Lecture 8 ① 27. 5. 13

HHG's in plasmas - 2 mechanisms { ROM
CWE

2 ways plasma electrons can absorb energy from laser: Brunel vs. resonance absorption

Laser field - oblique incidence + p-polarization

p polarizat. - E-field lies in incidence plane, & has component E_x, normal to plasma.

Resonance absorption - laser field excites local plasma osc. @ the same ω_L as the laser

- linear conversion process @ low intensity
- excited @ a point X_R where, $L_n = n_e (dn_e/dx)^{-1}$

$$\underbrace{\omega_L}_{\text{laser freq.}} = \underbrace{\sqrt{n_e(X_R)\, e^2 / m \varepsilon_0}}_{\text{local plasma frequency}} = \omega_p(x)$$

- For a given incidence angle, θ, optimal gradient scale length, L_0 exists that maximizes this absorption

* Brunel absorption - occurs @ higher intensities when the excursion amplitude, d, of plasma electrons > L_0

→ ① electrons are pulled out of the plasma
② accel. in the vaccuum
③ slammed back into plasma where they excite local plasma oscillations

} 3 step process of CWE

Brunel trajectories

⇒ Hence Brunel absorption is the relevant mechanism @ intensities needed for HHG.

Brunel's criterion: $d > L$

$d \approx v_{osc}/\omega_L$, where v_{osc} is the approx quiver velocity

$v_{osc} \approx \dfrac{eE}{\gamma m \omega_L}$ from last lecture: $a_0 = \dfrac{eE}{m\omega_L c}$

Lorentz factor

separates CWE from ROM regimes

$$\boxed{v_{osc} \approx \frac{a_0 c}{\gamma}, \quad \text{where } \gamma = \sqrt{1+a_0^2}}$$

electron excursion relative to laser wavelength, λ_L:

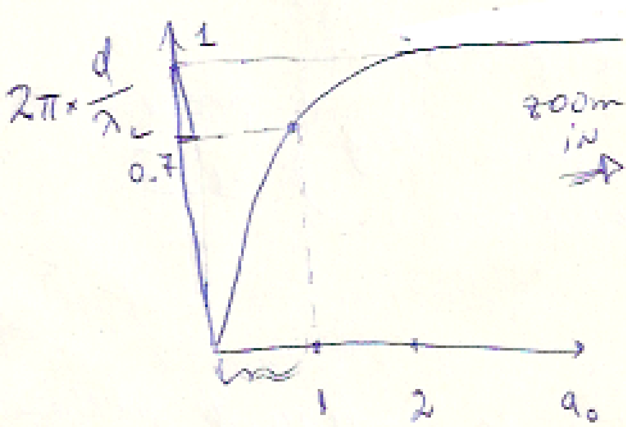
$$\frac{d}{\lambda_L} \Rightarrow c = \frac{\omega_L}{K_L} = \omega_L \times \frac{\lambda}{2\pi} \Rightarrow \lambda_L = \frac{c \cdot 2\pi}{\omega_L}$$

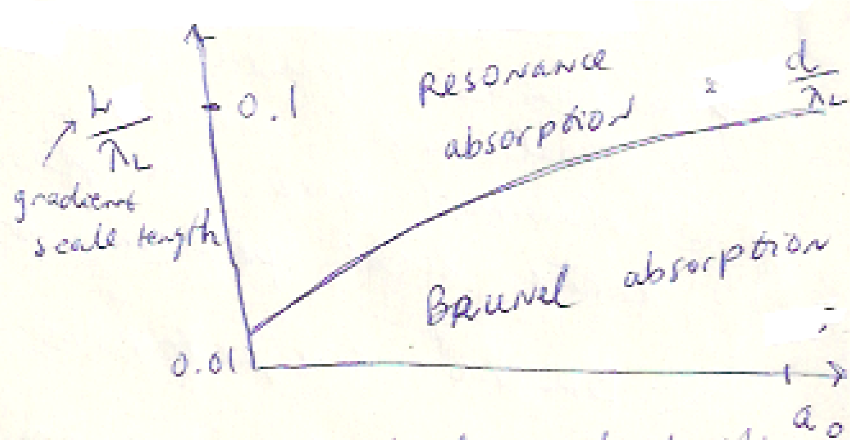
$$\frac{d}{\lambda_L} = \frac{d \cdot \omega_L}{c \cdot 2\pi} = \frac{v_{osc}}{c \cdot 2\pi} = \boxed{\frac{a_0}{2\pi\sqrt{1+a_0^2}} = \frac{d}{\lambda_L}}$$

$\Rightarrow$ electron excursion always smaller than λ_L,

$d \ll \lambda_L$ for $\underbrace{a_0 \ll 1}_{\text{resonance absorption regime}}$ or $\underbrace{a_0 \gg 1}_{\text{ROM regime (ultra-relativistic)}}$

However CWE dominates @ $a_0 \sim 1$





already the electron excursion distance is close to max for $a_0 \sim 1$

L_0 (optimal scale length) depends on freq, θ, but not intensity

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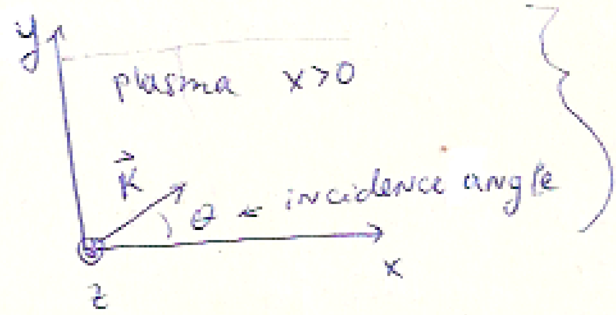
Analysis of HHG in plasma mirrors → transformation to 1-D



Lab frame → two-D problem in x & y

→ convert to "boosted frame" - frame moving with constant velocity along the y-axis, V_{drift} → reduces analysis to 1-D (along x-axis)

→ widely used in analysis & PIC codes

$V_{drift} = c \sin\theta \, \hat{y}$ $\quad \boxed{\frac{V_{drift}}{c} = \beta = \sin\theta}$ ← relativistic factor

$$\Gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\beta^2}} = \boxed{\frac{1}{\cos\theta} = \Gamma}$$

Lab frame: ω_L; $\vec{K}_L = \left(\frac{\omega_L \cos\theta}{c}, \frac{\omega_L \sin\theta}{c}, 0\right)$

Boosted frame, M: $\underbrace{\omega_m = \omega_L \cdot \cos\theta}_{\text{lowered frequency}}$; $\underbrace{\vec{K}_m = \left(\frac{\omega_L \cos\theta}{c}, 0, 0\right)}_{\text{1-D vector}}$

→ In frame M, electric field ⊥ to plasma surface, with plasma moving @ $V^{drift}_{plasma} = -c\sin\theta\,\hat{y}$

→ electron & ion plasma densities change by Γ in the moving frame - shrinking of length in relativistic motion

Lorentz transformations (Special Relativity)

boosted frame $\left\{\begin{aligned} t_m &= \Gamma \cdot (t - vx/c^2) \\ y_m &= \Gamma \cdot (y - vt) \\ x_m &= x \\ z_m &= z \end{aligned}\right\}$ Lab frame

→ $\underbrace{n_e^m(x)}_{\text{increase in density along y-axis}} = \frac{n_e^L(x)}{\cos\theta}$

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Calculating HHG spectra in boosted frame

⇒ HHG spectra is generated by transverse plasma currents: $\vec{j}_t = (0, j_y, j_z)$

→ In coulomb gauge: $\nabla \cdot A = 0$

wave equation: $\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial \vec{A}^2}{\partial t^2} = -\mu_0 j_t$

setting $j_t = 0$ gives the homogeneous solution to the eqn, or the vector potential of the incident laser field: $\vec{A}_L$

We're interested in the inhomogeneous solution, or the part of $\vec{A}$ produced by plasma currents, j_t

Inhomogeneous solution of wave-eqn.:

$$\underbrace{\vec{A}_r(x_0, t_0)}_{\text{vector pot. due to plasma currents}} = -\mu_0 \int_{x_0}^{\infty} dx \int_{-\infty}^{t_{ret}} dt\ \vec{j}_t(x,t), \text{ where } t_{ret} = t_0 - \frac{(x - x_0)}{c}$$

FROM $\underbrace{\vec{E}^r}_{\text{electric field polarized along } \hat{y} \text{ \& } \hat{z}} = -\frac{\partial \vec{A}_r}{\partial t}$ $\quad E^r(x_0, t_0) = \mu_0 \int_{x_0}^{\infty} dx\ \vec{j}_t[x, \underbrace{t_{ret}}_{f(t_0, x_0, x)}]$

Experimental observable ⇒ intensity of the high harmonic ⇒ $|\hat{E}^r(x_0, \omega)|^2$ for position x_0 in vacuum, where $\hat{E}_r(x_0,\omega)$ is a Fourier transform of $\vec{E}^r$ in time

⇒ spectra independent of x_0 in vacuum: $S(\omega) = |\hat{E}^r(x_0,\omega)|^2$

$$S(\omega) = \mu_0^2 \left| \int_{x_0}^{\infty} dx\ \hat{j}_t(x,\omega)\, e^{-i\omega x/c} \right|^2 = \mu_0^2 \left| \hat{\hat{j}}_t\left(k = \frac{\omega}{c}, \omega\right) \right|^2$$

⇒ the HHG spectra, $S(\omega)$ is given by a double Fourier transform of $j_t(x,t)$, s.t. $k = \frac{\omega}{c}$!

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So, how do we calculate $\underbrace{j_t(x,t)}_{\text{currents in plasma}}$ to get $\underbrace{\tilde{S}(\omega)}_{\text{HHG spectra}}$?

since $\boxed{S(\omega) \propto |\hat{j}_t(k=\frac{\omega}{c},\omega)|^2}$

Normally (for accurate results), simulations have to be used)
Here, we show how to analytically using a <u>fluid model</u> of plasma

transverse current due to ions:

$$\boxed{\begin{aligned} j_t^{ion} &= -Zen_i^m(x,t)\cdot c\sin\theta\,\hat{y} \\ &= -Zen_i^L(x,t)\cdot c\tan\,\hat{y} \end{aligned}}$$

⇒ due only to $v_{drift} = -c\sin\theta\hat{y}$

⇒ effect of the laser field assumed to be only on electrons

⇒ For $j_t^{electron}$, the laser field $\vec{A}$ is felt,

⇒ use conservation of canonical momentum in the transverse (y,z)-plane direction (Hamiltonian is independent of y & z (density gradient in x only))

⇒ hence $\dot{P}_z = -\frac{\partial H}{\partial z} = 0$; $\dot{P}_y = -\frac{\partial H}{\partial y} = 0$

$$\Rightarrow \vec{P}_t - e\vec{A} = \text{const} = \underbrace{P_t^0}_{\text{before the laser pulse}} = \underbrace{-m\cdot c\cdot\tan\theta\,\hat{y}}_{\text{same as for the ions}} \Rightarrow \text{because}$$

$$\Rightarrow \boxed{\vec{P}_t^0 = m\cdot\frac{1}{\cos\theta}\cdot(-c\sin\theta\,\hat{y})}$$

$$\vec{P}_t = m\gamma\vec{V}_t, \text{ where } \gamma = (1-(\beta_x^2 + \underbrace{\beta_t^2}_{\left(\frac{v_{tot}}{c}\right)^2}))^{-1/2}$$

$$\boxed{\vec{j}_t^e = -en_e^m V_t = \frac{-e n_e^L \vec{P}_t}{m\gamma\cos\theta}}$$

$$\boxed{\vec{P}_t = e\vec{A} - mc\tan\theta\,\hat{y}}$$

$$\boxed{j_t = j_t^e + j_t^i}$$

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$$j_t(x,t) = \overbrace{-\frac{e^2 n_e^L(x,t)}{m\cos\theta}\cdot\frac{\vec{A}(x,t)}{\gamma(x,t)}}^{\text{current driven directly by total radiation field}} - \overbrace{ec\tan\theta\left[\underbrace{Z n_i^L(x,t) - \frac{1}{\cos\theta}\,\frac{n_e^L(x,t)}{\gamma(x,t)}}_{\text{current induced because of plasma drift in M frame}}\right]\hat{y}}^{\text{obliquity current}}$$

conduction current

non-zero either because (i) $n_i^L(x,t) \neq n_e^L(x,t)$

or (ii) $\gamma(x,t) \neq \frac{1}{\cos\theta}$ $\Rightarrow$ the electron fluid has been accelerated by the laser field, resulting in an extra change in density by relativistic compression (Note, $\frac{1}{\cos\theta}$ gives a change in density due to v_{drift})

$$\Rightarrow n^M = \frac{n^L}{\cos\theta}$$

Conduction current - directly driven by the laser field $\propto \vec{A}(x,t)$

Obliquity current - only occurs for oblique laser incidence $\theta \neq 0$ $\Rightarrow$ corresponds to excitation of charge along the y-axis

$\Rightarrow$ means that even for s-polarized light ($\vec{E} \parallel z$), some p-polarized light can be radiated by the plasma.