20. 3. 2013 , Lecture 4

Strong field ionization

Simplifications: 1) monochromatic light

2) strong fields ⇒ classical

E & m after ionization

low frequency ⇒ atom "sees" only a time-dept. field, no spatial variation $\vec{A} \sim e^{ikx - \omega t} \simeq \vec{A}(t)$ (dipole approximation)

SFA (derived last class)

⇒ simplification ⇒ ignores Coulomb field after ionization

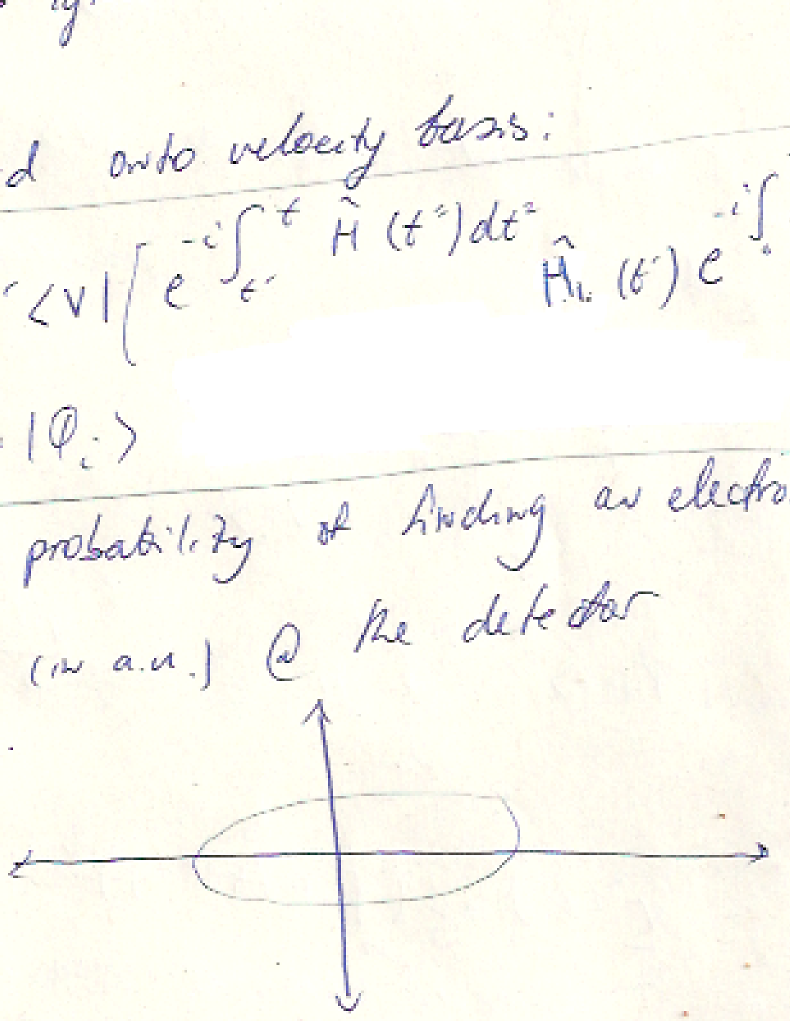
Wavefunction projected onto velocity basis:

$$\langle v | \Psi(t) \rangle = -i \int_0^t dt' \langle v | \left(e^{-i\int_{t'}^{t} \hat{H}(t'')dt''} \hat{H}_I(t') \, e^{-i\int_0^{t'} \hat{H}_0(t'') dt''} \right. \times |\varphi_i\rangle$$

electron momentum measured @ detector

$|\langle v | \Psi(t) \rangle|^2$ gives probability of finding an electron of momentum $m_e \vec{v} = \vec{v}$ (in a.u.) @ the detector

electron momenta distributions measure @ detector for LPL



$\hat{H}_0 \Rightarrow$ field free Hamiltonian

$\hat{V}_\cdot(t) \Rightarrow$ interaction with the laser field

$$\hat{H} = \hat{H}_0 + \hat{H}_L(t)$$

drop the $\hat{H}_0$ term $\Rightarrow$ interaction only with the laser field after ionization (the Coulomb field is a higher order perturbation)

$$\hat{H}_L(t) = \underbrace{(\vec{P} + \vec{A}(t))^2}_{KE = V^2} \Rightarrow \hat{H} \approx \hat{H}_L(t)$$

why? we ~~kke~~ know the Eigen functions of this Hamiltonian!

$\Rightarrow$ Volkov states $\Rightarrow |\vec{V}\rangle$, as $t \to \infty$ (after the laser pulse has passed $\langle x | \vec{V} \rangle = e^{i\vec{k}\cdot\vec{r}} \Rightarrow$ plane waves!

$$e^{-i\int_{t'}^{t} \hat{H}_L(t'')dt''} |\vec{V}(t)\rangle = e^{-\int_{t'}^{t} \frac{V^2}{2}(t'')dt''} |\vec{V}(t)\rangle$$

$$\langle V | \Psi(t) \rangle = -i\int_0^t dt' \, e^{-i\int_{t'}^{t} \frac{V^2(t'')}{2} dt''} \langle \vec{V}(t) | \hat{H}_L | g \rangle \times e^{iI_p t}$$

$$H = \frac{V^2(t'')}{2} = \frac{1}{2}(\vec{P} + \vec{A}(t''))^2$$

conserved canonical momentum

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$\vec{p}$ conserved because there is no spatial dependence in the Hamiltonian (b/c we dropped the Coulomb field & are using dipole approx)

$$\dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}} = 0 \qquad \vec{p} = \text{const}$$

Lets take LPL: $\vec{A}(t) = \underset{\text{laser freq}}{V_0 \sin(\omega t)}\, \hat{x}$ ← direction of polarization

$$\langle v | \Psi(t) \rangle = -i \int_0^t e^{-iS(t,t')} \times \underbrace{\langle \vec{v}(t')\, \hat{H}_L | g \rangle}_{\text{prefactor}}\, dt'$$

where $S(t,t') = \frac{1}{2} \int_{t'}^{t} dt'' \{ (p_x + V_0 \sin(\omega t''))^2 + \frac{p_\perp^2}{2} (t-t') - I_p t'$

$p_x = m v_x(t) - \vec{A}(t)$

since $m v_x(t) = p_x + \vec{A}(t)$ for all t

$$S(t,t') = \frac{1}{2} \int_{t'}^{t} dt'' \{ (\underbrace{m v_x(t) - V_0 \sin(\omega t)}_{p_x} + V_0 \sin(\omega t''))^2 + \frac{v_\perp^2}{2}(t-t') - I_p t'$$

SPA: $\langle v | \Psi(t) \rangle \sim e^{iS(t,t_0)}$ where $t' = t_0$ is s.t.

$$\left. \frac{\partial S(t,t')}{\partial t'} \right|_{t'=t_0} = 0$$

$$\frac{1}{2}(\underbrace{V_x(t) - V_0 \sin\omega t}_{p_x} + V_0 \sin\omega t')^2 + \underbrace{(I_p + \tfrac{1}{2}V_\perp^2)}_{I_p'} = 0$$

$$(\underbrace{\frac{V_x(t)}{V_0} - \sin(\omega t)}_{p_x} + \sin(\omega t'))^2 = \frac{2I_p'}{V_0^2} = \frac{2I_p' \cdot \omega^2}{F_0^2} = \gamma'^2$$

$$V_0 = \frac{F_0}{\omega} \quad \text{where} \quad F = -\frac{\partial \vec{A}}{\partial t} = -F_0 \cos(\omega t)$$

for $V_\perp = 0$ (no velocity transverse to polarization)

$$\gamma' = \gamma = \frac{\sqrt{2I_p}}{F_0} \cdot \omega \Rightarrow \text{keldysh gamma}$$

that divides tunnel ($\gamma \ll 1$) from multi-photon ($\gamma \gg 1$) ionization $\Rightarrow$ will assume $\gamma \ll 1$ in the derivation

$$\text{so:} \quad \underbrace{\overbrace{\left(\frac{V_x(t)}{V_0} - \sin\omega t\right.}^{p_x} + \left.\sin\omega t'\right)}_{\text{imaginary}} = \pm i\gamma'$$

t real $\Rightarrow$ t' must be complex $t' = t_R + i t_{im}$

$$\sin(\omega t') = \frac{e^{i\omega t'} - e^{-i\omega t'}}{2i} = \frac{1}{2i}\left[e^{i\omega t_R} e^{-\omega t_{im}} - e^{-i\omega t_R + \omega t_{im}}\right]$$

$$= \frac{1}{2}\left\{\underbrace{\frac{\cos(\omega t_R)}{i}\left[e^{-\omega t_{im}} - e^{\omega t_{im}}\right]}_{\text{imaginary part}} + \underbrace{\frac{i}{i}\sin(\omega t_R)\left\{e^{-\omega t_{im}} + e^{+\omega t_{im}}\right\}}_{\text{real part}}\right.$$

Lets say we want the probability of

$p_x = 0$

means $t_R = 0$ (ionization begins @ the peak of the E-field)

alternatively, for $p_x > 0$ $t_R < 0$ to cancel out the p_x

for $p_x < 0$ $t_R > 0$ " "

$\Rightarrow$ electrons with $p_x > 0$ @ detector are ionized before the peak

" " $p_x < 0$ " " " " after the peak

$\Rightarrow$ lets calculate the probability of finding $p_x = 0$; $p_\perp$ @ detector

$|\langle p_x = 0, p_\perp | \Psi(t \to \infty)\rangle|^2 \Rightarrow t_R = 0$

$\sin(\omega t') \approx i\omega t_{im} = i\gamma'$ $\quad t_{im} = \frac{\gamma'}{\omega}$

t_{im} is Keldysh time for $v_\perp = 0$

$\tau = \frac{\gamma}{\omega} = \frac{\sqrt{2I_p}}{F}$

$\Rightarrow$ assumed $\omega t_{im} \ll 1$

$\Rightarrow$ tunneling regime $\omega\tau \ll 1$ or $\gamma \ll 1$

We will see why this is a "tunneling regime" shortly: $t_0 = i t_m = \frac{\gamma'}{\omega}$

$$v_0^2\left(\frac{\gamma'}{2}\right)^2$$

$$S(t,t_0) = \frac{1}{2}\int_{\frac{i\gamma'}{\omega}}^{0} dt' \underbrace{(v_0 \sin(\omega t'))}_{\approx\, v_0(\omega t')}{}^2 - \left(I_p + \frac{v_\perp^2}{2}\right)\frac{i\gamma'}{\omega}$$

$$\approx \frac{1}{2}\int_{\frac{i\gamma'}{\omega}}^{0} v_0^2(\omega t')^2\, dt' - \left(I_p + \frac{v_\perp^2}{2}\right)\frac{i\gamma'}{\omega}$$

(where I am only considering imaginary part of S)

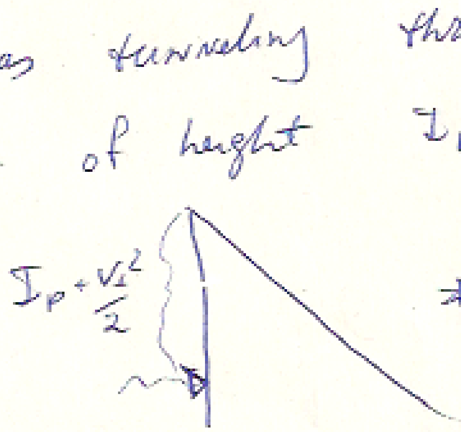
$$\mathrm{Im}(S(t,t_0)) = -\frac{1}{3}\frac{(2I_p')^{3/2}}{3F}$$

$$|\langle p_x = 0, p_\perp | \psi(t \to \infty)\rangle|^2 = e^{-2\,\mathrm{Im}\, S(t\to\infty,\, t_0)}$$

$$= e^{-2\frac{(2I_p + v_\perp^2)^{3/2}}{3F}} \approx e^{-2\frac{(2I_p)^{3/2}}{3F}}\, e^{-\frac{v_\perp^2}{2\sigma_\perp^2}}$$

where $\sigma_\perp = \sqrt{\frac{\omega}{2\gamma}}$

same as tunneling through a triangular barrier of height $I_p + \frac{v_\perp^2}{2}$



$\Rightarrow$ we know this by using a static solution for propagation through a triangular barrier!

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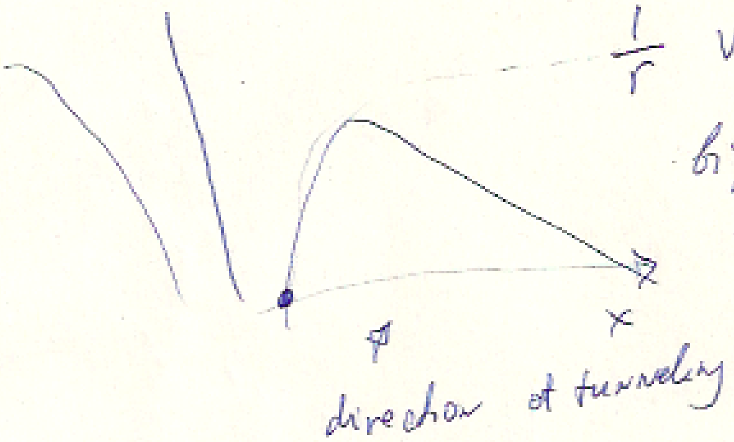
⇒ can be solved using WKB
or completely using Airy functions

⇒ so for $\gamma \ll 1$, the electron behaves as if it sees a static electric field

⇒ triangular barrier ⇒ why?
it is caused by the E-field,

$$V(x) = -F \cdot x$$

⇒ remember, the Coulomb field is neglected



$\frac{1}{r}$ varies little as r gets bigger so we can assume $\underbrace{V_C \approx \text{const}}$ for $r > r_0$

r_0: entrance to a barrier

↓ short-range potential approximation

Lets consider again the significance of γ

$$\gamma = \tau \cdot \omega = \frac{\sqrt{2I_p}}{F} \cdot \omega$$

average KE of electron : I_p (from Virial theorem: $\langle KE \rangle = -\frac{1}{2}\langle PE \rangle$)

average velocity of " : $\langle v \rangle = \sqrt{2I_p}$ ⇒ so Keldysh time is the time it would take to decelerate electron to zero velocity!