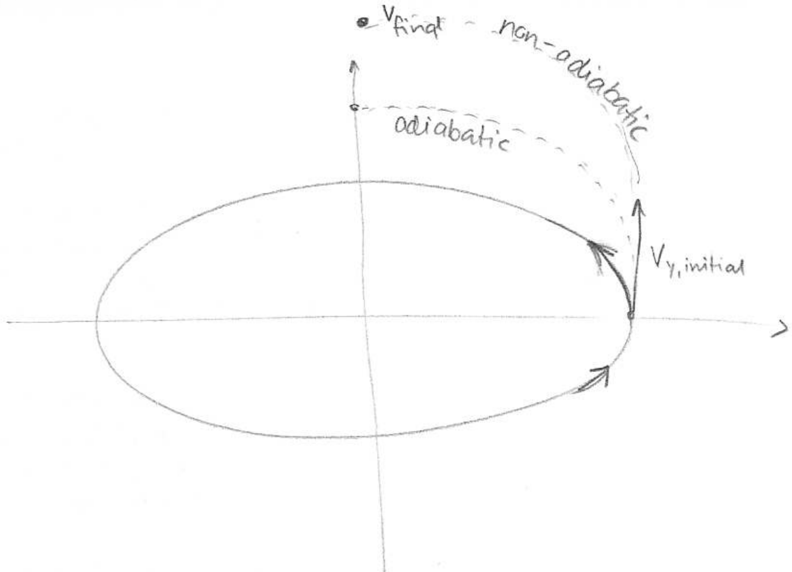
$$V_0 = \frac{\varepsilon F_{max}}{\omega}\left(\frac{\sinh \tau_0}{\tau_0}\right) > \frac{\varepsilon F_{max}}{\omega} \Rightarrow V_{y,\text{initial}} \neq 0$$

τ_0: dimensionless time. $i\frac{\tau_0}{\omega} \cong t_0^*$ from Saddle point approximation

most probable

$$V_{y,\text{initial}} = \frac{\epsilon F_0}{\sqrt{1+\epsilon^2}}\left(\frac{\sinh(\tau_0)}{\tau_0} - 1\right) \qquad (8)$$

Bramer



other effects: $\sigma_y > \sigma_\perp$

$\sigma_x > \sigma_\parallel$

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