

Ultrafast Laser Physics

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Chapter 4b: $\chi^{(2)}$ -nonlinearities with ultrashort pulses



- Second order nonlinear susceptibility and susceptibility tensor
- Overview of $\chi^{(2)}$ processes
- Crystal optics, tensors and effective nonlinearities
- Phase matching – with ultrashort pulses and realistic beams
- Quasi-phase matching
- Second harmonic generation and sum frequency mixing
- Difference frequency mixing and parametric amplification
- Ultrabroadband phase matching schemes
- Optical parametric chirped-pulse amplification
- Optical parametric oscillators
- Cascading of $\chi^{(2)}$ -nonlinearities



- Wave equation for homogenous, dielectric material:

$$\Delta \mathbf{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \mathbf{E} = \mu \mu_0 \frac{\partial^2}{\partial t^2} \mathbf{P}$$

- Linear optics:

$$P_i = \epsilon_0 \chi_{ij} E_j$$

- Nonlinear optics:

$$P_i = \epsilon_0 \chi_{ij}^{(1)} E_j + \epsilon_0 \chi_{ijk}^{(2)} E_j E_k + \epsilon_0 \chi_{ijkl}^{(3)} E_j E_k E_l + \dots$$

2nd order nonlinearities ←

3rd order nonlinearities ←

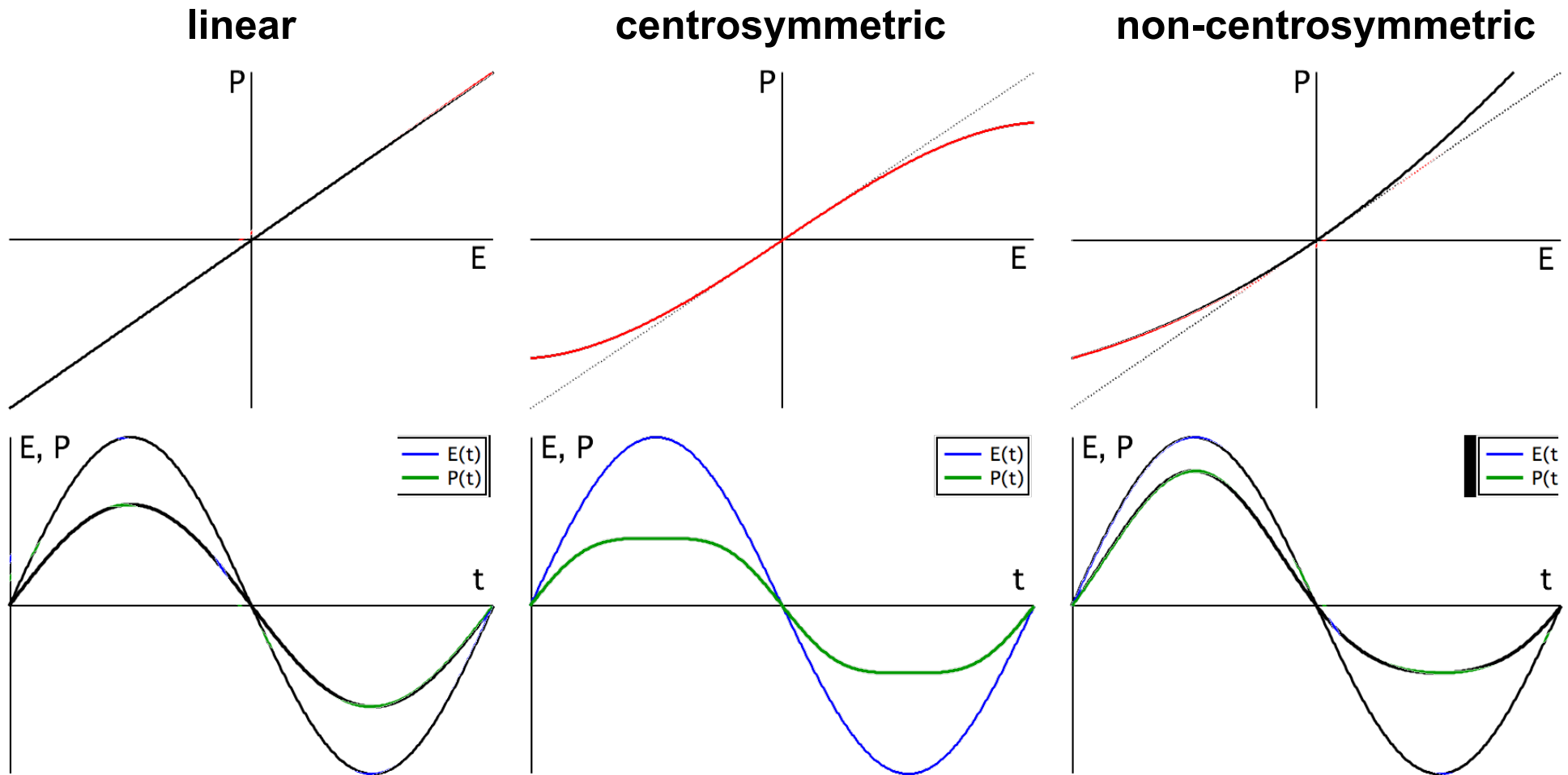
- Examples of 2nd order nonlinear processes:

- Second harmonic generation (SHG), sum-frequency generation (SFG)
- Difference-frequency generation (DFG), optical rectification

- Examples of 3rd order nonlinear processes:

- Third-harmonic generation (THG)
- Optical Kerr effect, self-phase modulation, self-focusing





- Third-order effects can occur in all materials, even isotropic systems (e.g., glass, air)
- Second-order nonlinearities occur only in non-centrosymmetric materials (ferroelectric materials)

- Interaction of two monochromatic waves

$$P_{NL} = \epsilon_0 \chi^{(2)} E^2 \quad E = E_1 + E_2$$

$$E_1 = E_{10} \cos(\omega_1 t - k_1 z)$$

$$E_2 = E_{20} \cos(\omega_2 t - k_2 z)$$

- Interaction at fixed position ($z = 0$)

$$P_{NL} = \epsilon_0 \chi^{(2)} (E_{10} \cos(\omega_1 t) + E_{20} \cos(\omega_2 t))^2$$

$$\begin{aligned}
 P_{NL} = & \frac{\epsilon_0 \chi^{(2)}}{2} (E_{10}^2 + E_{20}^2) \\
 & + \frac{\epsilon_0 \chi^{(2)}}{2} E_{10} E_{20} \cos((\omega_1 + \omega_2)t) \\
 & + \frac{\epsilon_0 \chi^{(2)}}{2} E_{10} E_{20} \cos((\omega_1 - \omega_2)t)
 \end{aligned}$$

Opt. rectification
Second harmonic 1 + Second harmonic 2
Sum frequency generation
Difference frequency generation



- Nonlinear optical polarization in 3D materials (crystals):

$$P_i^{NL} = \epsilon_0 d_{ijk} E_j E_k$$

- Product of E-vectors is symmetric in j and k , therefore

$$d_{ijk} = d_{ikj}$$

- This allows reduction of 27-element tensor to 18-element matrix:

$$d_{ijk} \rightarrow d_{im} \quad (i = 1, 2, 3 \quad m = 1 \dots 6)$$

j	k	m
1	1	1
2	2	2
3	3	3
2	3	4
3	2	4
1	3	5
3	1	5
1	2	6
2	1	6

$$\begin{pmatrix} P_x^{NL} \\ P_y^{NL} \\ P_z^{NL} \end{pmatrix} = \epsilon_0 \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{pmatrix} \cdot \begin{pmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_y E_z \\ 2E_z E_x \\ 2E_x E_y \end{pmatrix}$$

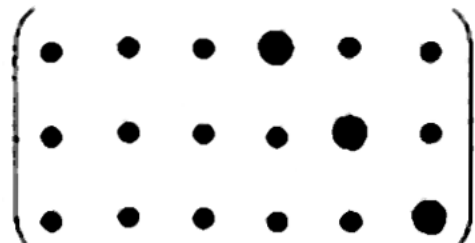
ETH Further reduction through crystal symmetries

- Material tensors have to be invariant against the symmetry operations of the point group of a crystal
- This allows to further reduce the number of independent tensor elements

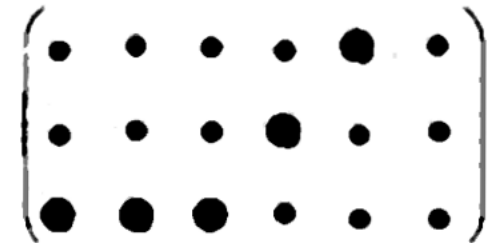
- zero modulus
- non-zero modulus
- equal moduli
- moduli numerically equal, but opposite in sign
- ⊙ a modulus equal to minus 2 times the heavy dot modulus to which it is joined

Orthorhombic

Class 222



Class mm2



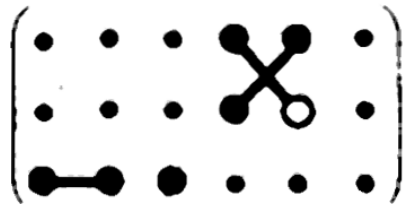
- For orthorhombic point groups: only 3 and 5 non-zero independent matrix elements



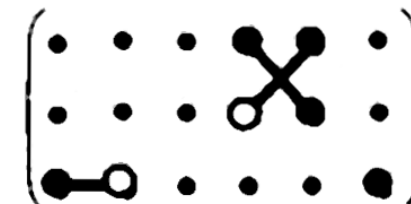
ETH One more example: tetragonal point groups

- zero modulus
- non-zero modulus
- equal moduli
- moduli numerically equal, but opposite in sign
- ⊙ a modulus equal to minus 2 times the heavy dot modulus to which it is joined

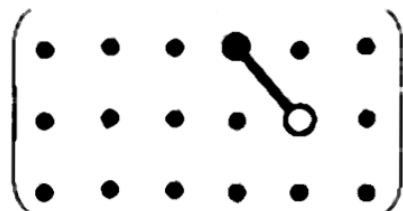
Class 4



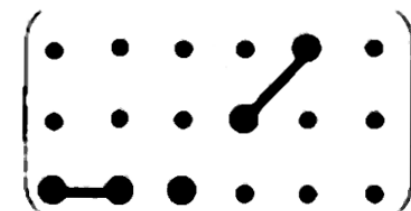
Class $\bar{4}$



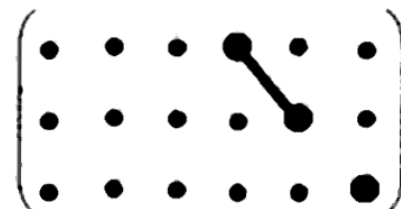
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Class 4mm



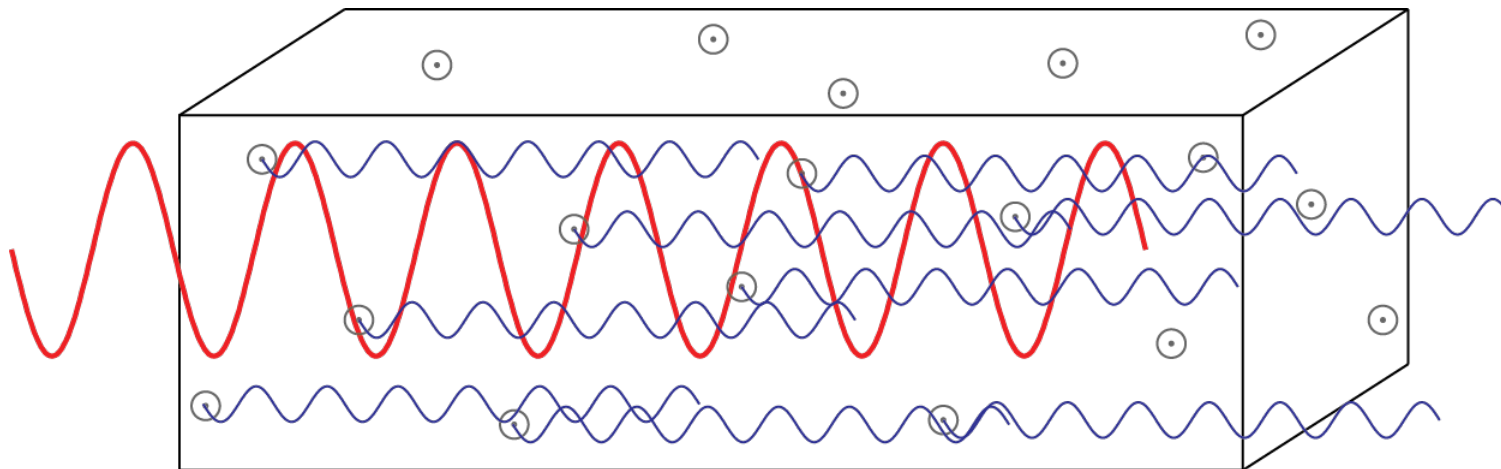
Class $\bar{4}2m (2 || x_1)$



Mixing processes to be observed macroscopically

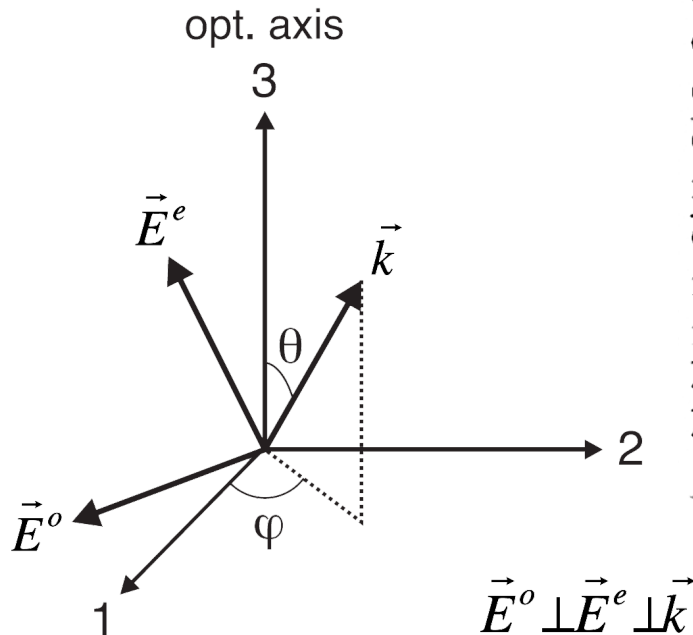
$$\begin{pmatrix} P_x^{NL} \\ P_y^{NL} \\ P_z^{NL} \end{pmatrix} = \epsilon_0 \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{pmatrix} \cdot \begin{pmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_y E_z \\ 2E_z E_x \\ 2E_x E_y \end{pmatrix}$$

- Microscopically, all wave-mixing processes mediated by non-zero tensor elements occur
- Macroscopically, mixing products can only build up significant power if the corresponding process is phase-matched (i.e., microscopic contributions add up constructively)



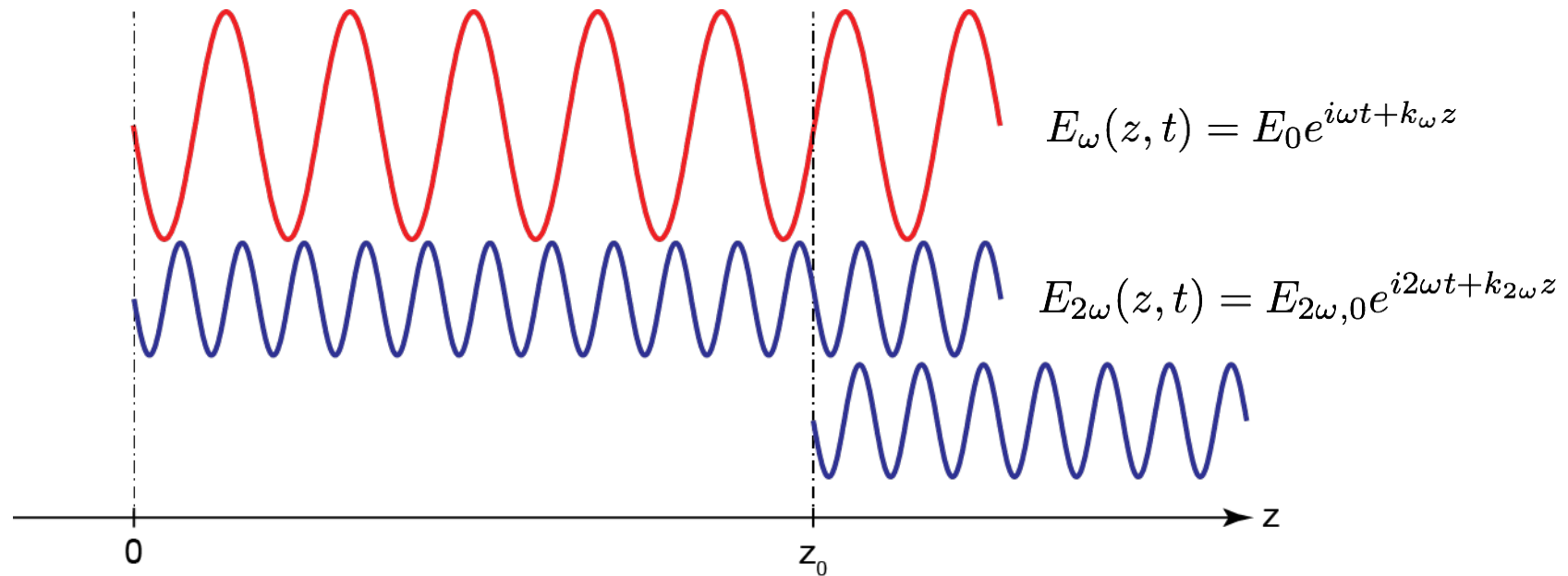
Back to 1D-problem

- For collinear beams, with one interaction selected (e.g., through phase-matching) and beam geometry fixed (polarization and propagation direction), the **nonlinear susceptibility can be projected down to a scalar effective nonlinearity d_{eff}**
- Assumptions: no dispersion, no absorption, uniaxial crystal



Crystal symm.	2 e-rays and 1 o-ray	2 o-rays and 1 e-ray
6, 4	0	$d_{15} \sin \theta$
622, 422	0	0
6mm, 4mm	0	$d_{15} \sin \theta$
$\bar{6}m2$	$d_{22} \cos^2 \theta \cos 3\varphi$	$-d_{22} \cos \theta \sin 3\varphi$
3m	$d_{22} \cos^2 \theta \cos 3\varphi$	$d_{15} \sin \theta - d_{22} \cos \theta \sin 3\varphi$
$\bar{6}$	$\cos^2 \theta (d_{11} \sin 3\varphi + d_{22} \cos 3\varphi)$	$\cos \theta (d_{11} \cos 3\varphi - d_{22} \sin 3\varphi)$
3	$\cos^2 \theta (d_{11} \sin 3\varphi + d_{22} \cos 3\varphi)$	$d_{15} \sin \theta + \cos \theta (d_{11} \cos 3\varphi - d_{22} \sin 3\varphi)$
32	$d_{11} \cos^2 \theta \sin 3\varphi$	$d_{11} \cos \theta \cos 3\varphi$
$\bar{4}$	$\sin 2\theta (d_{14} \cos 2\varphi - d_{15} \sin 2\varphi)$	$-\sin \theta (d_{14} \sin 2\varphi + d_{15} \cos 2\varphi)$
$\bar{4}2m$	$d_{14} (\sin 2\theta \cos 2\varphi)$	$-d_{14} \sin \theta \sin 2\varphi$

- Example: second-harmonic generation of collinear plane waves



- Second harmonic has to interfere constructively for all z_0 in crystal
- For fixed $t = 0$:

Second harmonic light generated at $z = 0$, propagated to z_0 :

$$E_{2\omega}(z_0) = E_{2\omega,0} e^{ik_{2\omega} z_0}$$

Second harmonic light generated at $z = z_0$:

$$E'_{2\omega}(z_0) \propto E_\omega(z_0)^2$$

- For constructive interference for all z_0 , phases of existing and newly generated light have to match

- Second harmonic has to interfere constructively for all z_0 in crystal
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Second harmonic light generated at $z = z_0$:

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- For constructive interference for all z_0 , phases of existing and newly generated light have to match

$$\varphi_{2\omega}(z_0) \stackrel{!}{=} \varphi'_{2\omega}(z_0)$$

$$k_{2\omega}z_0 \stackrel{!}{=} 2k_{\omega}z_0$$

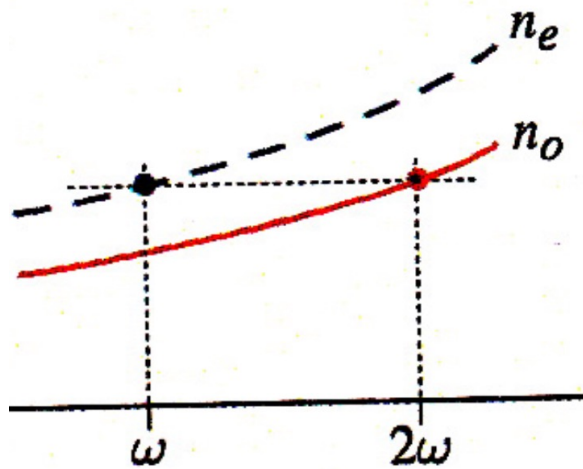
$$\frac{2\pi}{\lambda/2}n_{2\omega} \stackrel{!}{=} 2\frac{2\pi}{\lambda}n_{\omega}$$

$$n_{2\omega} \stackrel{!}{=} n_{\omega}$$

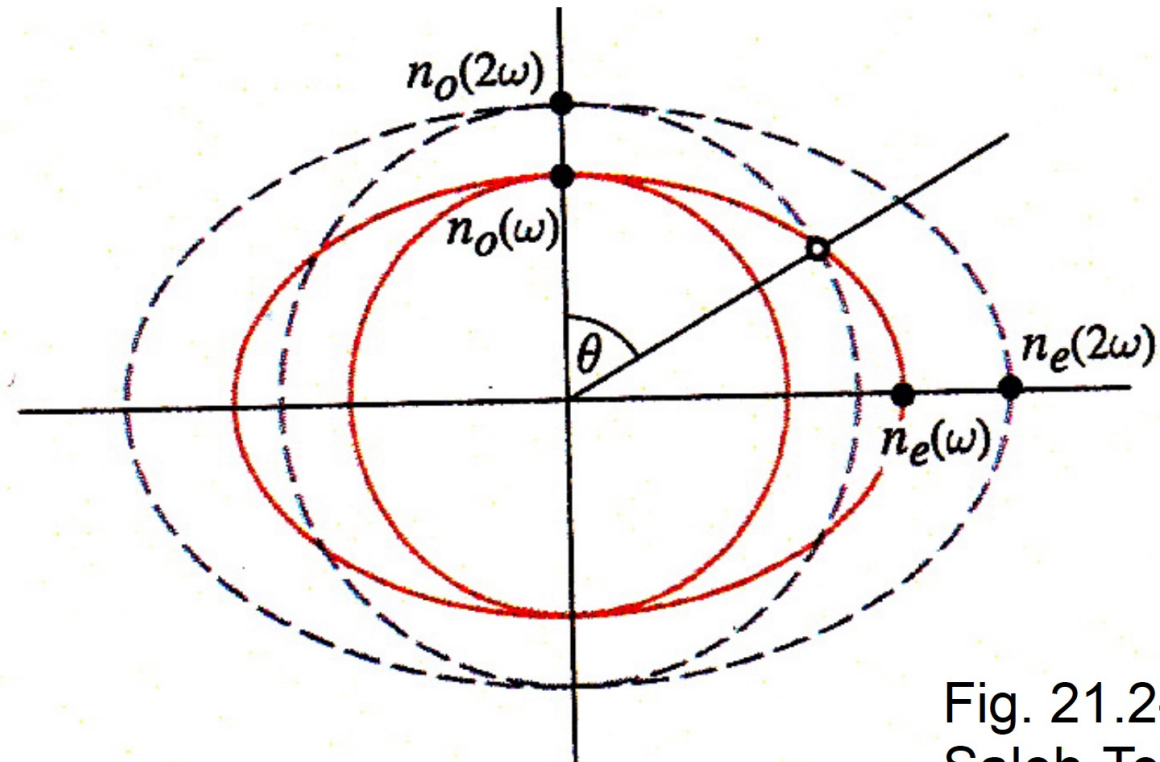
- This is in general not possible because of dispersion!

Birefringent phase matching

- This condition can be met at some wavelengths in suitable birefringent materials



(a)



(b)

Fig. 21.2-10
Saleh-Teich

- In this example, two fundamental photons polarized along extraordinary axis generate one second-harmonic photon along ordinary axis
- However, this condition is typically met exactly at only one set of interacting wavelengths

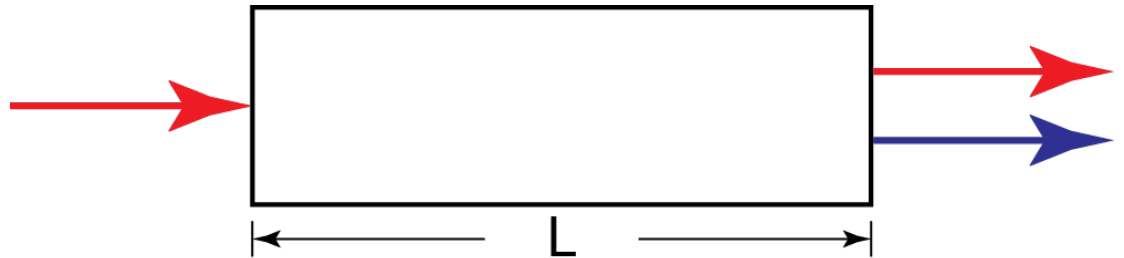


Phase mismatch and conversion efficiency (SHG)

- Coupled-wave equations (1D, collinear case, non-absorbing medium)

$$\frac{\partial E_\omega}{\partial z} = -i\omega \frac{1}{cn_\omega} d_{\text{eff}} E_{2\omega} E_\omega^* e^{-i\Delta k z}$$

$$\frac{\partial E_{2\omega}}{\partial z} = -i\omega \frac{1}{cn_{2\omega}} d_{\text{eff}} E_\omega^2 e^{i\Delta k z}$$



- Phase mismatch:

$$\Delta k := k_3 - k_1 - k_2 = k_{2\omega} - 2k_\omega$$

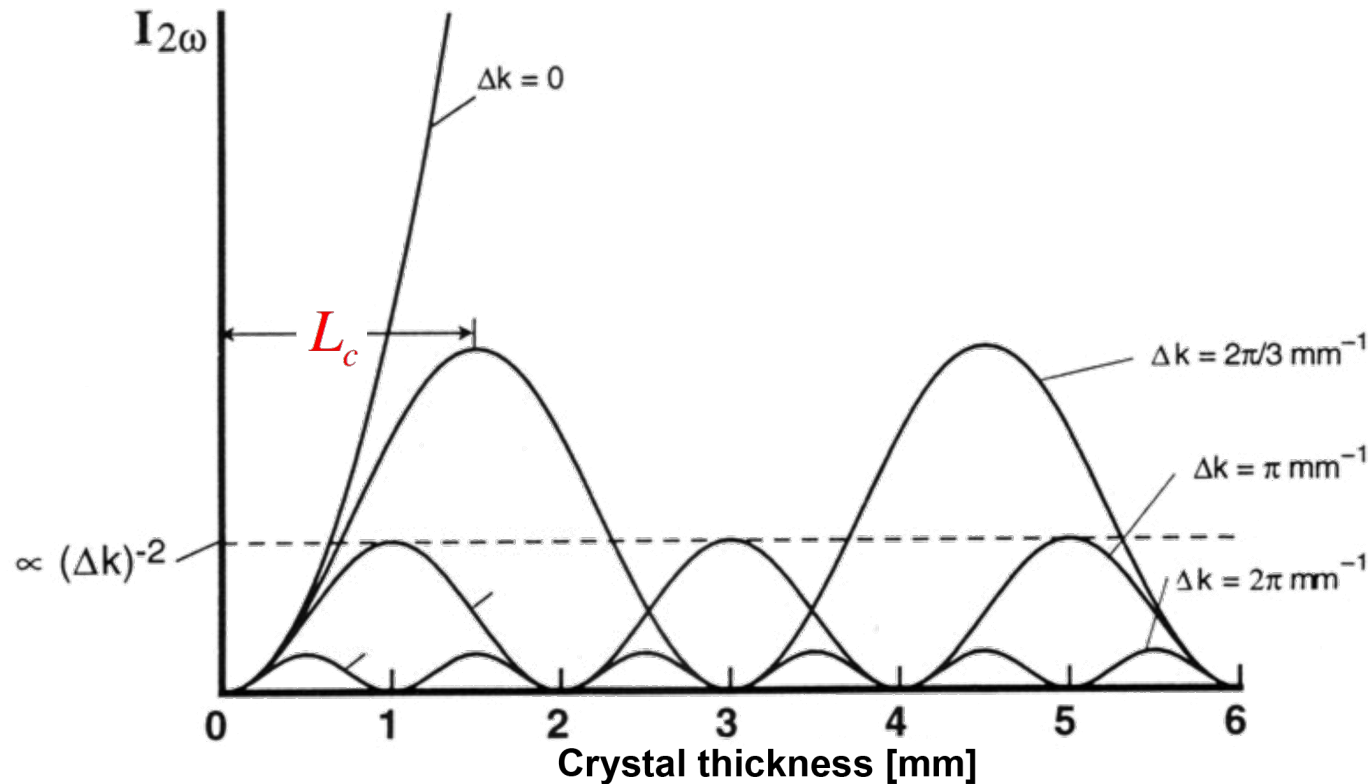
- Efficiency without depletion of fundamental, $I_{2\omega}(z=0) = 0$, $I_\omega(z) = \text{const}$
 \Rightarrow Integrate coupled wave equation for second harmonic along crystal length L

$$E_{2\omega}(L) = -i\omega \frac{1}{cn_{2\omega}} d_{\text{eff}} E_\omega^2 \frac{e^{i\Delta k L} - 1}{i\Delta k L} = -i\omega \frac{1}{cn_{2\omega}} d_{\text{eff}} E_\omega^2 \frac{\sin\left(\frac{\Delta k L}{2}\right)}{\frac{\Delta k L}{2}}$$

- With $I = \frac{1}{2} \epsilon_0 cn E^2$ it follows

$$\eta = \frac{I_{2\omega}}{I_\omega} = \frac{2\omega^2}{\epsilon_0 c^3} \left(\frac{d_{\text{eff}}^2}{n_{2\omega} n_\omega^2} \right) I_\omega L^2 \text{sinc}^2 \left(\frac{\Delta k L}{2} \right) \propto L^2 \text{sinc}^2 \left(\frac{\Delta k L}{2} \right)$$





- Signal grows quadratically for vanishing phase mismatch
- For non-zero mismatch, signal oscillates periodically
- First local maximum reached after so-called coherence length L_c

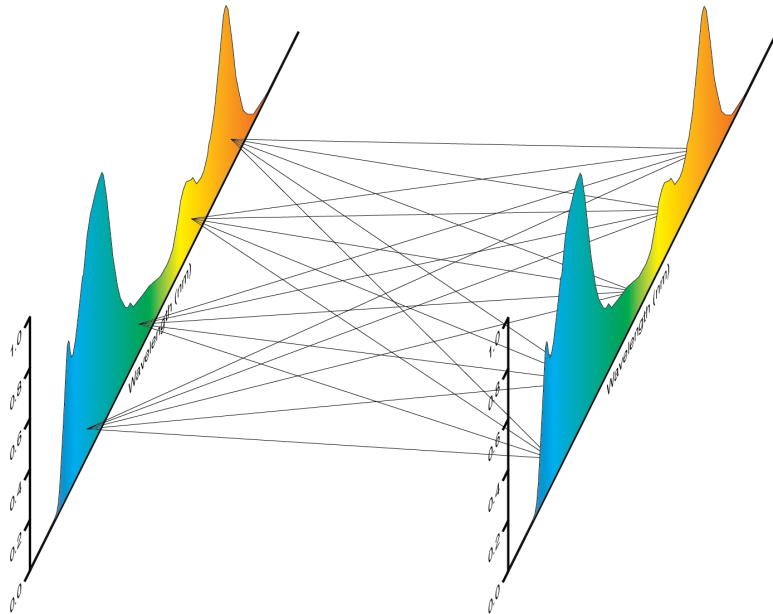
$$L_c := \frac{\pi}{\Delta k}$$

- With pulses and perfect phase matching, the second harmonic spectrum corresponds to the autoconvolution of the fundamental pulse spectrum

$$P_{\text{NL}}(t) \propto E_{\omega}(t)^2 \Rightarrow \tilde{P}_{\text{NL}}(\omega) \propto \tilde{E}_{\omega}(\omega) * \tilde{E}_{\omega}(\omega)$$

$$E_{2\omega}(t) \propto E_{\omega}(t)^2 \Rightarrow \tilde{E}_{2\omega}(\omega) \propto \tilde{E}_{\omega}(\omega) * \tilde{E}_{\omega}(\omega)$$

- As a result, every frequency component of the fundamental is mixed with every frequency
- Potentially, this can broaden spectrum / shorten pulses



- How well is each of these sum-frequency mixing processes phase-matched?

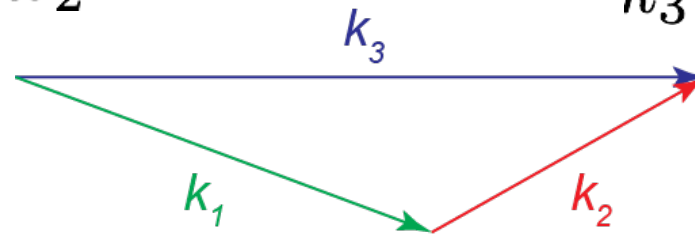
Sum frequency mixing ($\omega_3 > \omega_1, \omega_2$)

Energy conservation

$$\omega_3 = \omega_1 + \omega_2$$

Momentum conservation

$$\vec{k}_3 = \vec{k}_1 + \vec{k}_2$$



$$\Delta\vec{k} = \vec{k}_3 - \vec{k}_1 - \vec{k}_2$$

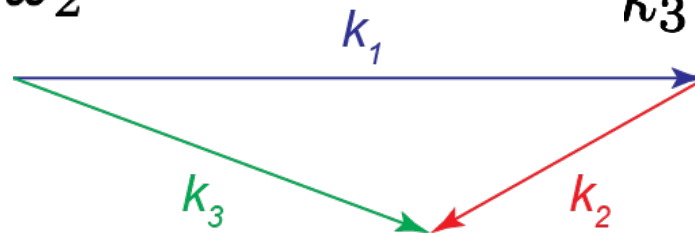
Difference frequency mixing ($\omega_1 > \omega_2, \omega_3$)

Energy conservation

$$\omega_3 = \omega_1 - \omega_2$$

Momentum conservation

$$\vec{k}_3 = \vec{k}_1 - \vec{k}_2$$



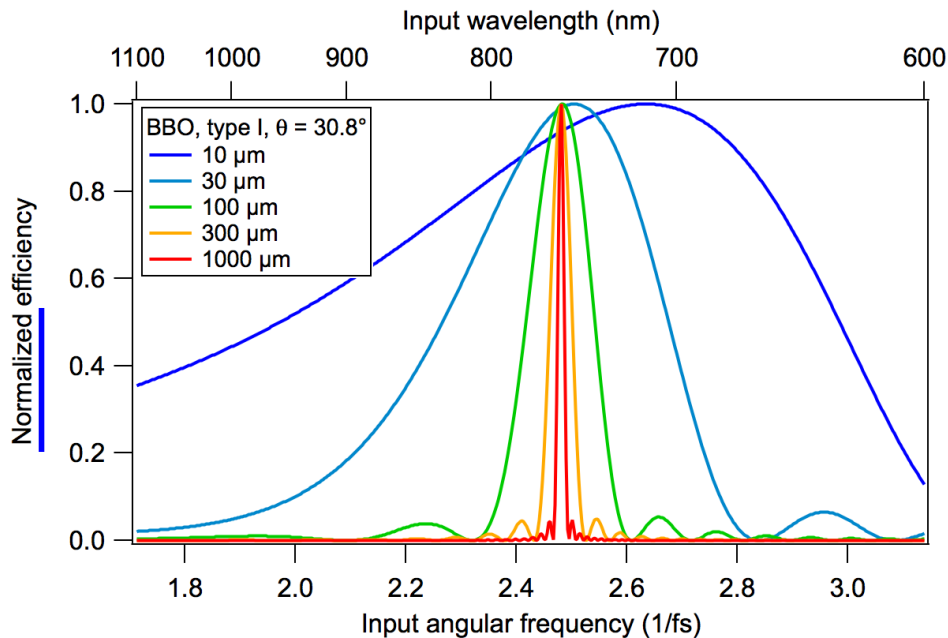
$$\Delta\vec{k} = \vec{k}_1 + \vec{k}_2 - \vec{k}_3$$

Phase matching bandwidth

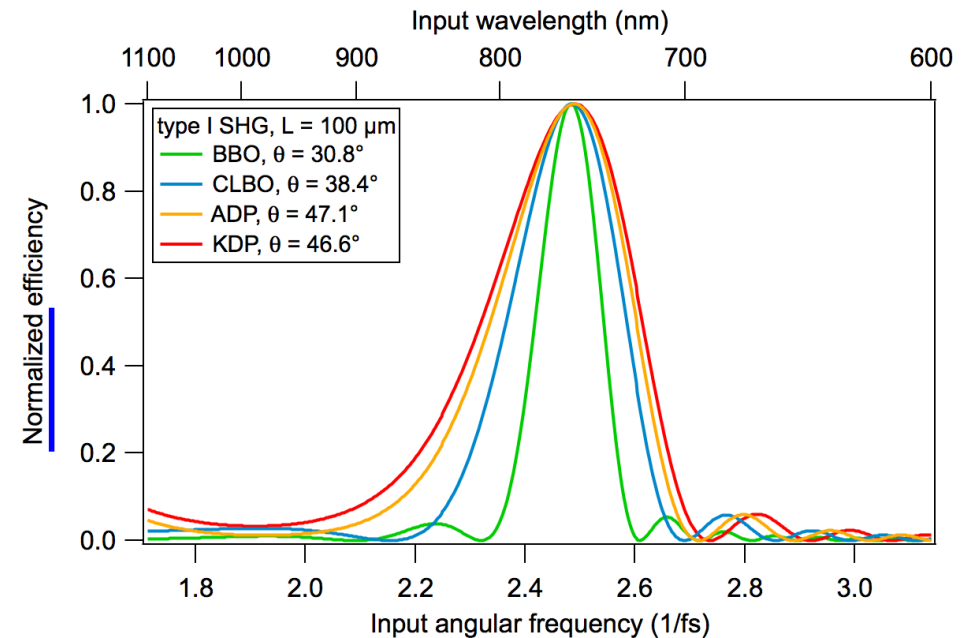
- Here, we estimate conversion bandwidth for a pulse by calculating pure SHG efficiency as a function of frequency (plane wave, no depletion, ignore all other sum-frequency mixing processes)

$$\eta = \frac{I_{2\omega}}{I_\omega} = \frac{2\omega^2}{\epsilon_0 c^3} \left(\frac{d_{\text{eff}}^2}{n_{2\omega} n_\omega^2} \right) I_\omega L^2 \text{sinc}^2 \left(\frac{\Delta k L}{2} \right) \propto L^2 \text{sinc}^2 \left(\frac{\Delta k L}{2} \right)$$

Different crystal thicknesses



Different crystal materials



- Let's revisit the phase-mismatch (here for SHG – can be generalized):

$$\begin{aligned} \Delta k(\omega) &= \Delta k(\omega_0) + \left. \frac{\partial}{\partial \omega} \Delta k(\omega_0) \right|_{\omega=\omega_0} (\omega - \omega_0) + \dots && \boxed{k_{2\omega}(\omega) = k_{\omega}(2\omega)} \\ &= k_{2\omega}(\omega_0) - 2k_{\omega}(\omega_0) + \left. \frac{\partial k_{2\omega}}{\partial \omega} \right|_{\omega=\omega_0} (\omega - \omega_0) - 2 \left. \frac{\partial k_{\omega}}{\partial \omega} \right|_{\omega=\omega_0} (\omega - \omega_0) + \dots \\ &= k_{2\omega}(\omega_0) - 2k_{\omega}(\omega_0) + 2 \left. \frac{\partial k_{\omega}}{\partial \omega} \right|_{\omega=2\omega_0} (\omega - \omega_0) - 2 \left. \frac{\partial k_{\omega}}{\partial \omega} \right|_{\omega=\omega_0} (\omega - \omega_0) + \dots \\ &= \frac{4\pi\omega_0}{c} \underbrace{(n_{2\omega_0} - n_{\omega_0})}_{\text{Can be made to vanish at typically only one frequency (phase velocities matched)}} + 2(\omega - \omega_0) \underbrace{\left(\frac{1}{v_g(2\omega_0)} - \frac{1}{v_g(\omega)} \right)}_{\text{Vanishes if group velocities match}} + \dots \end{aligned}$$

Vanishes if group velocities match

Can be made to vanish at typically only one frequency (phase velocities matched)

- Δk varies slowly around phasematched frequency (= large bandwidth), if group velocities are matched as well



Considerations for ultrabroadband phase matching

- Ultrabroadband birefringent phase-matching implies very thin crystal
- Very thin crystal implies low conversion efficiency
- Crystal thickness might become comparable with coherence length of other non-phasematched (and possibly unwanted) conversion processes

Point group 3m (e.g. BBO)

$$\begin{pmatrix} P_x^{NL} \\ P_y^{NL} \\ P_z^{NL} \end{pmatrix} = \epsilon_0 \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & d_{16} \\ d_{21} & d_{22} & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_y E_z \\ 2E_z E_x \\ 2E_x E_y \end{pmatrix}$$

Point group $\bar{4}2m$ (e.g. KDP, ADP,..)

$$\begin{pmatrix} P_x^{NL} \\ P_y^{NL} \\ P_z^{NL} \end{pmatrix} = \epsilon_0 \begin{pmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix} \cdot \begin{pmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_y E_z \\ 2E_z E_x \\ 2E_x E_y \end{pmatrix}$$

- May lead to signal background or interference effects
- Can sometimes be avoided by using crystal with higher symmetry (maybe in combination with polarizer to reject signals with 'wrong' polarization)

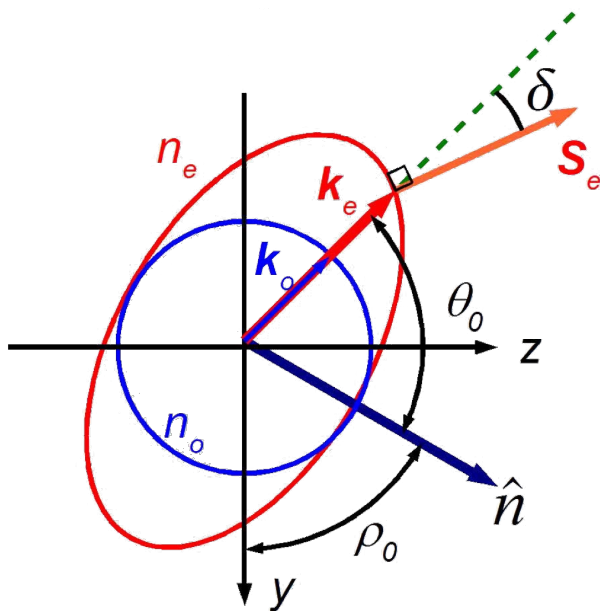
(L. Gallmann et al., *Opt. Lett.* **25** (4), 269 (2000))



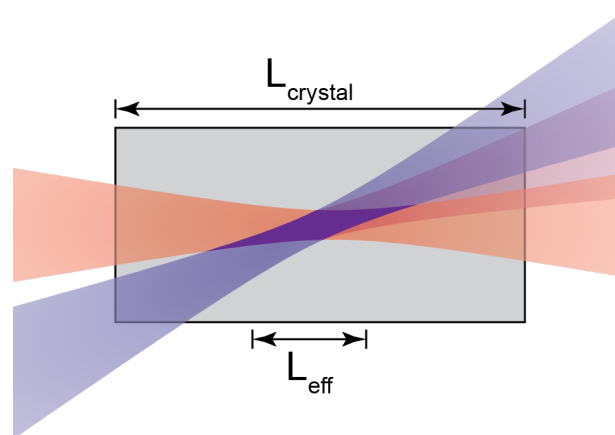
ETH Walk-off effects with real beams and pulses

- Spatial and/or temporal overlap of finite beams and pulses can be limited by the following effects
- This affects conversion efficiency and bandwidth

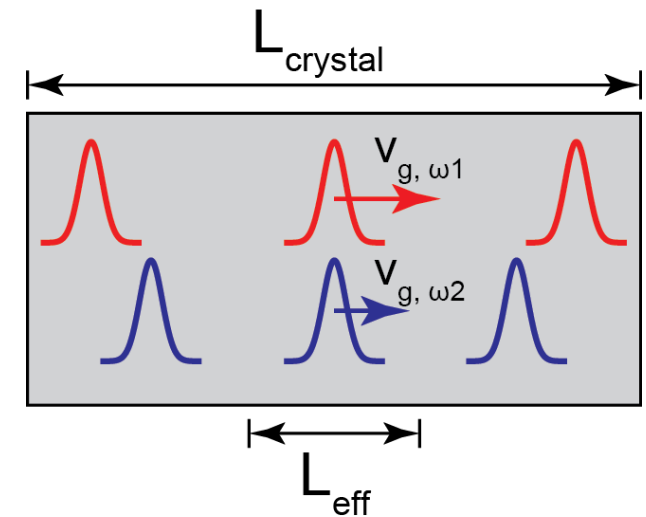
Spatial walk-off



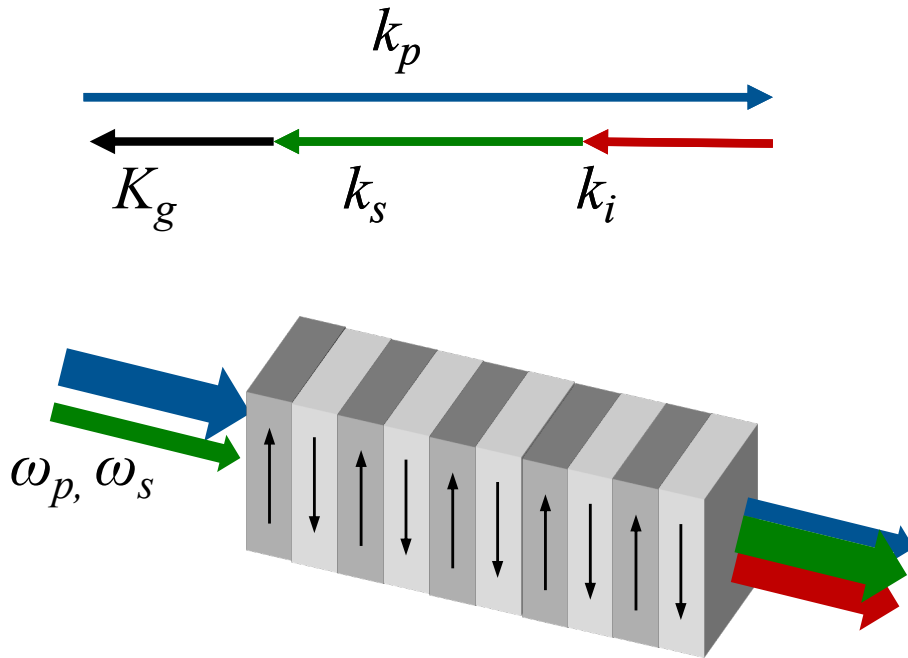
Noncollinear beams



Temporal walk-off (Group velocity mismatch)



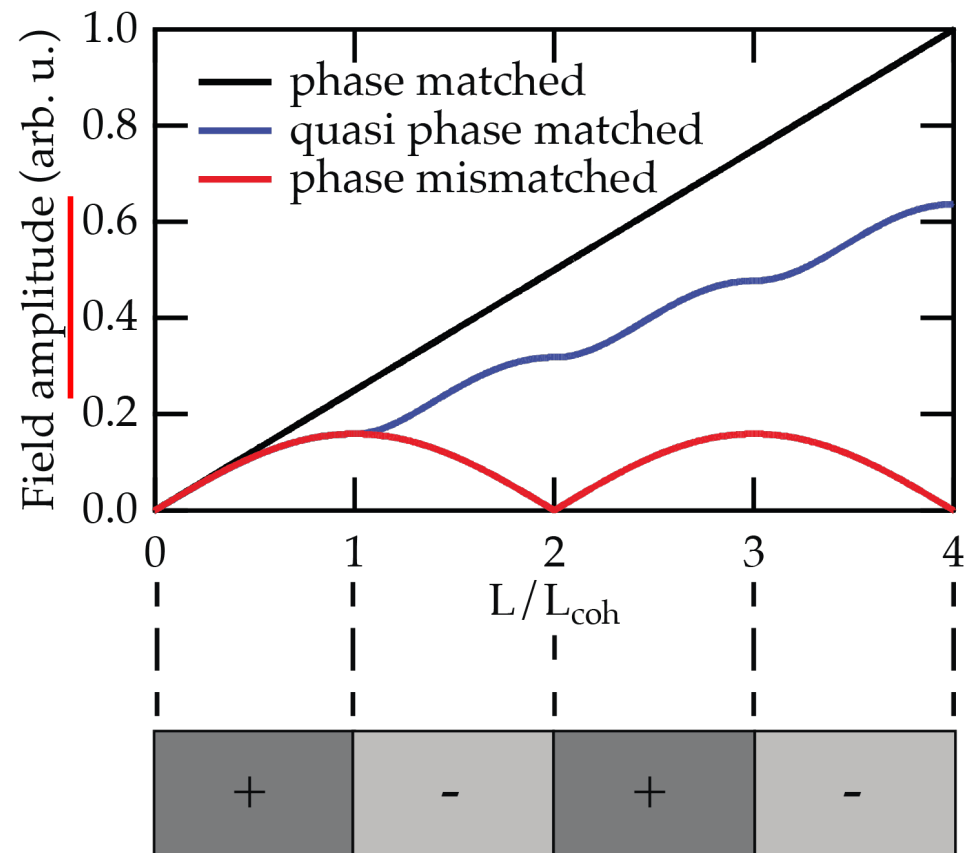
Quasi-phase-matching

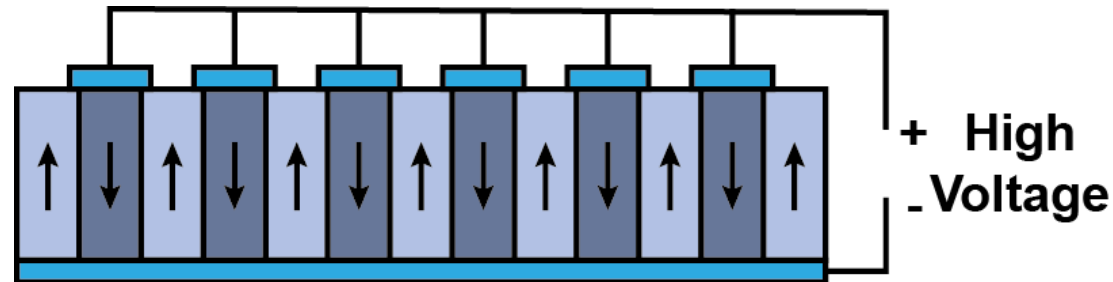


PP LiNbO₃ (PPLN)

- Orientation of ferroelectric domain is reversed every L_c
- This 'poling' changes sign of d_{eff}
- Turns destructive interference to constructive interference

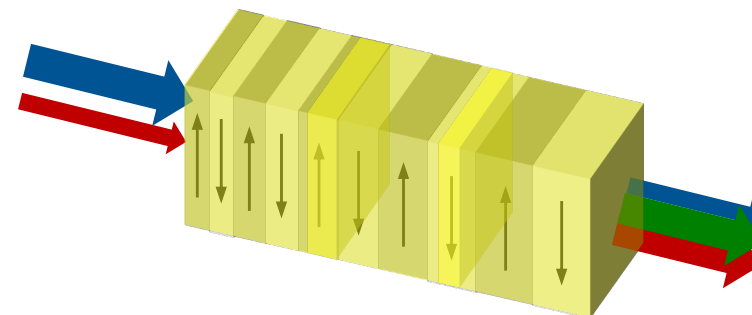
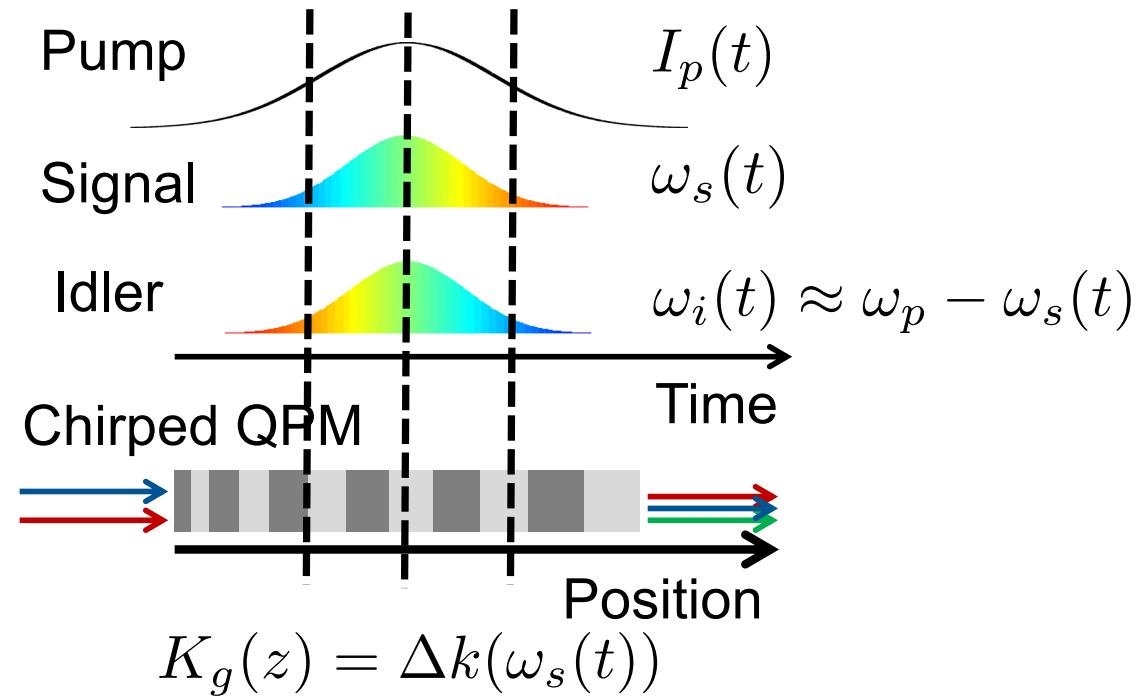
$$L_c := \frac{\pi}{\Delta k}$$





- No birefringence needed for phasematching
 - ⇒ Non-critical phasematching possible without spatial walk-off
- Can phase-match diagonal elements of nonlinear optical susceptibility tensor
 - ⇒ Are typically much larger than off-diagonal elements that are phasematched in birefringent phasematching
- Can phase-match materials that are not phase-matchable otherwise (e.g., GaAs)
- Electrodes for poling are manufactured lithographically, domain lengths are therefore precisely engineerable

How to increase the bandwidth?

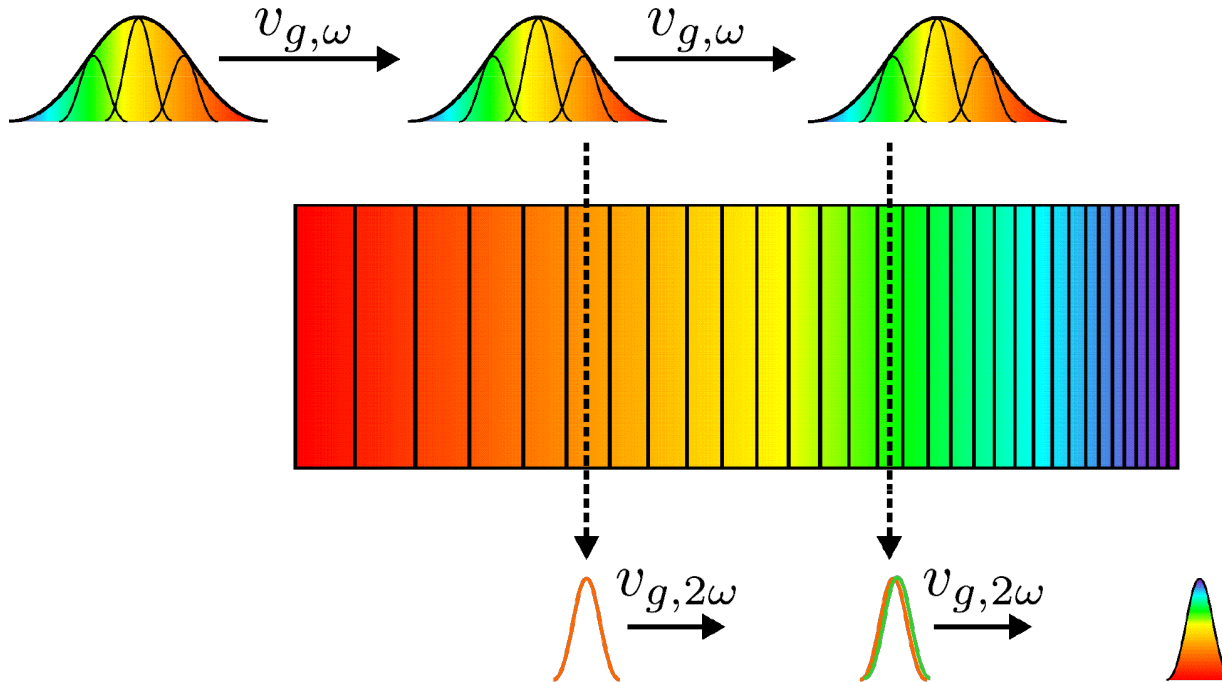


QPM-SHG pulse compression

Group velocity mismatch + spatial localization of conversion



frequency-dependent delay



Group velocity mismatch (GVM):

$$V_{g,\omega} \neq V_{g,2\omega}$$

Assumption: negligible group velocity dispersion (GVD)

compressed SH if pump chirp and grating chirp are matched

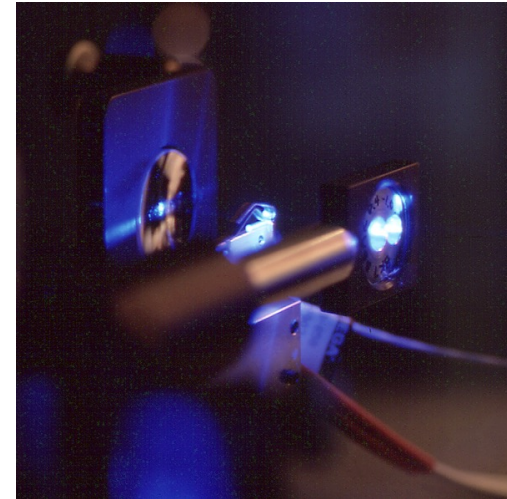
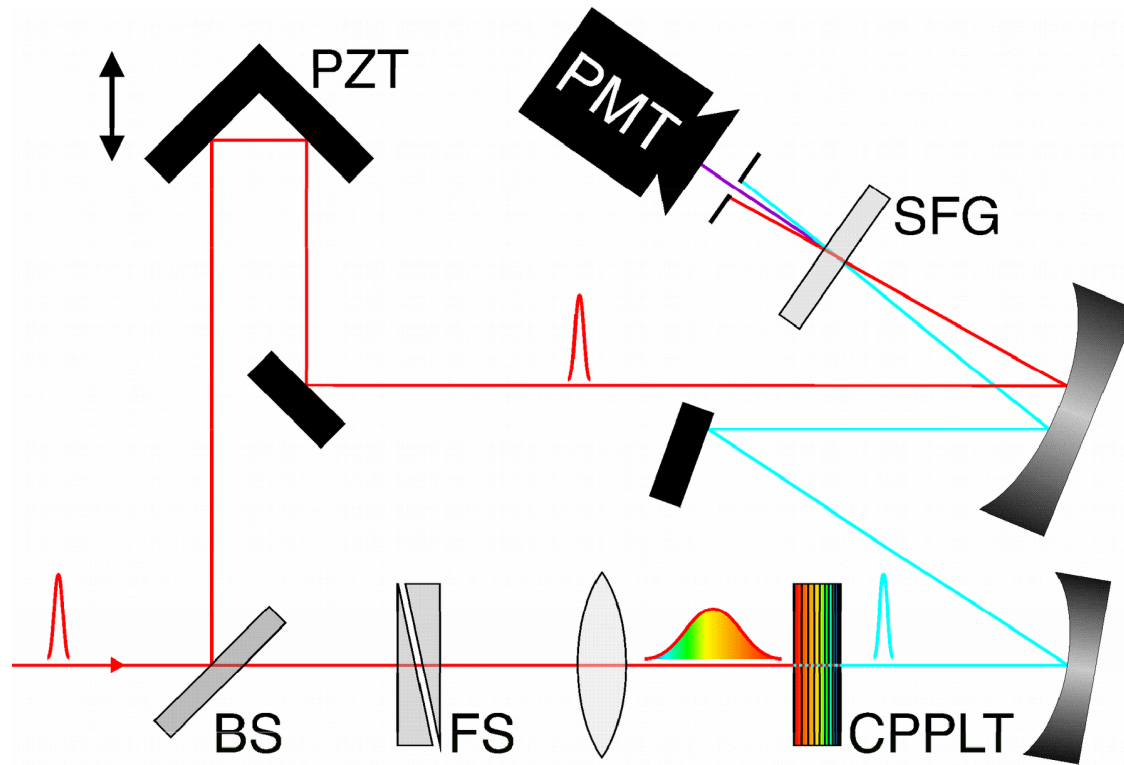
Arbore et al., *Opt. Lett.* **22**, 865 (1997)

Imeshev et al., *J. Opt. Soc. Am. B* **17**, 304 (2000)

- Fundamental and SH chirp engineerable

e.g.: positively chirped fundamental \Rightarrow negatively chirped SH

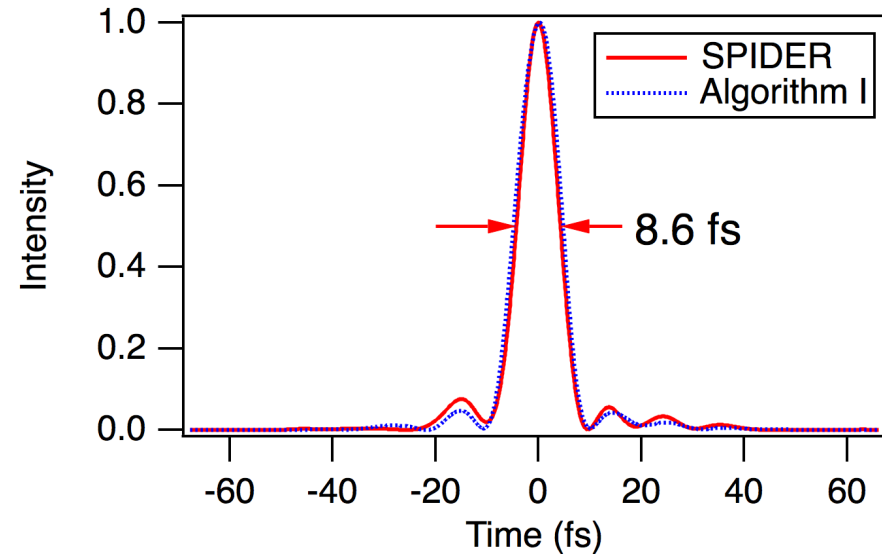
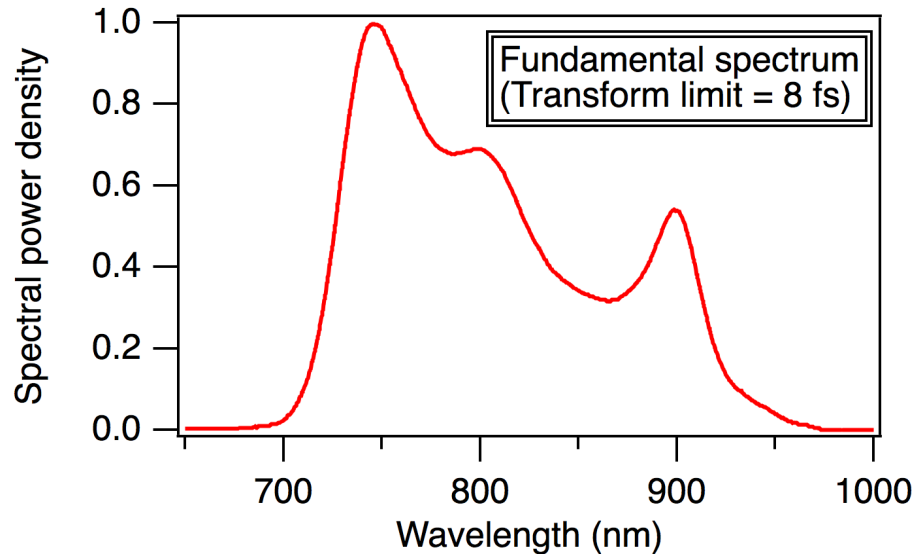




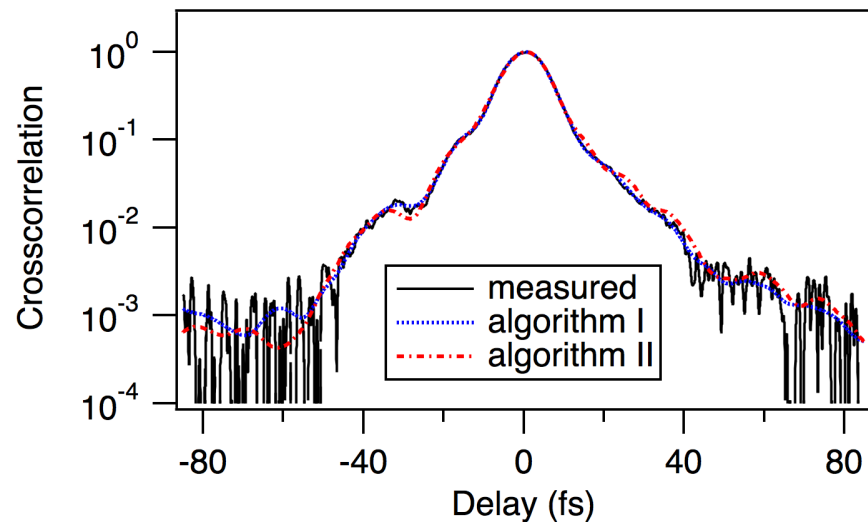
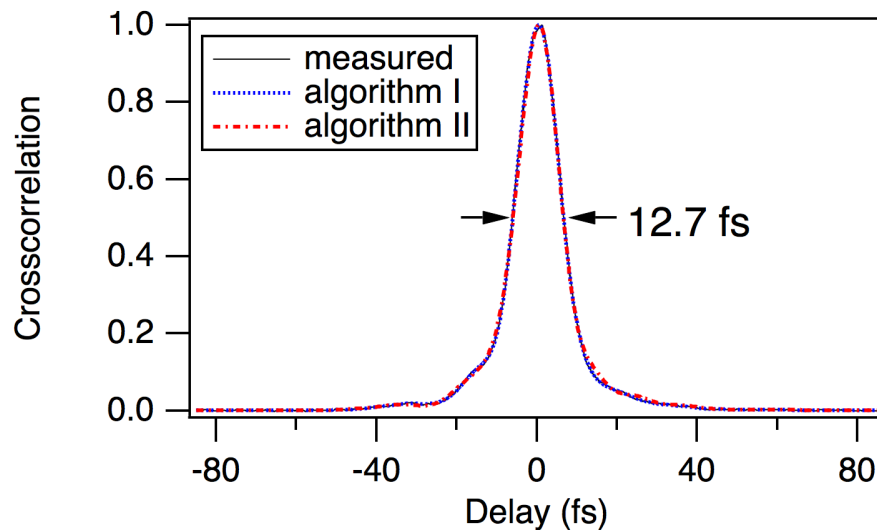
- $< 10 \mu\text{m}$ thick free-standing KDP crystal for crosscorrelation
- Solar-blind PMT for detection

Input pulse & crosscorrelation

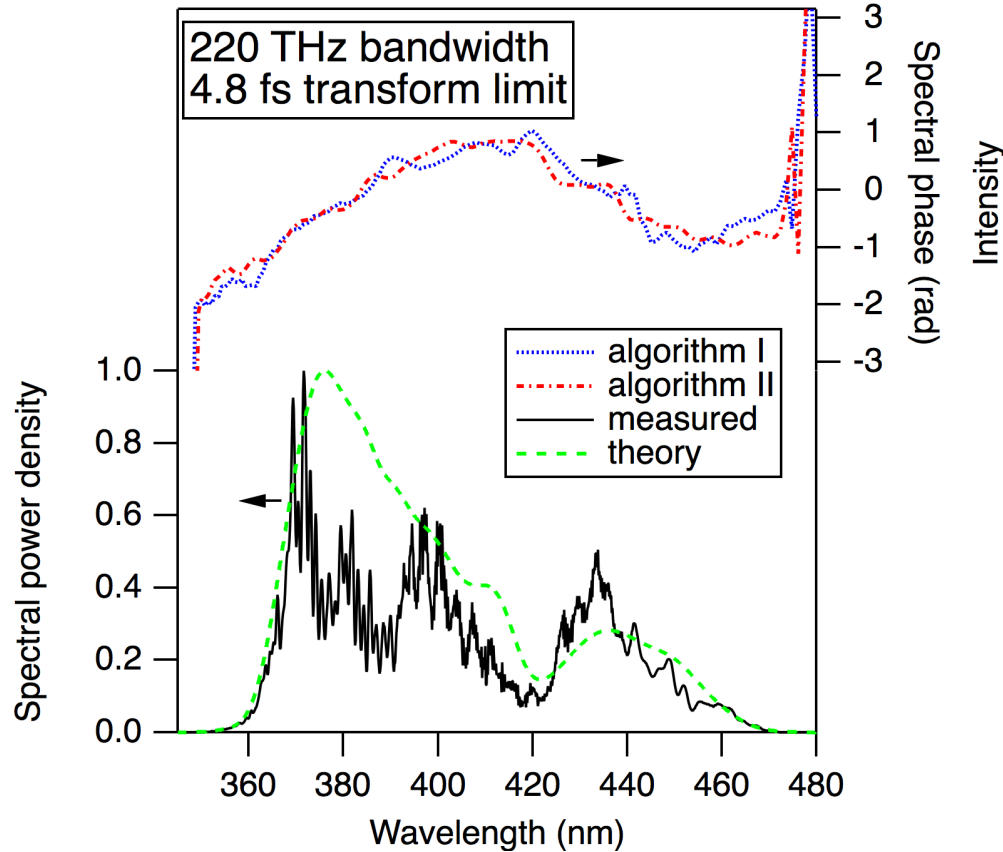
Fundamental pulse



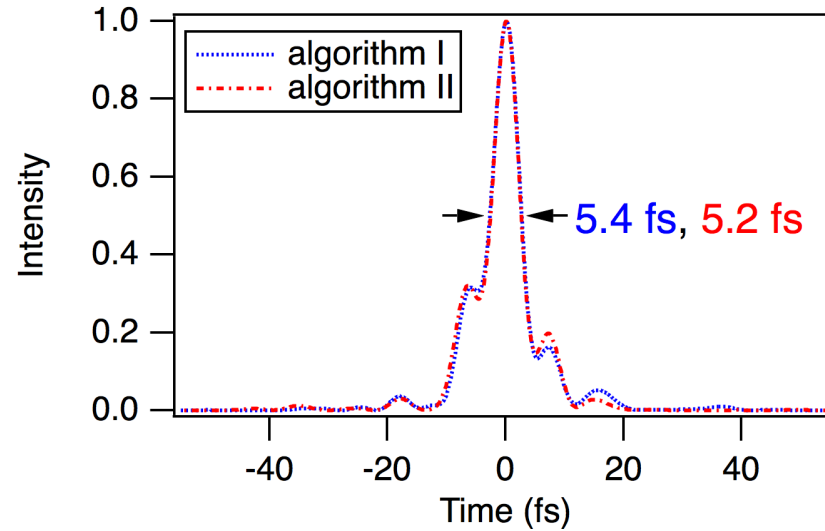
Crosscorrelation



SH spectrum and phase



SH pulse shape

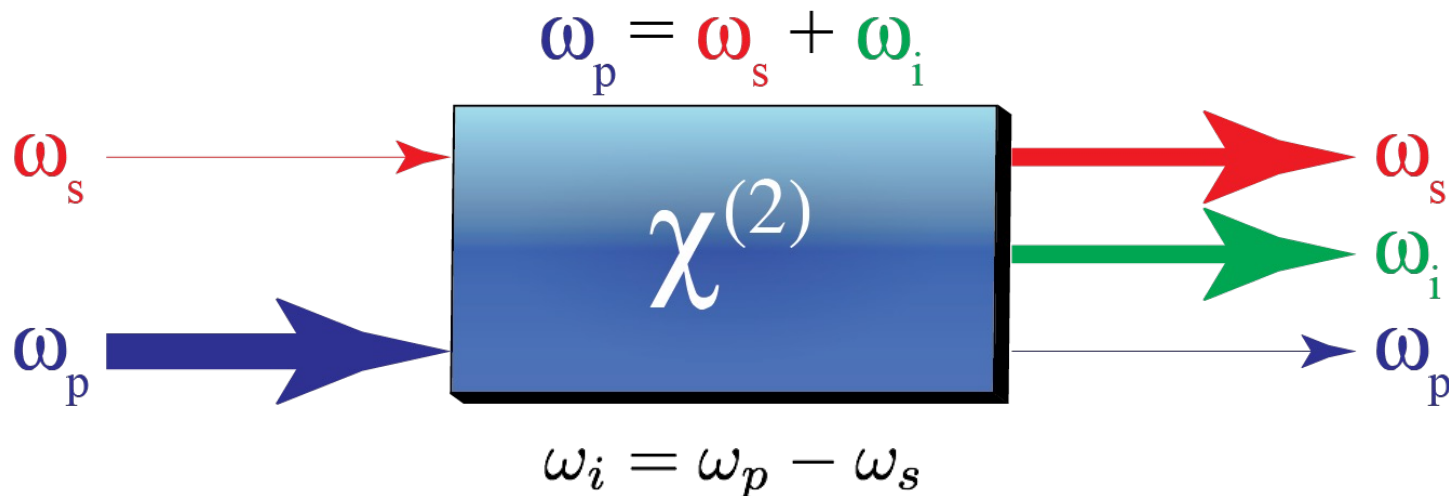


- 90 mW fundamental
=> 0.45 mW SH
- efficiency: 0.45 %/nJ
(theory: 2 %/nJ)

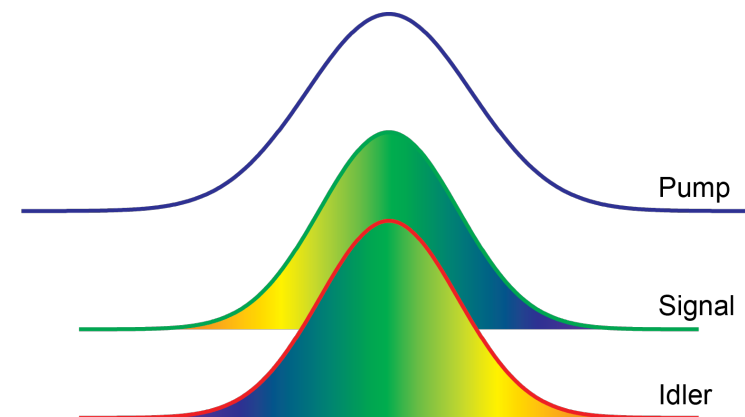
L. Gallmann et al., *Opt. Lett.* **26**, 614 (2001)

Optical parametric amplification (OPA)

- Optical parametric amplification is a special case of difference-frequency generation:
 - Process starts from strong pump wave and weak seed wave
 - Generates strong signal and idler waves (transfers energy from pump)

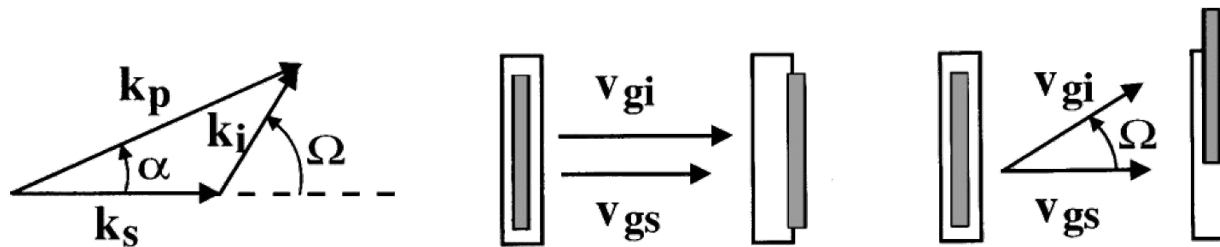
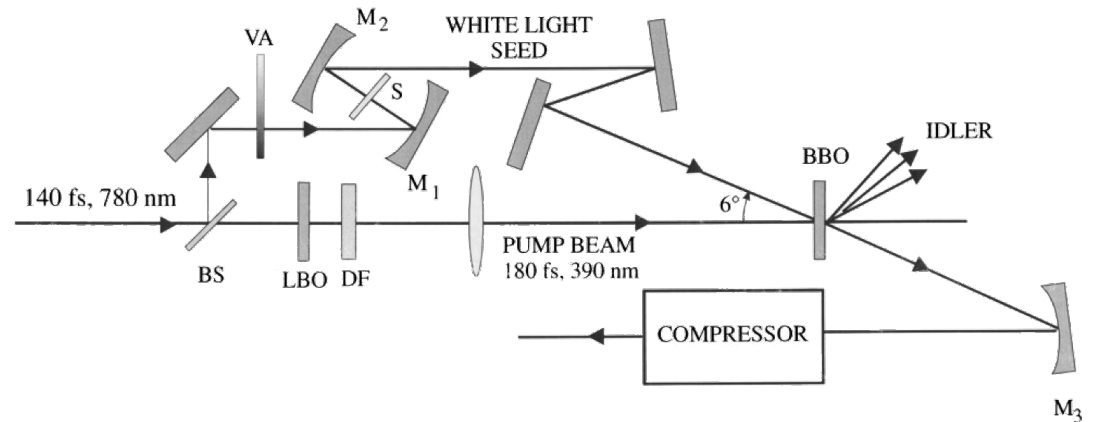
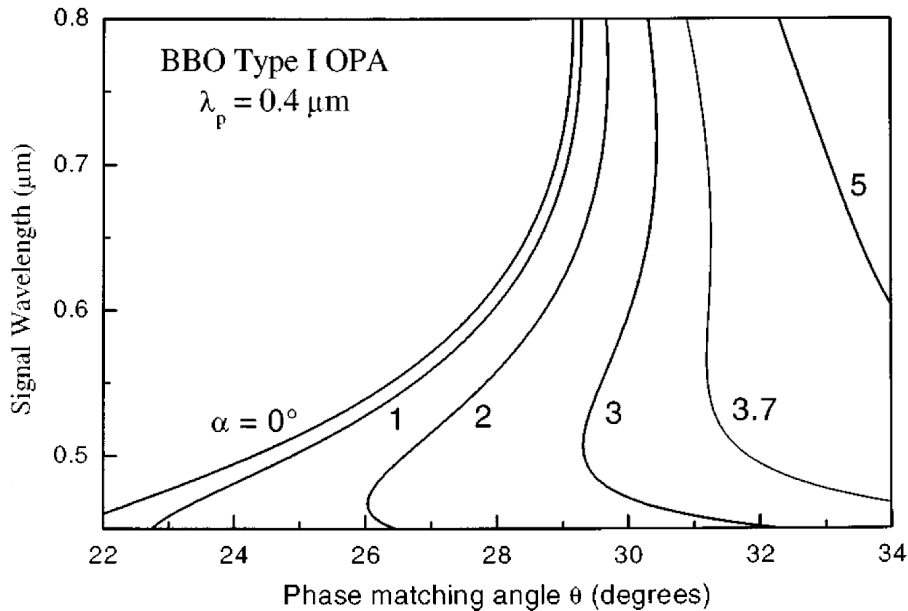


- OPA gain not limited to characteristic atomic or molecular transition (process only needs to be phase-matchable) – can thus provide **gain at “unusual wavelengths”**
- OPA can have **very large single-pass gain** (>50 dB easily possible)
- Amplification happens in OPA through instantaneous process, no energy is stored in crystal
 - ⇒ Pump pulse needs to **temporally coincide** with seed and have **similar pulse duration** (timing critical)
 - ⇒ **Low heat load** in crystal (there are no “non-radiative” transitions involved)
- **Back-conversion** of signal and idler photons into pump photons can happen through sum-frequency mixing (for strong saturation of the pump)
- Gain in OPA is proportional to **pump temporal intensity profile: may reduce bandwidth** of (chirped) amplified pulses



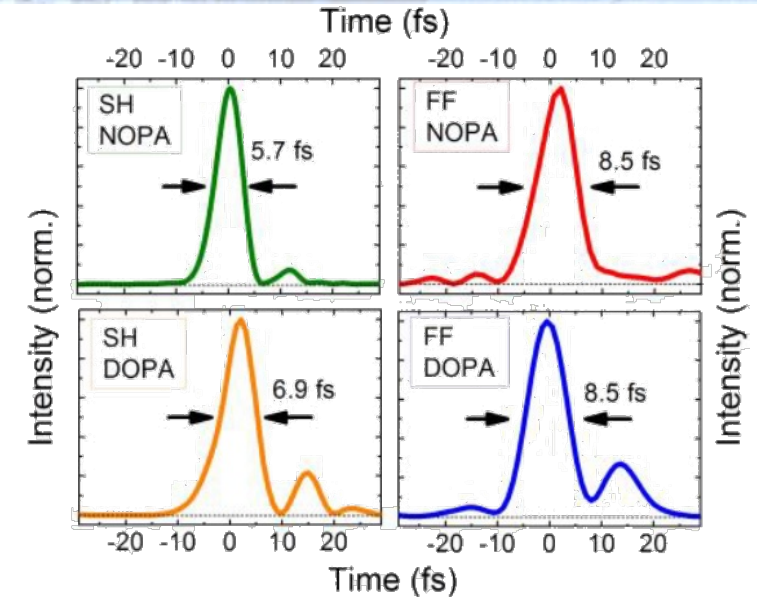
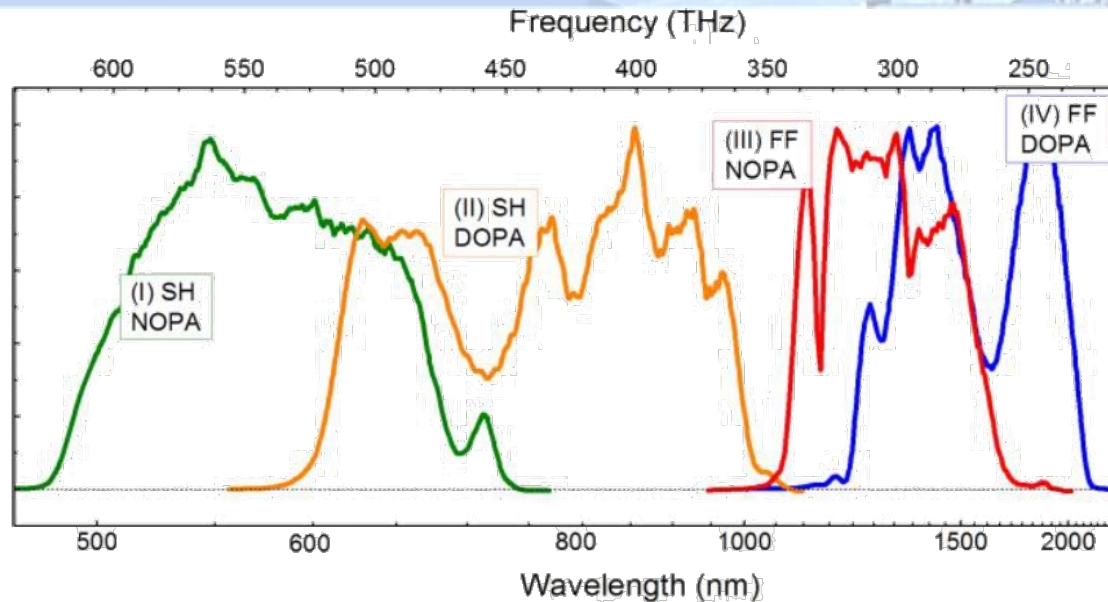
Ultrabroadband, noncollinear OPA

- Ultrabroadband amplification bandwidth can be achieved in a birefringently phase-matched OPA by group-velocity matching through non-collinear geometry
- Famous example: BBO-based OPA pumped at 400 nm, emitting in visible

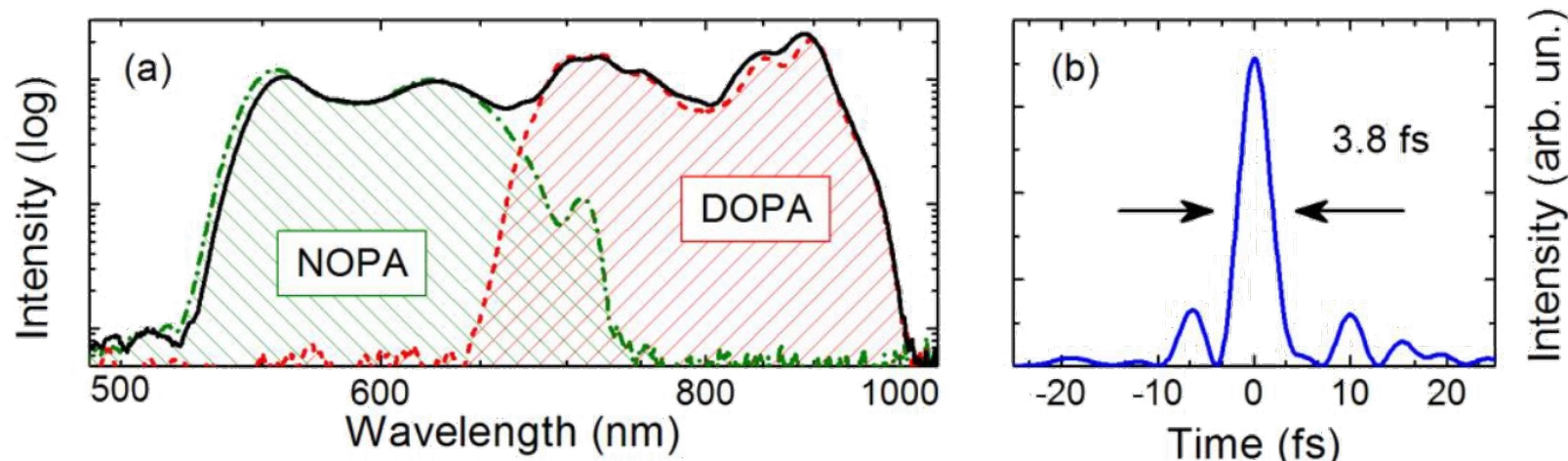


(G. Cerullo et al., *Rev. Sci. Instr.* **74**, 1 (2003))

ETH Waveform synthesis with ultrabroadband OPA

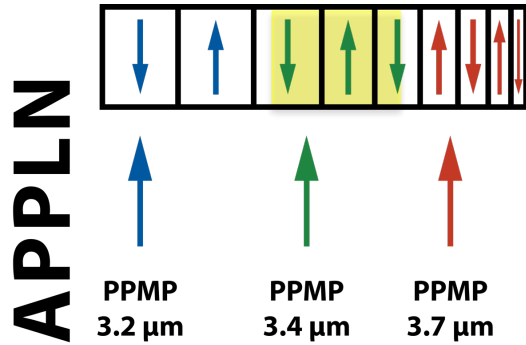


- Combination of two or more amplification channels can yield very short pulses
- Challenge: channels have to be phase stable with respect to each other



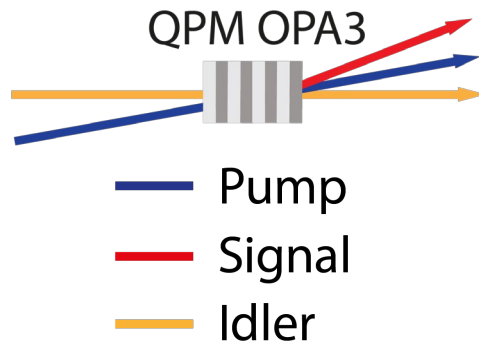
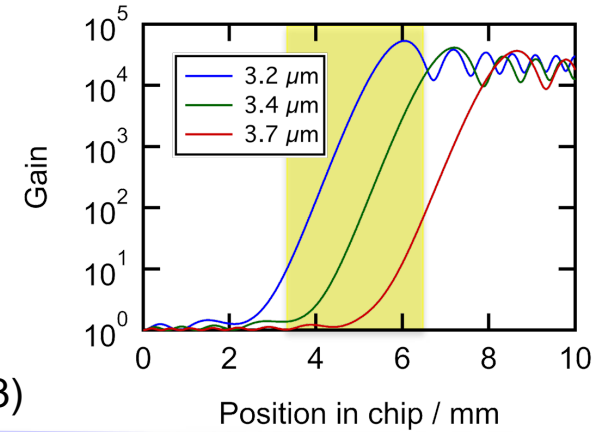
(D. Brida et al., *Nonlinear Optics 2013*, paper NW3A.1 (2013))

Ultrabroadband QPM amplification schemes



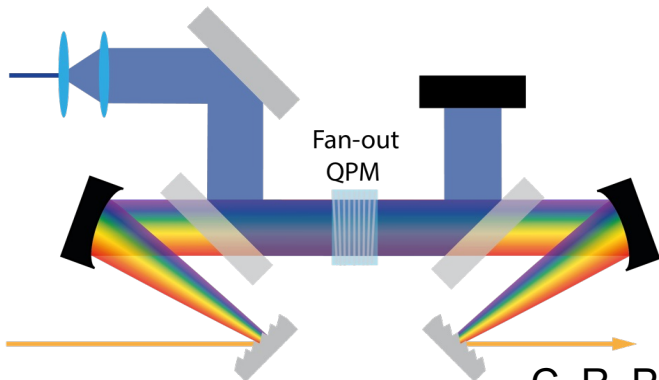
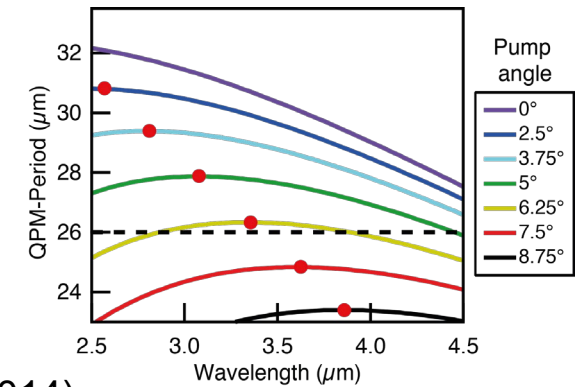
Aperiodic quasi-phase matching

B. W. Mayer et al., *Opt. Lett.* **38**, 4625 (2013)



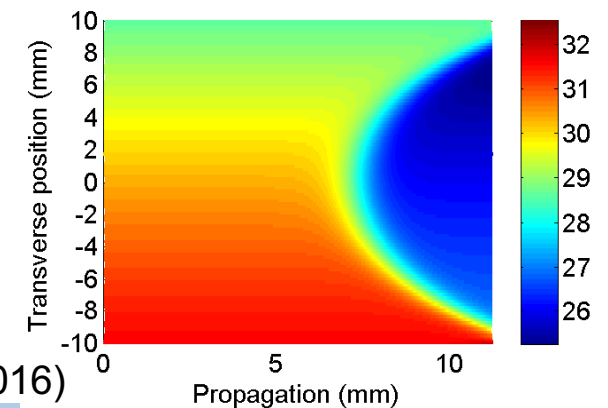
Non-collinear quasi-phase matching

B. W. Mayer et al., *Opt. Express* **22**, 20798 (2014)

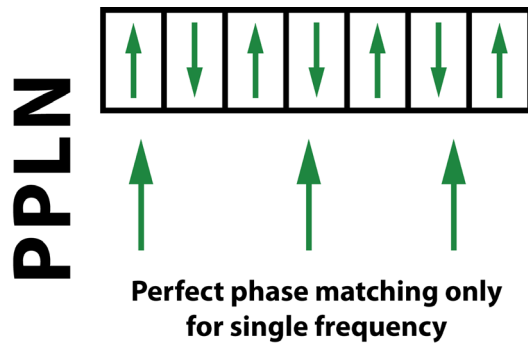


Fourier domain 2D QPM

C. R. Phillips et al., *Opt. Express* **24**, 15940 (2016)

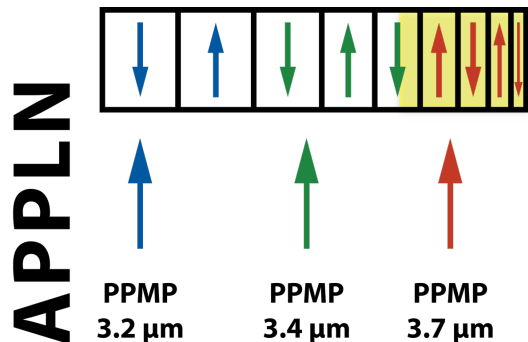


Concept of amplification in APPLN

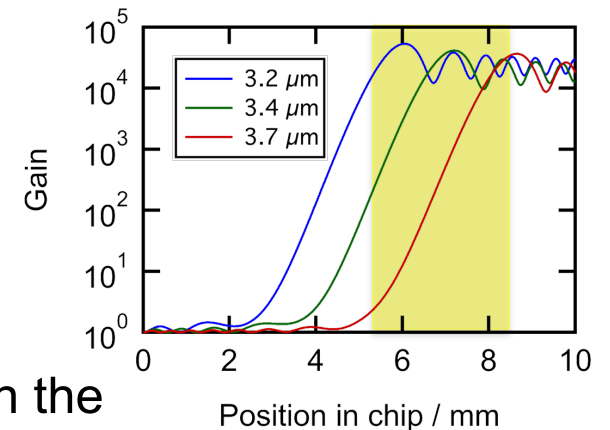
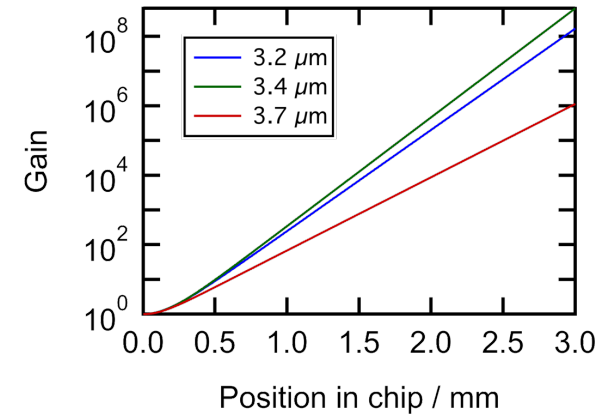


$$K_g = \text{const.}$$

Phase matching for single wavelength



$$K_g = K_g(z) = \underbrace{\kappa' \cdot z + \text{const.}}_{\text{linearly chirped grating}}$$



Each spectral component is amplified in a limited region in the grating around its phase matching point (PMP)

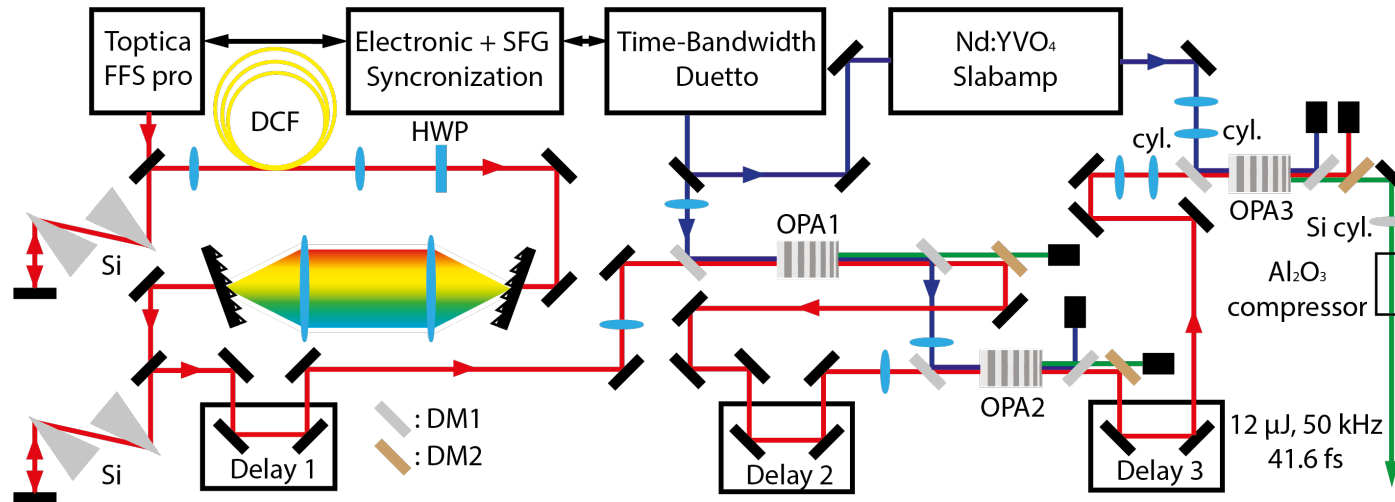
- Potential for very broadband phase matching bandwidths and customized gain profiles^{1,2}
- Potential for high conversion efficiency³

¹M. Charbonneau-Lefort, B. Afeyan, M. M. Fejer, *J. Opt. Soc. Am. B* **25**, 1402 (2008)

²C. R. Phillips, L. Gallmann, M. M. Fejer, *Opt. Express* **21**, 10139 (2013)

³C. R. Phillips, C. Langrock, D. Chang, Y. W. Lin, L. Gallmann, M. M. Fejer, *J. Opt. Soc. Am. B* **30**, 1551 (2013)

1.56- μm seeded femtosecond mid-infrared pulse generation



- Seed at 1.56 μm signal wavelength
- Dispersion compensation (stretcher/compressor) mainly on 1.5 μm – NIR seed side
- All-collinear setup
- Idler pulse compression with bulk material (sapphire)
- 3.4 μm idler extracted at last amplification stage

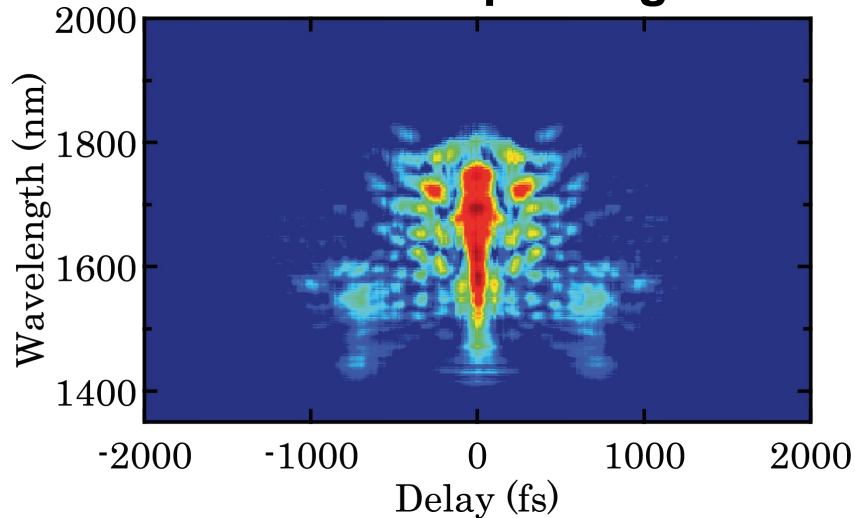
¹B. W. Mayer, C. R. Phillips, L. Gallmann, M. M. Fejer, U. Keller, *Opt. Lett.* **38**, 4625 (2013)

²C. Heese, A. E. Oehler, L. Gallmann, U. Keller, *Appl. Phys. B* **103**, 5 (2011)

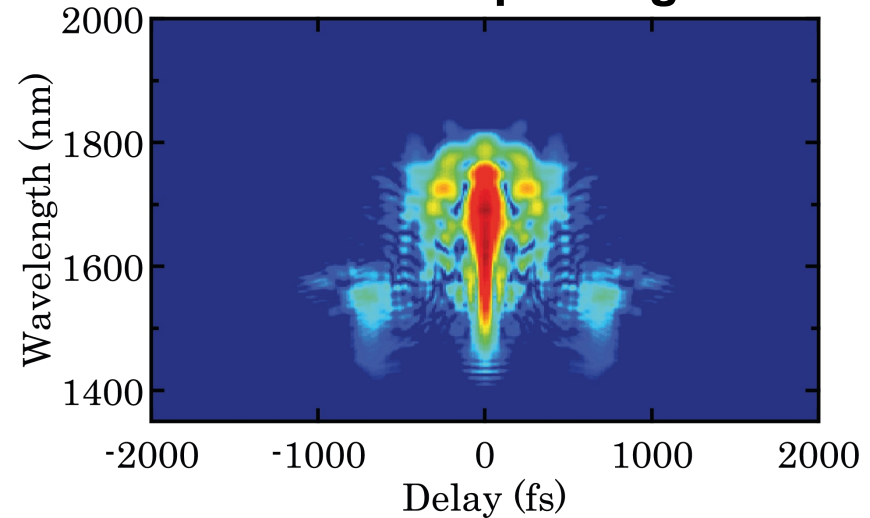
³C. R. Phillips, B. W. Mayer, L. Gallmann, M. M. Fejer, U. Keller, *Opt. Express* **22**, 9327 (2014)

Measured pulses with SHG FROG

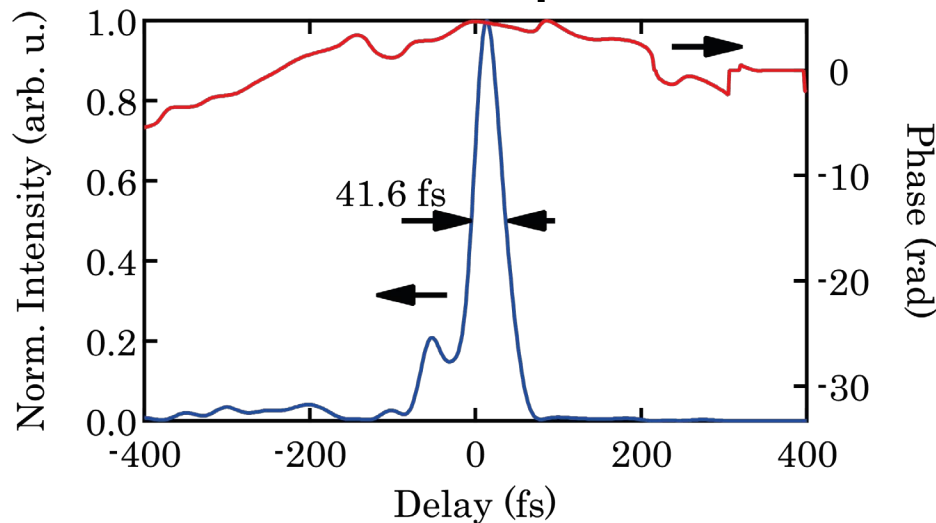
Measured spectrogram



Retrieved spectrogram



Retrieved pulse



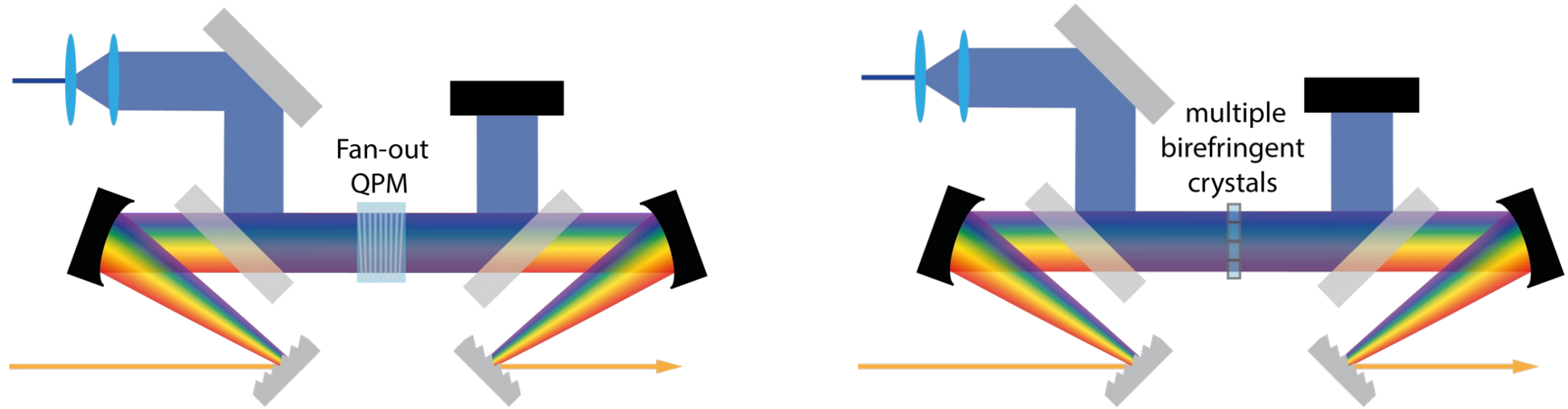
Output parameters

- Pulse energy: **12 μJ**
- Repetition rate: **50 kHz**
- Pulse duration: **41.6 fs**
- Center wavelength: **3.4 μm**
- Energy inside of crystal: **19 μJ**
- Int. conversion efficiency: **24.5%**

- Single cycle corresponds to 11.3 fs at 3.4 μm center wavelength

B. W. Mayer, C. R. Phillips, L. Gallmann, M. M. Fejer, U. Keller, *Opt. Lett.* **38**, 4625 (2013)

Fourier-domain OPA



- Concept proposed with fan-out PPLN:

- L. Chen, *et al.* "Ultrabroadband optical parametric chirped-pulse amplifier using a fan-out periodically poled crystal with spectral spatial dispersion" PRA **82**, 043843 (2010)

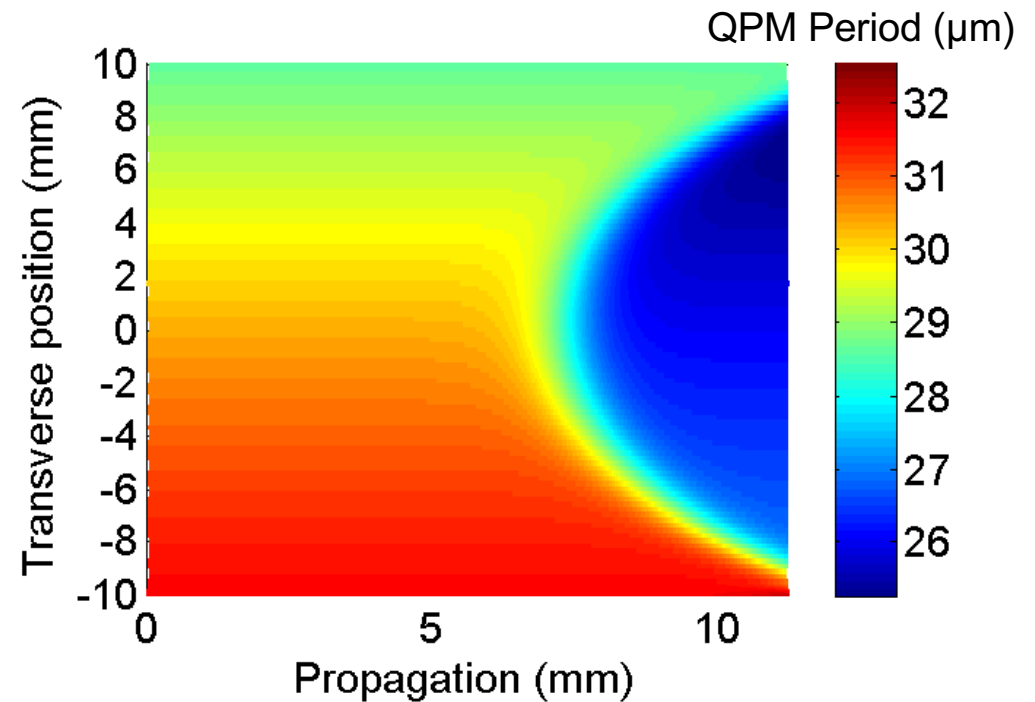
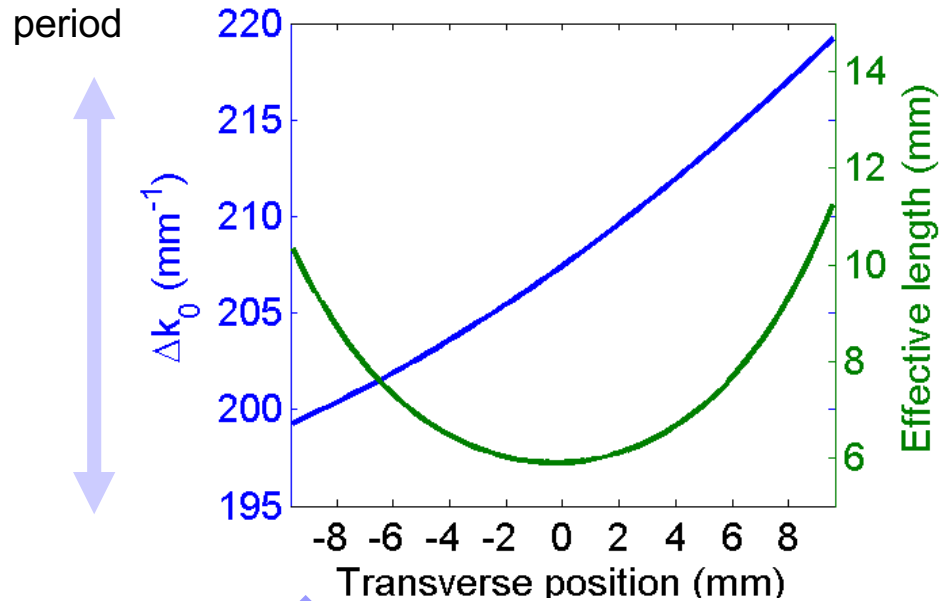
- Experiment with multiple birefringent phase-matching crystals:

- B. E. Schmidt *et al.* "Frequency domain optical parametric amplification," Nat. Commun. **5** (2014)

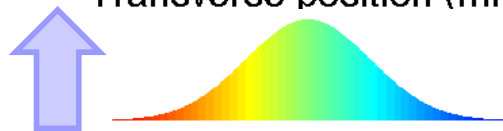
What can we do with a more general structure than conventional straight-domain fan-out?

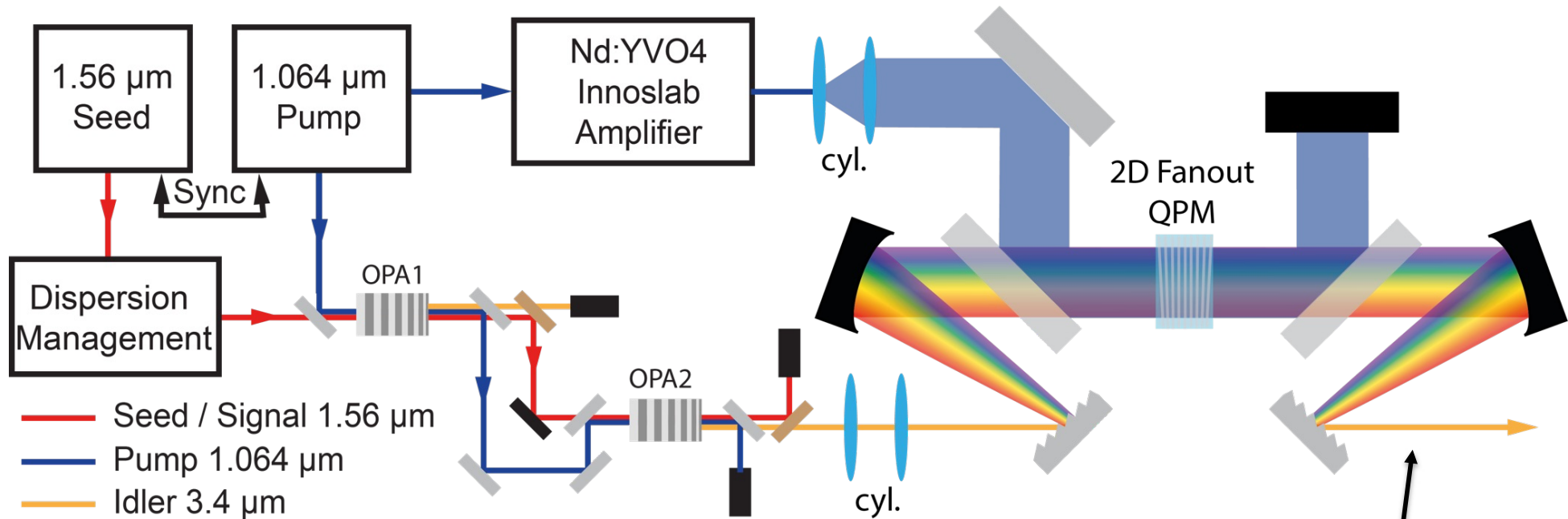
- Choose optimal QPM period trajectory vs. transverse position
- Vary interaction length for different frequencies (accounting for a gaussian transverse intensity pump beam profile)
 - Still monolithic / plane-parallel / non-critical phase-matching crystal

Varying QPM period



Propagation direction



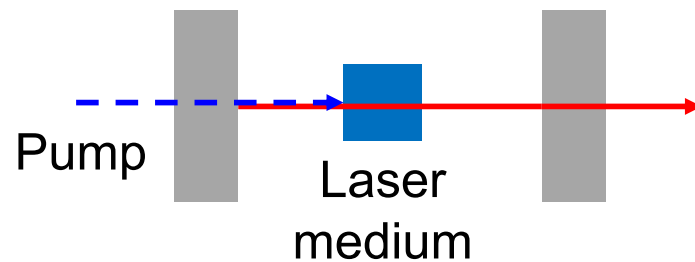


- Output power
 - **33 μJ (1.65 W @ 50 kHz)** after QPM grating
 - Corresponding to a **photon conversion efficiency of 32%**
 - **21 μJ (1.05 W @ 50 kHz)** including losses from the diffraction grating

C. R. Phillips et al., *Opt. Express* **24**, 15940 (2016)

Laser

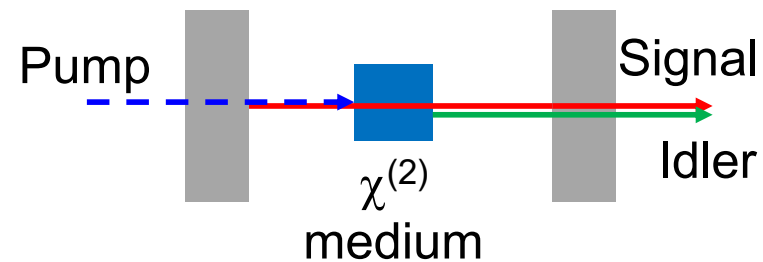
- Optical cavity
- Gain from stimulated emission
- “Pump” photons create a population inversion



$$\omega_{pump} > \omega_{laser}$$

OPO

- Optical cavity
- Gain from parametric amplification
- “Pump” photons converted directly into “signal” photons

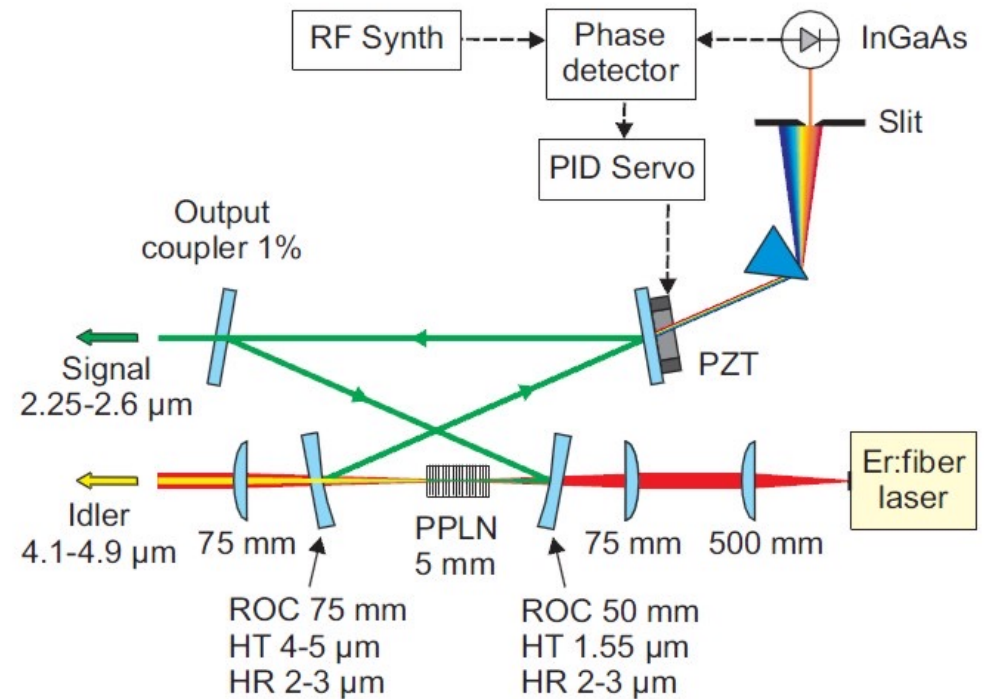


$$\omega_{pump} = \omega_{signal} + \omega_{idler}$$

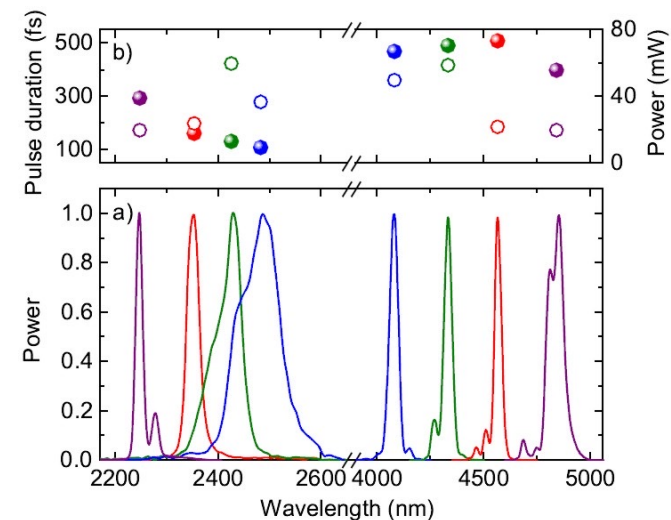
An OPO lets us generate light at wavelengths where we don't have a laser available

ETH Ultrafast optical parametric oscillator (OPO)

- Parametric amplification can also be used inside oscillators (i.e., with feedback from cavity)
- Instantaneous gain process: pump pulse train needs to be synchronized with pulse oscillating in cavity
- Benefit: source is not limited to existing laser transitions → OPOs are often widely tunable in wavelength
- OPOs are available from UV to far infrared

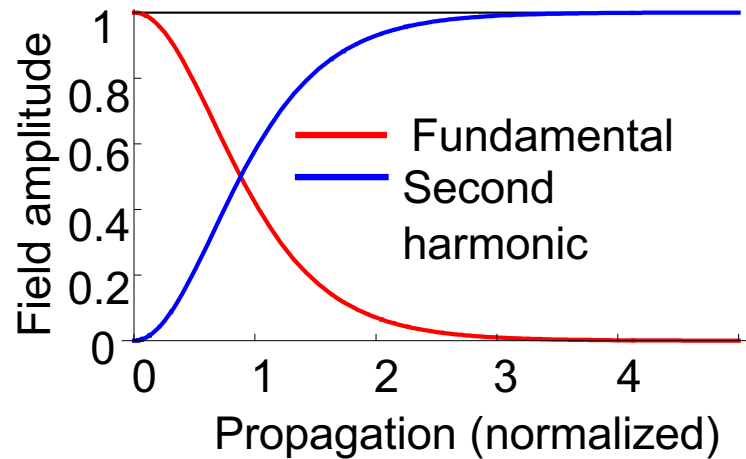


(image source: https://www.fisi.polimi.it/en/research/research_structures/laboratories/54057)



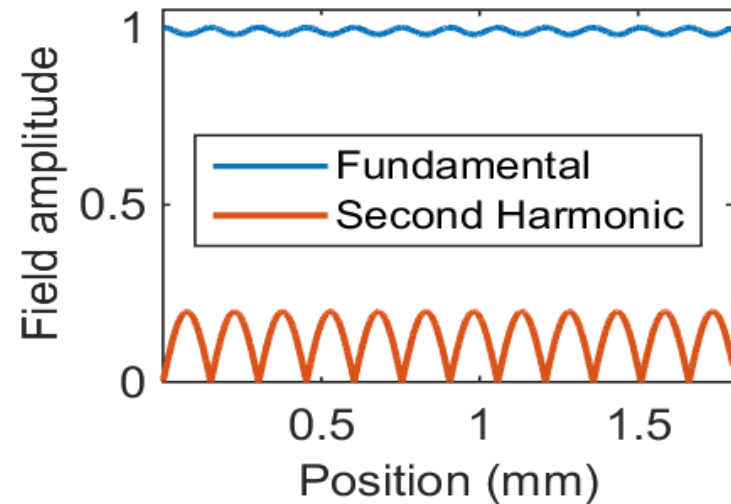
Normal cw SHG:

- Phase-matched, $\Delta k=0$
- Energy transfer from fundamental to SH



Cascaded quadratic nonlinearities:

- Large SHG phase-mismatch Δk
- Periodic transfer of energy



$$\frac{dE_{2\omega}}{dz} = -i \frac{\omega d_{\text{eff}}}{n_{2\omega} c} E_{\omega}^2 e^{i\Delta k z} \quad \Rightarrow \quad E_{2\omega} \sim -\frac{1}{\Delta k} \frac{\omega d_{\text{eff}}}{n_{2\omega} c} E_{\omega}^2 e^{i\Delta k z} \quad \Rightarrow \quad \frac{dE_{\omega}}{dz} = i \frac{1}{\Delta k} \frac{\omega^2 d_{\text{eff}}^2}{n_{\omega} n_{2\omega} c^2} |E_{\omega}|^2 E_{\omega}$$

$$\frac{dE_{\omega}}{dz} = -i \frac{\omega d_{\text{eff}}}{n_{\omega} c} E_{2\omega} E_{\omega}^* e^{-i\Delta k z}$$

Effective $\chi^{(3)}$ whose sign depends on $\Delta k!$

Solitons in the positive GDD regime