Lecture 9 ① 30.5.13

Last time: HHG spectra can be determined by a double (space x time) Fourier transform of transverse currents in the plasma: $j_t(x,t)$

$$S(\omega) = \mu_0^2 \left| \hat{j}_t\left(k=\frac{\omega}{c}, \omega\right) \right|^2$$

Calculated $j_t(x,t)$ analytically using a fluid model:

$$j_t(x,t) = \underbrace{\frac{-e^2 n_e^L(x,t)}{m\cos\theta} \times \frac{\vec{A}(x,t)}{\gamma(x,t)}}_{\text{conduction current}} \underbrace{- ec\tan\theta\left[Z n_i^L(x,t) - \frac{n_e^L(x,t)}{\cos\theta\,\gamma(x,t)}\right]}_{\text{obliquity current}} \hat{y}$$

Obliquity current: ① only occurs for $\theta \neq 0$ ($\tan(0)=0$)
② occurs due to distortion of electron density by laser field
③ Plays a crucial role in coherent wave emission (CWE)
↳ . Need oblique incidence for CWE

⇒ On the other hand, NO oblique current is needed to explain ROM.
⇒ Can assume $\theta = 0$ (no need for boosted frame)

⇒ attosecond pulses are produces with ROM when jets of electrons move toward the laser field (along $-x$) with $v \approx c$ ⇒ high freq. created be of Doppler shift

Basic (rough) mechanism behind ROM

* assume plasma surface moving with constant velocity $-v\hat{x}$ toward the laser field: $E(x,t) = E_0 \cos(\omega t - kx)$

→ the field experienced by the surface is then given by: $E_0 \cos(\omega t + k \cdot \underbrace{(vt)}_{x}) = E_0 \cos(\omega' t)$,

where $\omega' = \omega + k \cdot v = \omega \cdot (1 + \frac{v}{c})$, since $k = \frac{\omega}{c}$

Wavelength : $T' = \frac{2\pi}{\omega'} = \frac{\lambda'}{c}$ → $\frac{2\pi \cdot c}{\lambda'} = \omega'$

The wavelength of the reflected light in a lab frame: $\lambda'' = \lambda' - v\underbrace{(\lambda'/c)}_{T'}$ → $\omega'' = \frac{2\pi \cdot c}{\lambda''}$

Freq. reflected by the osc. mirror surface:

$$\omega'' = \frac{\omega'}{(1-\frac{v}{c})} = \frac{\omega(1+\frac{v}{c})}{(1-\frac{v}{c})}$$

$$\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$$

Doppler shift of the reflected frequency:

$$\boxed{\frac{\omega''}{\omega} = \frac{1+\beta}{1-\beta} = (1+\beta)^2 \cdot \gamma^2}$$ where $\beta = \frac{v}{c}$

$$\lambda'' = \lambda'(1-\frac{v}{c}) \rightarrow \omega'' = \frac{2\pi \cdot c}{\lambda'(1-\frac{v}{c})} = \frac{\omega'}{(1-\frac{v}{c})}$$

→ Well-known Doppler shift induced upon reflection from a moving mirror $\frac{\omega''}{\omega} \rightarrow 4\gamma^2$ as $v \rightarrow c$

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physical insight into ROM: $\frac{\omega^{\circ}}{\omega} = (1+\beta)^2 \gamma^2$
(assuming surface moving with constant
velocity v, & $\theta = 0$ incidence angle)

$1+\beta$ term: 1 frequency shift in the frame of the
moving mirror ⇒ $\omega' = \omega(1+\beta)$ ⇒ can at most increase
by a factor of 2

2 Now, consider the mirror as a moving source ⇒
emission observed @ $\omega^{\circ} = \frac{\omega'}{1-\beta}$

⇒ $\omega^{\circ} \to \infty$ as $\beta \to c$, therefore most of
the high harmonic effect comes from 2

⇒ Cut-off under these simple assumptions goes as $\sim \gamma^2$

⇒ to fully solve for the spectra, need to calculate
all currents, $\vec{j}$

⇒ However, at ROM intensities $\sim 10^{19} \frac{W}{cm^2}$, distortions
are so large, that a simple fluid model fails

⇒ Huge distortions both in $n_e^{L}(x,t)$ & $\vec{A}(x,t)$

<u>Alternative model</u>: γ-spikes model or BGP theory
developed by Baeva-Gordienko-Pukhov (BGP), see
2006, PRE, 74, 046404 for detail

⇒ does not calculate $\vec{j}$, but uses a totally
different approach, based on <u>boundary</u> conditions

BGP theory - assumes an existence of an "apparent reflection point" (ARP), with position $x_{ARP}(t)$ s.t. for all time t, $E_y^i(x_{ARP}(t), t) + E_y^r(x_{ARP}(t), t) = 0$ where E_y^i & E_y^r are incident & reflected fields (respectively) along the target surface.

$\Rightarrow$ means $E_y^{TOT} = 0$ @ x_{ARP} for all t!

$\Rightarrow$ generalizes the usual boundary condition @ the surface of a perfect fixed mirror to a moving mirror

$\Rightarrow$ using this boundary condition, BGP derive 2 main, "universal" properties of emitted harmonic spectrum in the ultra-relativistic intensity limit

1. harmonic spectra predicted to decay as $n^{-8/3}$, with harmonic order n

2. the cut-off frequency, ω_c scales as γ^3 (rather than γ^2 like in the simple example above), where γ is the maximum Lorentz factor

$\Rightarrow$ This actually corresponds to the general scaling for a charge in ultrarelativistic motion.

$\Rightarrow$ hence electrons @ plasma surface behave as a bunch of ultrarelativistic electrons radiating coherently

$\Rightarrow$ coherence is due to the fact that the Doppler-induced freq upshift occurs only during a small fraction of the laser cycle (when electrons are accelerated to maximum velocity)

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Subsequent experiments seem to validate the theory.

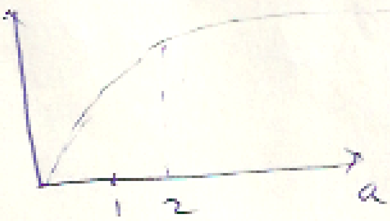
Dromey et al: 2006 Nat. Phys. 2 456;
"   2007 PRL 99, 085001

→ showed the $n^{-8/3}$ scaling

Short-comings of the theory: $E_y^i(x_{ARP}) \neq E_y^r(x_{ARP})$
at HHG-relevant regimes
Normally $|E_y^r(t)| > \max|E_y^i|$

$\boxed{\text{Comparison CWE / ROM}}$

→ from last class: $\to d/\lambda_L$
electron excursion amplitude



d saturates quickly, already $a_0 \sim 1$ → means CWE has almost constant generation efficiency in a large intensity range.

Generation efficiency for ROM saturates @ much higher intensity → efficiency for ROM grows in the 10^{18} - 10^{20} $\frac{W}{cm^2}$ range

→ ROM contributes at higher intensities, CWE @ lower (two regimes are roughly separated by a_0, with $a_0 < 1$ → CWE & $a_0 > 1$ ROM).

⑥

⇒ different influence of gradient scale length in CWE vs ROM: $L = \left| \frac{n_e(x)}{dn_e/dx} \right|$ or $n_e(x) \propto e^{+x/L}$

Optimal L for CWE very short $\sim \frac{\lambda_L}{50}$

Optimal L for ROM $\sim \frac{\lambda_L}{5}$

⇒ short gradients favor CWE (low intensities)

⇒ long gradient favor ROM (high intensities)

⇒ both CWE & ROM have an optimal incidence angle of ~55° (from simulations

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Testing of BGP $n^{-8/3}$ law with numerics: