Lecture ① 15.5.13

Shafir, 2012: "Resolving the time when an electron exits a tunnelling barrier"

- Not tunneling time, but the time the electron appears @ tunnel exit
- Claims the results are different from "semiclassical model", but agree with the stationary phase solution from SFA
- Used a "classical retrieval method"

⇒ 2 color field : $\vec{F} = \hat{x} F_\omega \cos(\omega t) + \hat{y} F_{2\omega} \cos(2\omega t + \varphi)$ only this is considered

⇒ perpendicular polarized: $F_{2\omega} \ll 1$

⇒ weak and harmonic used as a "probe" - probes the dynamics without substantially altering it.

- Reconstruction assumes motion along $\hat{y}$ is classical + neglects coulomb field along $\hat{y}$

⇒ motion along y is completely decoupled from x & has no spatial dependence : $\ddot{y} \propto F_0 \cdot \epsilon \sin(2\omega t + \varphi)$

- Recently Lein & Zhao did simulation of the experiment, showing that using this classical reconstruction method will "introduce substantial errors"

(in other words we should not be seeing such excellent agreement btwn theory & experiment as shown in Shafir

"the error of the classical retrieval did not become apparent in Shafir, because φ was fixed using Q.O model"

Also in Lein:
experimental uncertainties: Intensity & delay btwn the two
colors ϕ

→ delay not actually measured but extracted using QO model (the same theory that is being confirmed by the experiment

→ the point of Lein paper is still confirming the QO (or stationary solution) model, but that a non-classical reconstruction procedure is needed to do so.

Reconstruction → Displacement Gate - PRL 97, 253903 (2006)
Velocity Gate

Displacement gate → comes from requirement that the drift velocity of the electron along $\hat{y}$ is cancelled by initial velocity @ tunnel exit.

→ think of 2 decoupled motions, along $\hat{x}$ & $\hat{y}$

electron leaves exit point @ $t=t_i$ & comes back @ $t=t_R$ → determined by motion along $\hat{x}$

However, electron must also come back along $\hat{y}$.
→ there is only one velocity @ exit point: v_{0y} that insures this happens

the probability of this velocity is given by:

$$P(v_{0y}) \propto e^{-v_{0y}^2/2\sigma_\perp^2}$$ ← can be usual ADK probability (exact $\sigma_\perp$ doesn't matter)

→ the velocity distribution is centered around 0.

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→ not equivalent to adiabatic assumption, because its close to linearly polarized light

→ t_i & t_R are given by the harmonic n, "realize stationary phase points of the general quantum expression... & are dictated by the electron motion only in the strong fundamental field & the core potential" (page 17, supplemental)

Each harmonic n: t_i^n; t_R^n

→ ϕ only changes the intensity of the harmonic n by changing V_{0y} (the initial velocity required to cancel the drift)

→ this also makes sense if the 2nd harmonic is to only act as a probe (not significantly alter the dynamics)

{Displacement gate experiment - PRL 97, 253903 (2006)

→ Cleaner because there is no unknown ϕ

→ uses low E field

→ Only provides a "consistency check" for SFA, because they could not decouple t_i (ionization time) from t_R (return time)

→ already in the conclusion suggests t_i & t_R can be independently extracted using a weak 2nd harmonic

(4)

$$v_y(t) = P_y + A_{2\omega}(t)$$

P_y: conserved canonical momentum

$A_{2\omega}(t)$: vector potential along the y-axis

$$P_y = v_{0y} - A_{2\omega}(t_i)$$

$$F_y = F_{2\omega}\cos(2\omega t + \varphi) \Rightarrow A_{2\omega} = -\frac{F_{2\omega}}{2\omega}\sin(2\omega t + \varphi)$$

$F_{2\omega} = \varepsilon F_0 \Rightarrow$ field amplitude ; $\varepsilon \ll 1$

condition for $\Delta y = 0$ or $y(t_i) = y(t_R)$

$$\Rightarrow \int_{t_i}^{t_R} v_y(t)\,dt = 0 \Rightarrow \int_{t_i}^{t_R} (v_{0y} - A_{2\omega}(t_i) + A_{2\omega}(t))\,dt = 0$$

$\Rightarrow$ solving for $v_{0y} = -\varepsilon \frac{F_0}{2\omega}\left[\sin(2\omega t_i + \varphi) + \frac{\cos(2\omega t_R + \varphi) - \cos(2\omega t_i + \varphi)}{2\omega(t_R - t_i)}\right]$

$\Rightarrow$ Required initial offset velocity for each harmonic : $v_{0y}^n(t_r, t_i, \varphi)$

Intensity of harmonic n should be $\propto G_y^n(t_r^n, t_i^n, \varphi)$

$$= e^{-v_{0y}^2/2\sigma_\perp}$$

$\Rightarrow$ maximal intensity should occur at a φ that maximizes G_y^n (or minimizes v_{0y}^2)

$\Rightarrow$ hence $\sigma_\perp$ exact value does not matter

$$\frac{dG_y^n}{d\varphi} = 0 \Rightarrow \frac{\partial G_y^n}{\partial \varphi} + \frac{\partial G_y^n}{\partial t_R^n}\cdot\frac{\partial t_R^n}{\partial \varphi} + \frac{\partial G_y^n}{\partial t_i^n}\cdot\frac{\partial t_i^n}{\partial \varphi} = 0$$

can assume $\frac{\partial G_y^n}{\partial \varphi} = \frac{\partial (v_{0y}^2)}{\partial \varphi} = 0$ if t_i & t_R are independent of φ

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⇒ so this is displacement gate ⇒ uses experimentally measured ϕ^n_{max} ⇒ angle corresponding to the maximal intensity for each harmonic, to obtain:

$$\left.\frac{\partial G^n_y(t^n_R, t^n_i, \phi^{n}_{max})}{\partial \phi}\right|_{\phi = \phi^n_{max}} = 0$$

} this is solved separately for each harmonic n

⇒ however, we have 2 unknowns t_R & t_i & only 1 equation

⇒ Shafir therefore takes another experimental observable "velocity gate" to get a 2nd equation

⇒ "that way t_R & t_i are obtained independently, experimentally, without using any theory" (except to calibrate ϕ)

⇒ Right? Wrong! ⇒ by their own classical reconstruction procedure, ~~deux~~ t_i is completely determined if t_R & y_R are known.

Velocity gate $G_v(t_R, t_i, \phi) = \dfrac{v_y(t_R, t_i, \phi)}{v_x(t_R, t_i, \phi)}$ ← velocities @ recombination time

$$v_y = \underbrace{v_{0y} - A_{2w}(t_i)}_{P_y} + A_{2w}(t_R)$$ ~~since~~

~~$v_y(t_R) \approx v_{0y}$~~

substituting for v_{0y} & using $v_x(t_R) \approx \sqrt{2E_{t_R}}$ (since most of the velocity is directed along x: $v_x(t_R) \gg v_y(t_R)$
(valid assumption, except for low $n \approx I_p/\omega$)

where $E(t_R) = N\omega - I_p$

$$G_v(t_R, t_i, \phi) = \frac{eF_0/2\omega}{\sqrt{2(N\omega - I_p)}}\left[\sin(2\omega t_R + \phi) + \frac{\cos(2\omega t_R + \phi) - \cos(2\omega t_i + \phi)}{2\omega(t_R - t_i)}\right]$$

$$\Rightarrow G_v(t_R, t_i, \phi) = \frac{v_y(t_R | t_i; \phi)}{v_x(t_R | t_i; \phi)} \propto \sqrt{\frac{\text{Intensity even } n}{\text{Intensity odd } n}}$$

→ recollision angle : θ

→ $\frac{\sin\theta}{\cos\theta} = \tan\theta$ $\boxed{\begin{array}{l} v_x(t_R) = A\cos\theta \\ v_y(t_R) = A\sin\theta \end{array}}$

$I_{even}(N\omega) = I(N\omega)\sin^2(\theta)$; $I_{odd}(N\omega) = I(N\omega)\cos^2\theta$

<u>2nd Equation</u>: $\left.\frac{\partial G_v}{\partial \phi}\right|_{\phi^v_{max}} = 0$, where ϕ^v_{max} maximizes the recollision angle for each harmonic

$\Rightarrow$ so, for each n, 2 eqns, for two unknowns: t_i, t_R $\Rightarrow$ so extract t_i & t_R from 2 "independent" observables.

$\Rightarrow$ The problem is the system is overdetermined! (also different ϕ actually correspond to different trajectories)

$\Rightarrow$ 1) fix the phase. 2) For each harmonic, can determine $v_y(t_R)$: $v_y \approx \sin\theta \cdot \sqrt{2(N\omega - I_p)}$

3) this will atomatically give t_i & V_{y0}

why? $\Rightarrow$ 2nd order ODE: if @ any time you specify velocity & position, the system is known for <u>all</u> times.

@ $t = t_R$, you know $v_y(t_R)$, so you can by integrating backwards obtain when the system comes back to the same position or $t = t_i$ (Note: the absolute position does not matter because there is no spatial dependence in the force eqn for $y(t)$ → the Coulomb field is neglected)

⇒ need to establish self-consistency

$v_y(t_R) =$ ⇒ 3 eqns, but only 2 unknowns, does it mean that eqns contradict each other or just contain the same information?

⇒ no need to find ϕ^v_{max}

⇒ For the 2 gate reconstruction procedure to work, need t_R & t_i independent of ϕ, otherwise

$$\left.\frac{\partial G_y(t_R, t_i, \phi)}{\partial \phi}\right|_{\phi_y^{max}} = 0 \quad \& \quad \left.\frac{\partial G_v(t_R, t_i, \phi)}{\partial \phi}\right|_{\phi_v^{max}} = 0$$

doesn't make sense, unless ϕ_y^{max} & ϕ_v^{max} are the same; if t_R & t_i depend on ϕ, can't substitute the same t_R & t_i into eqns where ϕ's are different (

⇒ inconsistencies in the paper: = a shift in this phase (ϕ) would cause an absolute shift of the total reconstructed curve of $t_i(N)$ and $t_R(N)^2$ (page 7 supplemental)

in other words, changing ϕ, will shift t_i & t_R!