Ultrafast Laser Physics

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Chapter 5: Relaxation oscillations in lasers
Diode-pumped solid-state lasers

Longitudinal pumping

Transversal pumping
Diode-pumped solid-state lasers

[Diagram showing layers of material with labels like proton implant, metal contact, p-GaAs (cap), p-GaAlAs, GaAlAs (active layer), n-GaAlAs, n-GaAs (substrate), and metal contact.

Graph showing absorption and emission spectra with peaks at different wavelengths.


Wavelength (nm) on the x-axis from 300 to 900, Absorption (OD) on the y-axis from 0.0 to 0.6.]
Diode-pumped solid-state lasers

Three-level system

Yb-doped

Four-level system

Nd-doped

Ti:sapphire
Laser rate equation

- ideal four-level system
- single mode laser
- homogeneous broadened gain material

\[ N \text{ inversion } N = N_2 - N_1, \quad N_1 \approx 0 \]

\( n \) photon number (intracavity)

\[ W_{stim} \equiv W_{12} = W_{21} = Kn \]

\( KN \) spontaneous emission rate

\( KnN \) stimulated emission rate

\( \gamma_c \) cavity photon decay rate

\( \gamma_L \) spontaneous decay rate of level 2

\[
\frac{dn}{dt} = K(n + 1)N - \gamma_c n
\]

\[
\frac{dN}{dt} = R_p - KnN - \gamma_L N
\]
Above threshold, i.e. $r > 1$

\[ \begin{align*}
  n_s &\approx \frac{\tau_c}{\tau_L} N_{th} (r - 1) = \frac{\gamma_L}{K} (r - 1) \\
  N_s &\approx N_{th} = \frac{\gamma_c}{K}
\end{align*} \]

Below threshold, i.e. $r < 1$

\[ \begin{align*}
  n_s &= \frac{r}{1-r} \\
  N_s &\approx r N_{th}
\end{align*} \]
Linearized rate equations

\[ n(t) = n_s + n_1(t), \quad n_1(t) \ll n_s \]
\[ N(t) = N_s + N_1(t), \quad N_1(t) \ll N_s \]

\[ \frac{dn_s}{dt} = 0, \quad \frac{dN_s}{dt} = 0 \]

\[ \frac{dn_1(t)}{dt} = \gamma_L (r - 1) N_1(t) \]

\[ \frac{dN_1(t)}{dt} = -\gamma_L r N_1(t) - \gamma_c n_1(t) \]

Solution: Ansatz

\[ n_1(t) \propto e^{st}, \quad N_1(t) \propto e^{st} \]

\[ s = s_{1,2} = -\frac{r \gamma_L}{2} \pm \sqrt{\left(\frac{r \gamma_L}{2}\right)^2 - \gamma_L \gamma_c (r - 1)} \]
Over-critical damping (no relaxation oscillations)

\[ n(t) = n_s + n_1(t), \quad n_1(t) \ll n_s \]
\[ N(t) = N_s + N_1(t), \quad N_1(t) \ll N_s \]

Solution: Ansatz
\[ n_1(t) \propto e^{st}, \quad N_1(t) \propto e^{st} \]

\[ s = s_{1,2} = -\frac{r\gamma_L}{2} \pm \sqrt{\left(\frac{r\gamma_L}{2}\right)^2 - \gamma_L\gamma_c(r - 1)} \]

s is real and negative:
\[ \left(\frac{r\gamma_L}{2}\right)^2 > \gamma_L\gamma_c(r - 1) \]

two solutions: two over-critically damped relaxation constants:

\[ r\gamma_L \gg \gamma_c \quad \Rightarrow \quad \begin{cases} s_1 = -r\gamma_L \\ s_2 = -\gamma_c \frac{r - 1}{r} \end{cases} \]

Stimulated decay rate of excited atoms
Photon decay rate inside laser cavity

\[ s_2 = -\gamma_c \frac{r - 1}{r} \quad \Rightarrow \quad s_2 \approx -\gamma_c, \quad \text{for} \quad r \gg 1 \]
Example HeNe Laser (p. 10)

$\lambda = 632.8$ nm, $\tau_L \approx 100$ ns, $2l = 0.02$ (2% output coupler), $T_R = 2$ ns

$$\Rightarrow \quad \tau_c = \frac{T_R}{2l} \approx 100 \text{ ns} \quad \Rightarrow \quad \tau_L \approx \tau_c, \quad \text{or} \quad \gamma_L \approx \gamma_c$$

For $r \gg 1$ (when HeNe is pumped sufficiently far above threshold)

Condition for over-critical damping is fulfilled:

$$r \gamma_L \gg \gamma_c$$

HeNe laser falls back into steady state after about 100 ns
Under-critical damping (relaxation oscillations)

\[ n(t) = n_s + n_1(t), \quad n_1(t) \ll n_s \]
\[ N(t) = N_s + N_1(t), \quad N_1(t) \ll N_s \]

Solution: Ansatz

\[ n_1(t) \propto e^{st}, \quad N_1(t) \propto e^{st} \]

\[ s = s_{1,2} = -\frac{r\gamma_L}{2} \pm \sqrt{\left(\frac{r\gamma_L}{2}\right)^2 - \gamma_L\gamma_c(r - 1)} \]

s is complex:

\[ \left(\frac{r\gamma_L}{2}\right)^2 < \gamma_L\gamma_c(r - 1) \]

relaxation oscillations with an attenuation factor:

\[ n(t) = n_s + n_1 e^{-\gamma_{relax} t} \cos(\omega_{relax} t) \]
\[ \gamma_{relax} = \frac{r\gamma_L}{2} \]

\[ \gamma_c \gg r\gamma_L: \quad \omega_{relax} \approx \sqrt{\frac{r - 1}{\tau_L} \frac{1}{\tau_c}} = \sqrt{\frac{1}{\tau_{stim}} \frac{1}{\tau_c}} \]
Relaxation oscillations in the time domain and frequency domain (microwave spectrum analyzer - not optical frequencies!)

\[ n(t) = n_s + n_1 e^{-\gamma_{relax} t} \cos(\omega_{relax} t) \]

\[ \gamma_{relax} = \frac{r \gamma L}{2} \]

\[ \gamma_c \gg r \gamma L : \quad \omega_{relax} \approx \sqrt{\frac{r - 1}{\tau_L} \frac{1}{\tau_c} \tau_{stim} \frac{1}{\tau_c}} \]
Relaxation oscillations

Example: diode-pumped cw Nd:YLF laser

![Diagram of a diode-pumped cw Nd:YLF laser setup]

\[ f_{\text{relax}} = 116 \text{ kHz} \]
Measurement of small signal gain

\[ g = \frac{g_0}{1 + 2I/I_{sat}} \]

\[ f_{relax} \approx \frac{1}{2\pi} \sqrt{\gamma_L \gamma_c (r - 1)} = \frac{1}{2\pi} \sqrt{\frac{r - 1}{\tau_L \tau_c}} = \frac{1}{2\pi} \sqrt{\frac{g_0}{l} - 1} = \frac{1}{2\pi} \sqrt{\frac{2g_0 - 2l}{\tau_L T_R}} \]

\[ \tau_c = \frac{T_R}{2l} \]

\[ 2g_0 \approx 4\pi^2 \tau_L T_R f_{relax}^2 + 2l \]

\[ r = \frac{g_0}{l} \]

\[ G_0 \approx e^{2g_0} = \exp \left( 4\pi^2 \tau_L T_R f_{relax}^2 + 2l \right) \]
**Measurement of small signal gain**

Measured relaxation oscillation frequency for different output couplers, for \( g_0 \gg l \)

\[
f_{\text{relax}} \approx \frac{1}{2\pi} \sqrt{\frac{2g_0 - 2l}{\tau_L T_R}} \quad \text{as} \quad g_0 \gg l \rightarrow \approx \frac{1}{2\pi} \sqrt{\frac{2g_0}{\tau_L T_R}}
\]

relaxation oscillations independent of output coupler

\[
2g_0 \approx 4\pi^2 \tau_L T_R f_{\text{relax}}^2 + 2l
\]

\[
G_0 \approx e^{2g_0} = \exp \left(4\pi^2 \tau_L T_R f_{\text{relax}}^2 + 2l\right)
\]

Example: diode-pumped Nd:YLF laser