Lecture 3

Last Lecture

SFA : Neglects Coulomb potential after ionization (Volkov states).

solution of the form:

$$\langle v | \Psi(t) \rangle = \Psi(v,t) = -i \int_{t_0}^{t} dt' \underbrace{\langle v' | \hat{V}(t') | \Psi_i \rangle}_{\text{prefactor}} e^{-iS_v(t,t')}$$

$t' \Rightarrow$ ionization time (path integral type of formulation where all possible "paths" are added)

$v' \; e^{i\int_{t'}^{t} \hat{H}(t'')\,dt''} \quad \Psi(v,t)$

$\hat{H}(t) \Rightarrow$ atomic pot + laser field

ground state

$$e^{-i\int_{t_0}^{t'} \underbrace{\hat{H}_0(t'')}_{\text{atomic potential no laser field}} dt''}$$

atomic potential no laser field: $|\Psi_g(t)\rangle = e^{iI_p t} |\Psi(t-t_0)\rangle$

$\hat{H}(t'')$ approximated by $\hat{H}_F(t'')$ (Volkov states), where

$$\hat{H}_F = \frac{1}{2m} (\underbrace{\hat{p} - \frac{q}{c}\vec{A}}_{m\vec{v}})^2 = KE$$

with Volkov solution, $\hat{H}_F$,

$$\boxed{S(t,t') = \frac{1}{2} \int_{t'}^{t} \underbrace{(v_x - A(t) + A(t''))^2}_{\frac{1}{2} m v^2(t'')} dt'' - I_p t' + \frac{v_\perp^2}{2}(t-t')}$$

②

for strong fields: $S >> 1$,

$$\Psi(v,t) \sim -i \langle v(t_a) | \hat{V}(t_a) | \Psi_i \rangle \, e^{-iS_v(t,t_a)}$$

where $\dfrac{\partial S_v(t,t')}{\partial t'} = 0$ for $t' = t_a$

$$\Rightarrow \tilde{S}_v = S_v(t, t_a) \Rightarrow e^{-i\tilde{S}_v} \sim \Psi(v,t)$$

Linear polarization: $\vec{A}(t) = v_0 \sin(\omega t)\, \hat{x}$

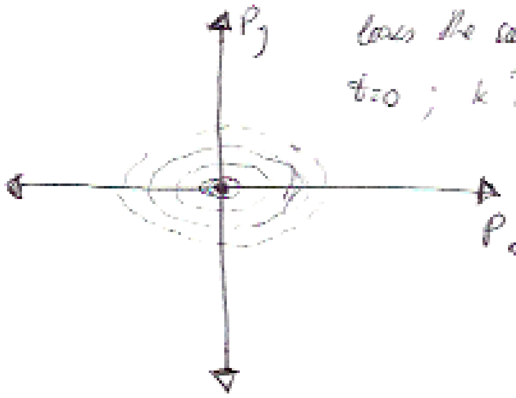
ponderomotive energy $\left.\right\}$ $U_p = \dfrac{1}{4} m \left(\dfrac{E_0}{\omega}\right)^2 = \langle KE \rangle_{electron}$

$$V_{max} = \frac{E_0}{\omega} = V_0 \quad \text{since } \vec{E} = \frac{\partial \vec{A}}{\partial t} = \underbrace{V_0 \omega}_{E_0} \cos(\omega t)\, \hat{x}$$

Peak of the field @ $t=0 \Rightarrow \vec{E} = E_0 \hat{x}$; $\vec{A}(t) = 0$

@ $t=0 \Rightarrow \underbrace{P}_{\text{canonical ang. mom.}} = m(V_x - A_0) = mV_x$ $\Rightarrow$ velocities measured @ $t=0$ are the same as after the laser pulse has passed $\underbrace{\vec{A}(t\gg 0)}_{=0}$

so $|\Psi(v, t=0)|^2 = |\Psi(v, t\to\infty)|^2$ (at the detector same probability distribution)



looks the same @ $t=0$; & large t

$\Rightarrow$ most probable ionization @ $t=0$ (peak of the field) &

$$P_x = P_y = P_z = 0$$

$P(v_y; t) = P(v_y; t')$ since $A_y = 0$; $v_y = \text{const.}$

PPP (adrian) $\Rightarrow$ shift in $p_y = mv_y$ suggests a small ellipticity: $A_y \neq 0$

Probability of ionization & velocity distributions (using SFA) for Linear polarization

$$S_v(t, t') = \frac{1}{2}\int_{t'}^{t} dt'' \left[V_x - V_0 \sin(\omega_L t) + V_0 \sin(\omega_L t'')\right]^2 - I_p t' + \frac{V_\perp^2}{2}(t - t')$$

$(V_\perp^2 = V_y^2 + V_z^2)$

from $\frac{\partial S_v}{\partial t'} = 0$ $\Rightarrow$ since $\Psi(v,t) \sim e^{iS} \times \underset{\text{prefactor}}{\underline{\quad}}$

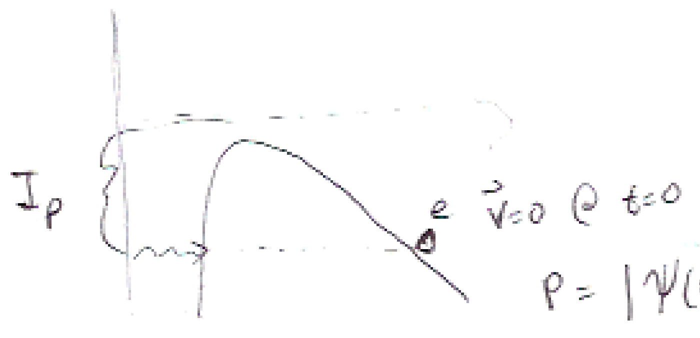
$$\Rightarrow \left(\frac{V_x}{V_0} - \sin(\omega_L t) + \sin(\omega_L t')\right)^2 + \tilde{\gamma}^2 = 0$$

where $\boxed{\tilde{\gamma} = \frac{(2I_p + V_\perp^2)^{1/2}}{F} \cdot \omega_L}$

Remember: Keldysh parameter $\Rightarrow$ $\gamma = \frac{\sqrt{2I_p}}{F} \cdot \omega_L$ ($V_\perp$ just adds to the ionization potential)

$\Rightarrow$ (will make it easy to derive $\sigma_\perp$ once we have $|\Psi(v_x, v_\perp = 0, t)|^2$)

Lets derive $|\Psi(v=0, t=0)|^2$ (ionization probability @ the peak with $V=0$)



$P = |\Psi(v=0, t=0)|^2$

$$\Rightarrow \frac{\partial S(t=0, v=0, t')}{\partial t'} = 0 \quad \Rightarrow \quad \sin^2(\omega t') = -\gamma^2$$

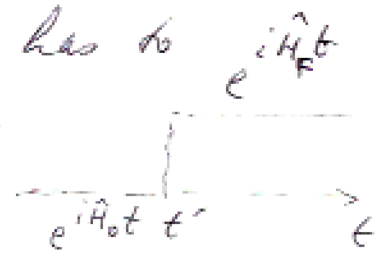
$$\sin(\omega t') = \pm i\gamma$$

for $\omega t' \ll 1 \Rightarrow \sin(\omega t') \approx \omega t' \approx i\gamma$

$\Rightarrow t' = i\frac{\gamma}{\omega} = i\tau_{\text{keldysh}} \Rightarrow$ imaginary keldysh time

$t' \Rightarrow$ dominant contribution to ionization

Possible Interpretation: for the electron to appear in the continuum with $v=0$ @ $t=0$, it has to "start" tunneling @ $t' = i\tau_{kel}$



Imaginary tunneling time or the artifact of SFA?

Let's go back to: $\frac{\partial S}{\partial t'} = 0 \Rightarrow \frac{1}{2}\underbrace{\left[v_x - v_0\sin(\omega_L t) + v_0\sin(\omega_L t')\right]}_{KE(t') = \frac{1}{2}m v_x(t')^2} = -I_p$

(setting $v_\perp = 0$

$\Rightarrow$ SFA sets total energy = KE @ the time of ionization!
(ignores atomic potential, only counts $\vec{A}$)

$\Rightarrow$ leads to imaginary $V_x(t)$ that therefore must be decelerated to zero with imaginary impulse

$\Rightarrow$ imaginary time!

$V_x(t') = i\sqrt{2I_p}$ $\quad \dot{V}_x = \underbrace{q E_x}_{\text{electric field}} = V_0 \cdot \omega_L \cos(\omega_L t'')$ $\quad F = V_0 \cdot \omega_L$

$V_x(0) = 0$

$V_x(0) - V_x(t') = \Delta V_x = \int_{t'}^{0} q E_x \, dt'' = +V_0 \sin(\omega_L t'') \Big|_{t'}^{0} = -i\sqrt{2I_p} \approx -F t'$

$$t' = i\frac{\sqrt{2I_p}}{F} = i\tau_{kel} \qquad \tau_{kel} = \frac{\sqrt{2I_p}}{F}$$

ionization probability: $\Psi(v=0, t=0) \sim e^{-i S(v=0, t=0, t'=i\tau_{kel})}$

@ the peak of the field

$$S_0 = S(v=0, t=0, t'=i\tau_{kel}) \approx \frac{1}{2}\int_{i\tau_{kel}}^{0} dt'' \cdot (v_0 \omega_L t'')^2 - iI_p\tau_{kel}$$

$$= +\frac{i}{2}\underbrace{v_0^2\omega_L^2}_{F^2} \times \underbrace{\frac{|\tau_{kel}|^3}{3}}_{\frac{(2I_p)^{3/2}}{3F^3}} - iI_p\tau_{kel} = i\left(\frac{1}{2}\cdot\frac{(2I_p)^{3/2}}{3F} - \frac{(2I_p)^{3/2}}{2F}\right)$$

$$\boxed{S_0 = \frac{-i}{2}\left(\frac{2}{3}\frac{(2I_p)^{3/2}}{F}\right)}$$

Probability of ionization: $P(v=0, t=0) = \Psi_0^* \Psi_0$ where $\Psi_0 = \Psi(v=0, t=0)$

$$\boxed{P \sim e^{-\frac{2(2I_p)^{3/2}}{3F}}}$$

$\Rightarrow$ estimates probability of ionization with exponential accuracy. ("most ionization events do happen with $v=0$ @ the peak of the field.)

Corrections: non-zero velocities $v_\perp$; v_x

Coulomb corrections (prefactor)

$$\langle v|\Psi(t)\rangle = \Psi(v,t) \sim \langle v(t)|\hat{V}(t_a)|\Psi_i\rangle e^{-iS_v(t,t_a)}$$