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Last time: production of HHG by evaluating

$x(t) = \langle \Psi(t) | X | \Psi(t) \rangle$

different HHG harmonics are phase-locked & form at attosecond pulse

What does the as pulse look like?

$\Rightarrow$ note, short wavelength, so dipole approx likely to fail

$$\vec{E}(t, \vec{r}) = \frac{1}{2\pi} \int d\omega \, \vec{E}(\omega, \vec{r}) \, e^{-i\omega t}$$

amplitude of each harmonic & phase

Each harmonic obeys the Maxwell's eqn:

$$\nabla^2 \vec{E}(\omega, \vec{r}) + \frac{\omega^2}{c^2} \vec{E}(\omega, \vec{r}) = -\frac{1}{\varepsilon_0} \frac{\omega^2}{c^2} \vec{P} - \frac{1}{\varepsilon_0} \nabla(\nabla \cdot \vec{P}) \quad (1)$$

where $\vec{P}(\omega, \vec{r})$ is the polarization term:

$$\vec{P}(\omega, \vec{r}) = \varepsilon_0 \chi^{(1)}(\omega, \vec{r}) \, \vec{E}(\omega, \vec{r})$$

linear susceptibility (may vary in space)

$$\vec{P}(t, \vec{r}) \quad \frac{1}{2\pi} \int d\omega \, \vec{P}(\omega, \vec{r}) \, e^{-i\omega t}$$

$\Rightarrow$ Note, $\chi^1(\omega, \vec{r})$ may have a more complicated evolution in time at peak intensities $> 10^{16} \frac{W}{cm^2}$ (such as in XFEL) inserting $\vec{E}(\omega,\vec{r}) \propto e^{i\phi(\omega,x)}\,\hat{z}$ into Eqn. (1), we get

$$i\frac{\partial^2\phi}{\partial x^2} - \left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\omega}{c}\right)^2 = -\frac{\omega^2}{c^2}\chi^{(1)}$$

For slow variation of material optical properties:

$$\left|\frac{\partial^2\phi}{\partial x^2}\right| \ll \left|\frac{\partial\phi}{\partial x}\right|^2 \Rightarrow \frac{\partial\phi}{\partial x} \approx \frac{\omega}{c}\sqrt{1+\chi^{(1)}}$$

assumes material that is not dispersive

$$\phi(\omega,x) = \frac{\omega}{c}\int_{-\infty}^{x} dx'\sqrt{1+\chi^{(1)}(x')} \equiv \underbrace{\frac{\omega}{c}}_{k}\int_{-\infty}^{x} dx'\,\underbrace{n(x')}_{\text{local refractive index}}$$

asymptotic phaseshift due to propagation through a medium: $\delta(\omega) = \lim_{x\to\infty}\frac{\omega}{c}\int_{-\infty}^{x} dx'\,[n(x')-1]$

delay in propagation, as compared to vacuum ; $\tau = \delta/\omega$

$$\underbrace{E_{free}(t-\tau,x)}_{\text{shifted wave-packet}} = \frac{1}{2\pi}\int d\omega\, E_{free}(\omega,x)\,e^{-i\omega t}\,\underbrace{e^{i\omega\tau}}_{\text{phase-shift}}$$

aborbing medium: $e^{i\delta} = \underbrace{e^{iR(\delta)}}_{\text{phase-shift}}\;\underbrace{e^{-\operatorname{Im}\delta}}_{\text{damping term}}$

Suppose an as pulse passing through material of length, L, with constant refractive index, n

$\delta = \frac{\omega}{c}(n-1)L \Rightarrow \tau = \frac{(n-1)L}{c} \Rightarrow$ linear scaling with both L & n

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For dispersive material $\Rightarrow$ $n(\omega, x)$ $\Rightarrow$ refractive index is frequency-dependent

$$\vec{E}(t, x) = \frac{1}{2\pi} \int d\omega \underbrace{E_{free}(\omega, x)}_{\text{free propagation}} e^{-i\omega t} e^{i\delta(\omega)}$$

$\Rightarrow$ wave-packet is deformed w.r.t. free propagation

Expanding around the central freq. of the pulse:

$$\delta(\omega) = \delta(\omega_0) + \sum_{n=1}^{\infty} \frac{1}{n!} \left. \frac{\partial^n \delta}{\partial \omega^n} \right|_{\omega_0} (\omega - \omega_0)^n$$

over-all phase of the pulse

$n=1$ term determines the delay of the pulse

$\tau_{GD} = \frac{\partial \delta}{\partial \omega}$ $\Rightarrow$ delay in the propagation of a harmonic frequency ω

Similar to the Eisenbud-Wigner time delay

derived for mono-chromatic wave-packets:

$\Rightarrow$ τ_{EW} used to calculate delays for single photon ionization

$\Rightarrow$ Gives a shift (delay) of the peak of the wave-packet due to an interaction with a potential

Wigner, Physical Review, V. 98 (1), 1955

Eisenbud - Wigner, τ_{EW}, time delay derivation

(see below)

First, Lets take XUV wave-packet traveling through plasma of length L: $n_p(\omega) = \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2}$ for $\omega >> \omega_p$;

$$n_p(\omega) \simeq 1 - \frac{1}{2}\left(\frac{\omega_p}{\omega}\right)^2$$

$\omega_p = \sqrt{Ne^2/\epsilon_0 m}$ is plasma frequency

N - concentration of free electrons in plasma

$$\delta_p(\omega) = \frac{\omega}{c}\left[n_p(\omega) - 1\right] L \simeq -\frac{1}{2}\frac{\omega_p^2}{c}\frac{L}{\omega}$$

⇒ negative asymptotic phase seems to imply super luminal propagation, because refractive index $n_p(\omega) < 1$

⇒ However: $\tau_{GD} = \frac{\partial \delta_p}{\partial \omega} \approx \frac{1}{2}\frac{\omega_p^2}{c}\frac{L}{\omega^2}$ ⇒

light pulse is delayed by plasma & travels slower than c.

For phase-matching, plasma density can be altered by changing the intensity of the laser (changes N)

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spherical potential:

Incident wavepacket:

$$\Psi_{inc} = r^{-1}\left(e^{-(k+k')r - i(\nu+\nu')t} + e^{-i(k-k')r - i(\nu-\nu')t} \right)$$

$k' << k$; $\nu' << \nu$ $\Rightarrow$ two spherical waves of nearly identical momentum, $k \pm k'$ & energy $\nu \pm \nu'$

peak of the wave-packet: $2k'r + 2\nu' t = 0$

Group velocity: $\frac{d\nu}{dk} = \frac{\nu'}{k'}$

Outgoing components incur phase-shifts $\eta + \eta'$ and $\eta - \eta'$, respectively.

$$\Psi_{out} = r^{-1}\left(e^{i(k+k')r - i(\nu+\nu')t + 2i(\eta+\eta')} + e^{i(k-k')r - i(\nu-\nu')t + 2i(\eta+\eta')} \right)$$

peak of the wave-packet: $2k'r + 2\nu' t + 4\eta' = 0$

$$r = -2\eta'/k' + (\nu'/k')t = -2\frac{d\eta}{dk} + \underbrace{\frac{d\nu}{dk}t}_{r_{free}}$$

$\Rightarrow$ the outgoing wave is retarded by $-2\frac{d\eta}{dk}$

additional time delay incurred for incoming & outgoing wave-packet: $2\tau_{EW} = \frac{\Delta r}{v} = \underbrace{\frac{2d\eta}{dk}}_{\Delta r} \cdot \underbrace{\frac{1}{\hbar k/m}}_{v} = 2\hbar\frac{\partial\theta}{\partial E}$

$$\tau_{EW} = \hbar\frac{\partial\theta}{\partial E}$$