SESAM modelocked solid-state lasers

Lecture 1
Passive modelocked solid-state lasers

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Ultrafast Optics 2013
Davos, Switzerland, March 2-8, 2013

Outline of Lectures

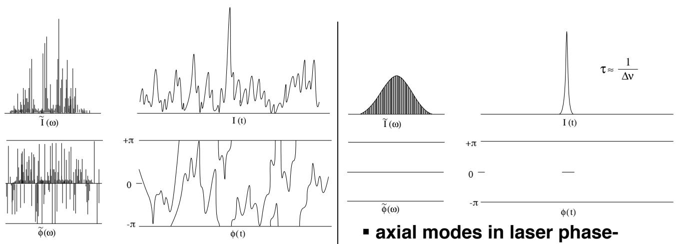
- **Lecture 1:** Passive modelocked solid-state lasers
- **Lecture 2:** SESAMs

- **Useful references:**
Best review with extended tables for different lasers
U. Keller, **Ultrafast solid-state lasers,** Landolt-Boernstein, Group VIII/1B1, Laser Physics and Applications. Subvolume B: Laser Systems. Part 1. Edited by G. Herziger, H. Weber, R. Proprawe, Springer-Verlag, Berlin, Heidelberg, New York, October, pp. 33-167, 2007
ISBN 978-3-540-26033-2
free download from http://www.ulp.ethz.ch/research/UltrafastSolidStateLasers

20 years of ultrafast solid-state lasers: a personal review
U. Keller, "Ultrafast solid-state laser oscillators: a success story for the last 20 years with no end in sight,"
Appl. Phys. B, vol. 100, pp. 15-28, 2010
free download Ref. 300, http://www.ulp.ethz.ch/publications/paper/2010

Modelocking

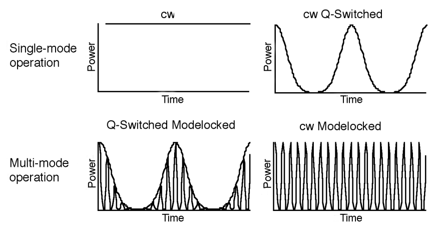
by forcing all modes in a laser to operate phase-locked, "noise" is turned into ideal ultrashort pulses



- **axial modes in laser not phase-locked**
- **noise**

- **axial modes in laser phase-locked**
- **ultrashort pulse**
- **inverse proportional to phase-locked spectrum**

Different mode of operation



cw: continuous wave
τ_p: pulse duration
T_R: roundtrip time
f_{rep}: pulse repetition frequency

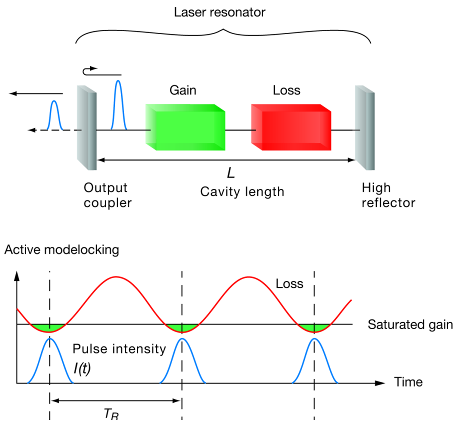
Q-switching: single axial mode $\tau_p >> T_R, \quad \text{and} \quad f_{rep} << \frac{1}{T_R}$

(fundamental) modelocking: **one** pulse per cavity roundtrip, multi axial modes, phase-locked

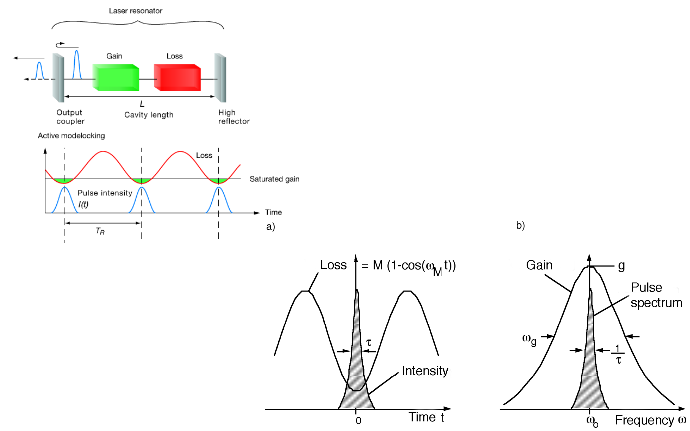
$$\tau_p << T_R, \quad \text{and} \quad f_{rep} = \frac{1}{T_R}$$

harmonic modelocking: $f_{rep} = n\frac{1}{T_R}, \quad n = 2,\ 3,\ \ldots$

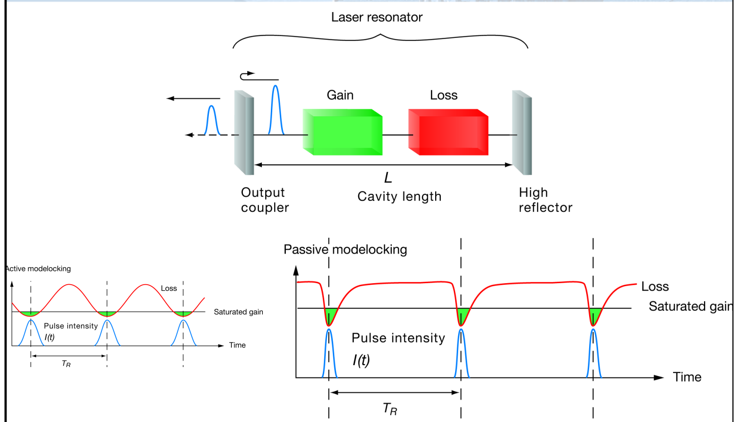
Active Modelocking



Balance between loss modulation and gain



Passive Modelocking



Loss modulation for a slow and fast saturable absorber

$$\frac{dq(t)}{dt} = -\frac{q(t)-q_0}{\tau_A} - \frac{q(t)P(t)}{E_{sat,A}}$$

slow saturable absorber: $\tau_p \ll \tau_A$

neglect recovery within pulse duration

$$\frac{dq(t)}{dt} \approx -\frac{q(t)P(t)}{E_{sat,A}}$$

$$P(t) = E_p f(t), \quad \text{where} \quad \int_0^{T_R} f(t)\,dt = 1$$

$$q(t) = q_0 \exp\left[-\frac{E_p}{E_{sat,A}} \int_0^t f(t')\,dt'\right]$$

fast saturable absorber: $\tau_p \gg \tau_A$

follows "immediately" incoming power

$$0 = -\frac{q(t)-q_0}{\tau_A} - \frac{q(t)P(t)}{E_{sat,A}}$$

$$P_{sat,A} = \frac{E_{sat,A}}{\tau_A}, \quad \Rightarrow \quad \frac{P(t)}{P_{sat,A}} = \frac{I_A(t)}{I_{sat,A}}$$

$$q(t) = \frac{q_0}{1 + I_A(t)/I_{sat,A}}$$

Loss modulation for a slow saturable absorber

$$\frac{dq(t)}{dt} = -\frac{q(t)-q_0}{\tau_A} - \frac{q(t)P(t)}{E_{sat,A}}$$

slow saturable absorber: $\tau_p \ll \tau_A$

neglect recovery within pulse duration

$$\frac{dq(t)}{dt} \approx -\frac{q(t)P(t)}{E_{sat,A}}$$

$P(t) = E_p f(t)$, where $\int_0^{T_R} f(t)\,dt = 1$

$$q(t) = q_0 \exp\left[-\frac{E_p}{E_{sat,A}}\int_0^t f(t')\,dt'\right]$$

loss a pulse experiences through this saturable absorber:

$$q_p(E_p) = \int_0^{T_R} q(t) f(t)\,dt = q_0 \frac{F_{sat,A}A_A}{E_p}\left[1-\exp\left(-\frac{E_p}{F_{sat,A}A_A}\right)\right]$$

does not depend on pulse shape because: $\tau_p \ll \tau_A$

absorbed energy:

$$E_{abs} = 2q_p(E_p)E_p$$

Loss modulation for a fast saturable absorber

$$\frac{dq(t)}{dt} = -\frac{q(t)-q_0}{\tau_A} - \frac{q(t)P(t)}{E_{sat,A}}$$

fast saturable absorber: $\tau_p \gg \tau_A$

follows "immediately" incoming power

$$0 = -\frac{q(t)-q_0}{\tau_A} - \frac{q(t)P(t)}{E_{sat,A}}$$

$P_{sat,A} = \frac{E_{sat,A}}{\tau_A}$, $\Rightarrow \frac{P(t)}{P_{sat,A}} = \frac{I_A(t)}{I_{sat,A}}$

$$q(t) = \frac{q_0}{1+I_A(t)/I_{sat,A}}$$

loss a pulse experiences through this saturable absorber:

assuming **soliton pulse shape** and **fully saturated ideally fast absorber**

$$I(t) = I_p \,\text{sech}^2(t/\tau)$$

$$q_p \approx \frac{q_0}{3}$$

loss now depends on pulse shape!

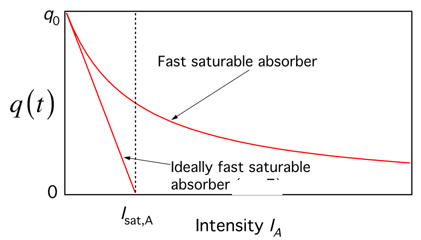
Ideally fast saturable absorber

$$q(t) = \frac{q_0}{1+I_A(t)/I_{sat,A}} \approx q_0\left(1-\frac{I_A(t)}{I_{sat,A}}\right) = q_0 - \gamma_A I_A(t)$$

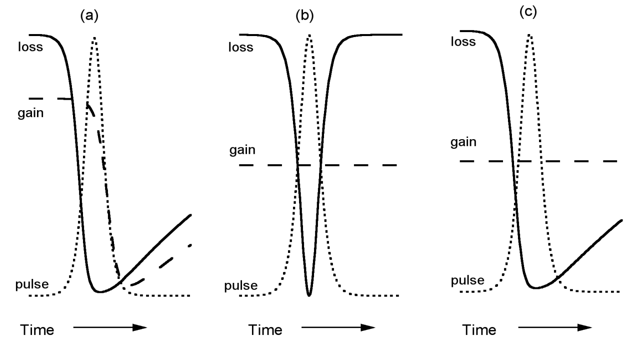
$$\gamma_A \equiv \frac{q_0}{I_{sat,A}}$$

fast saturable absorber

ideally fast saturable absorber



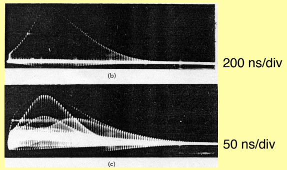
Pulse-shaping in passive modelocking



U. Keller, Ultrafast solid-state lasers, Landolt-Börnstein, Group VIII/1B1, edited by G. Herziger, H. Weber, R. Proprawe, pp. 33-167, 2007, ISBN 978-3-540-26033-2

Ultrashort pulse generation with modelocking

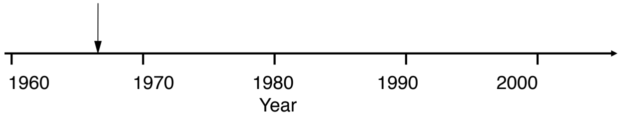
A. J. De Maria, D. A. Stetser, H. Heynau
Appl. Phys. Lett. **8**, 174, 1966



Nd:glass
first passively modelocked laser
Q-switched modelocked

Q-switching instabilities continued to be a problem for solid-state lasers until 1992 (i.e. for 26 years!)

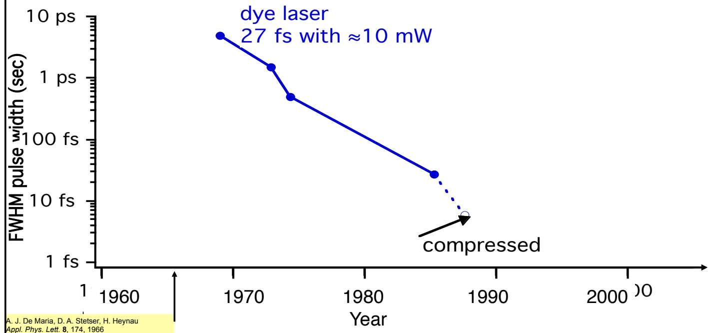
Theoretical investigations in the 1970th confirmed:
" … such solid-state lasers cannot be passively modelocked …"

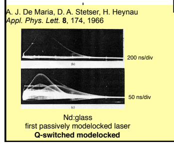


Flashlamp-pumped solid-state lasers

Diode-pumped solid-state lasers
(first demonstration 1963)

Ultrashort pulse generation with modelocking



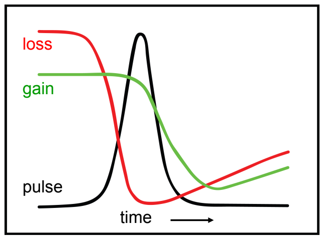


E. P. Ippen, C. V. Shank et al, *Appl. Phys. Lett.* 21, 348, 1972

J. A. Valdmanis et al., *Opt. Lett.* 10, 131, 1985 (27 fs)

R. L. Fork et al., *Opt. Lett.* 12, 483, 1987 (6 fs)

Slow saturable absorber and dynamic gain saturation



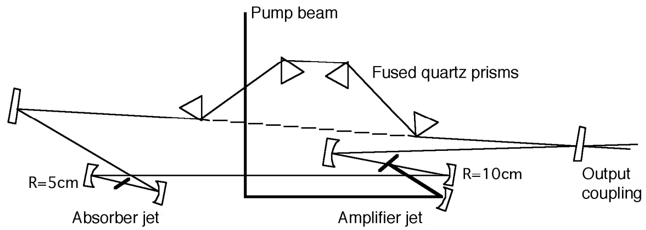
Conditions for stable modelocking:

1) at the beginning loss is larger than gain: $l_0 > g_0$

2) absorber saturates faster than the gain: $E_{sat,A} < E_{sat,L} \quad \Leftrightarrow \quad \frac{A_A}{\sigma_A} < \frac{A_L}{\sigma_L}$

3) absorber recovers faster than the gain: $\tau_A < \tau_L$

Colliding pulse modelocking (CPM)

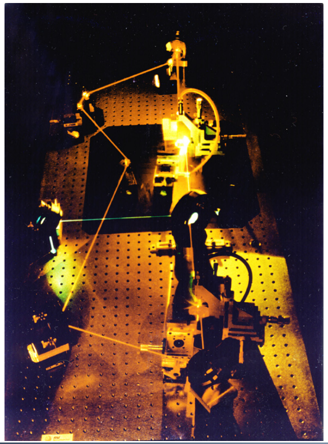


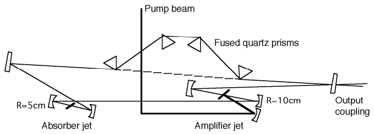
Amplifier jet: Rhodamine 6G (R6G)

Saturable absorber jet: DODCI (3,3-Diethylocadicarbocyanin)

Rhodamine 6G (R6G)	$\sigma_L = 1.36 \cdot 10^{-16}$ cm^2	$\tau_L = 4$ ns
DODCI	$\sigma_A = 0.52 \cdot 10^{-16}$ cm^2	$\tau_A = 2.2$ ns
Photo-isomer	$\tilde{\sigma}_A = 1.08 \cdot 10^{-16}$ cm^2	$\tilde{\tau}_A \approx 1$ ns

Colliding pulse modelocking (CPM)





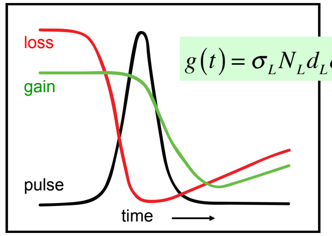
CPM Rhodamine 6G-laser	Typical	Best [86Val]
FWHM pulse duration τ_p	100 fs	27 fs
Emission wavelength λ_L	620 nm	620 nm
Average power P_{av}	2 · 25 mW	2 · 20 mW
Resonator length L	3.3 m	3.3 m
Repetition rate f_{rep}	100 MHz	100 MHz
Pulse peak power in resonator P_{peak}	100 kW	190 kW
Output coupling transmission T_{out}	2.5 %	3.5 %
Pump power	5 W at 514 nm	4 W at 514 nm
Amplifier jet thickness (at focus) and pressure		80 µm 300 µm nozzle, 40 psi
Absorber jet thickness (at focus) and pressure		30 – 50 µm 120 µm nozzle, 15 psi
Prisms for GDD-compensation		Quartz glass

Slow saturable absorber and dynamic gain saturation

$$q(t) = \sigma_A N_A d_A e^{-E_p(t)/E_{sat,A}}$$

$$P(t) = E_p f(t)$$

$$E_p(t) = A_L \int_{-\infty}^{t} |A(t')|^2 \, dt'$$



$$g(t) = \sigma_L N_L d_L e^{-E_p(t)/E_{sat,L}}$$

Assume slow saturable absorber and slow gain saturation

net gain window: $g_T(t) = g(t) - q(t) = g_0 e^{-E_p(t)/E_{sat,L}} - q_A e^{-E_p(t)/E_{sat,A}} - l_R$

resonator loss: $l_0 = q_A + l_R$ $\quad q_A = \sigma_A N_A d_A$

s-parameter: $s \equiv \dfrac{E_{sat,L}}{E_{sat,A}}$

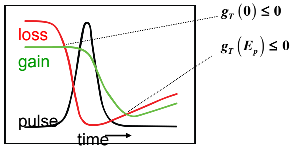
Slow saturable absorber and dynamic gain saturation

$$g_T(t) = g(t) - q(t) = g_0 e^{-E_p(t)/E_{sat,L}} - q_A e^{-E_p(t)/E_{sat,A}} - l_R$$

$$\downarrow \quad e^x \approx 1 + x + \frac{x^2}{2}$$

$$g_T(E) \approx (g_0 - q_A - l_R) + (q_A - \frac{g_0}{s}) \cdot E + \frac{1}{2}\left(\frac{g_0}{s^2} - q_A\right) \cdot E^2$$

For stable pulse generation we need to have $g_T < 0$ at both beginning and end of the pulse.
(see later that this does not always need to be the case - see soliton modelocking)



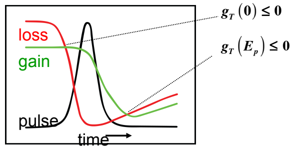
Shorter pulses for larger $g_T^{\max}$

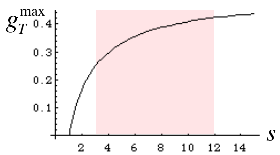
$$g_T^{\max} = g_T(E_{\max}), \quad \text{with} \quad \left.\frac{\partial g_T}{\partial E}\right|_{E=E_{\max}} = 0$$

Slow saturable absorber and dynamic gain saturation

$$g_T(E) \approx (g_0 - q_A - l_R) + (q_A - \frac{g_0}{s}) \cdot E + \frac{1}{2}\left(\frac{g_0}{s^2} - q_A\right) \cdot E^2$$

For stable pulse generation we assume to have $g_T < 0$ at both beginning and end of the pulse.





Shorter pulses for larger $g_T^{\max}$

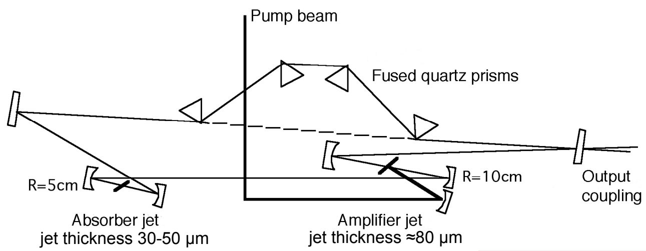
$$g_T^{\max} = g_T(E_{\max}), \quad \text{with} \quad \left.\frac{\partial g_T}{\partial E}\right|_{E=E_{\max}} = 0$$

$$g_T^{\max} = \frac{l_0}{2} \frac{(1-1/s)^2}{(1-1/s^2)}$$

optimize s-parameter: goal in CPM

$$3 < s = \frac{\sigma_A A_L}{\sigma_L A_A} < 12$$

Colliding pulse modelocking (CPM)



Amplifier jet: Rhodamine 6G (R6G)

Saturable absorber jet: DODCI (3,3-Diethyloxadicarbocyanin)

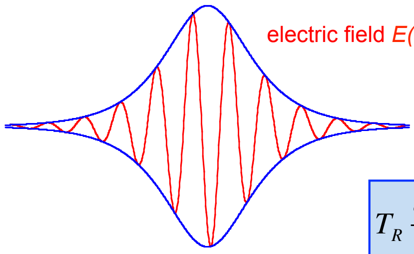
$$3 < s = \frac{\sigma_A A_L}{\sigma_L A_A} < 12$$

Optimization of s-parameter:
- mode size: factor of 4
- colliding pulse in absorber (bi-directional pulse propagation in ring laser): factor of 2 - 3

need thin absorber < spatial extend of pulses (100 fs pulses) $d_A < \frac{c}{n}\tau_p \approx 20\ \mu m$

s ≈ 12

Haus master equation



$$T_R \frac{\partial A(T,t)}{\partial T} = \sum_i \Delta A_i = 0$$

$$E(t) = A(t)e^{i\omega_0 t}$$

Haus master equation

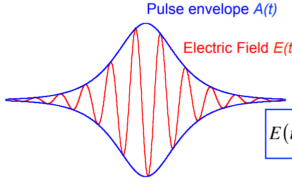
No change in pulse envelope per cavity roundtrip

Monochromatic plane wave

$$E(z,t) = E_0 e^{i(\omega t - k_n z)}$$

Vacuum		Dispersive material
Frequency:	ν	ν
Period:	$T = 1/\nu$	$T = 1/\nu$
Phase velocity:	$\upsilon_p = c$	$\upsilon_p = c_n = c/n$
Wave number:	$k = \frac{\omega}{c}$	$k_n = \frac{\omega}{\upsilon_p} = \frac{\omega}{c} n = kn$
	$k = \frac{2\pi}{\lambda}$	$k_n = \frac{2\pi}{\lambda_n} = kn$
Wavelength:	λ	$\lambda_n = \frac{\lambda}{n}$

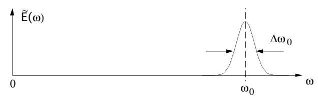
Laser pulse

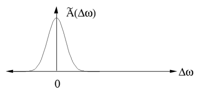


$$E(t) = \frac{1}{2\pi}\int \tilde{E}(\omega)e^{i\omega t}d\omega$$

$$E(t) = A(t)e^{i\omega_0 t} \quad \text{where} \quad A(t) = \frac{1}{2\pi}\int \tilde{A}(\Delta\omega)e^{i\Delta\omega t}d\Delta\omega$$

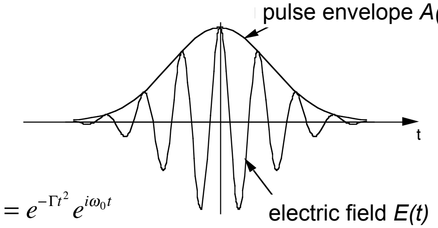
$$E(t) = \frac{1}{2\pi}\int \tilde{E}(\omega_0 + \Delta\omega)e^{i(\omega_0+\Delta\omega)t}d\Delta\omega = \frac{1}{2\pi}e^{i\omega_0 t}\int \tilde{A}(\Delta\omega)e^{i\Delta\omega t}d\Delta\omega$$





$$\tilde{A}(\Delta\omega) = \tilde{E}(\omega_0 + \Delta\omega)$$

Example: Gaussian pulse



$$E(t) = e^{-\Gamma t^2} e^{i\omega_0 t}$$

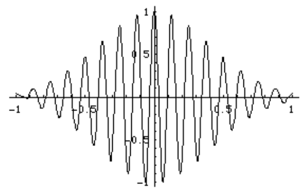
$$A(t) = e^{-\Gamma t^2}\ ,\quad \Gamma \equiv \Gamma_1 - i\Gamma_2$$

$$\tau_p = \sqrt{\frac{2\ln 2}{\Gamma_1}}$$

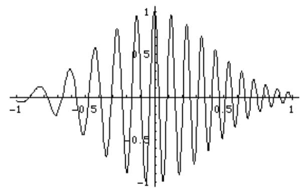
$$\omega(t) \equiv \frac{d\phi_{tot}(t)}{dt} = \omega_0 + 2\Gamma_2 t$$

Chirped Gaussian Pulse

$$E(t) = A(t)\exp(i\omega_0 t) = \exp(-\Gamma t^2)\exp(i\omega_0 t)$$



$$\Gamma \equiv \Gamma_1 - i\Gamma_2$$
$$\Gamma_2 = 0$$



$$\Gamma \equiv \Gamma_1 - i\Gamma_2$$
$$\Gamma_2 \neq 0$$

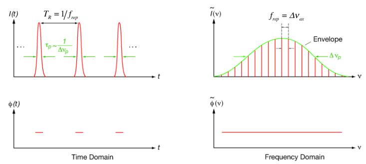
$$\phi_{tot}(t) \equiv \omega_0 t + \Gamma_2 t^2$$

$$\omega(t) \equiv \frac{d\phi_{tot}(t)}{dt} = \omega_0 + 2\Gamma_2 t$$

Time-Bandwidth Products

$I(t)$ $(x \equiv t/\tau)$	τ_p/τ	$\Delta\nu_p \cdot \tau_p$
1. Gaussian $I(t) = e^{-x^2}$	$2\sqrt{\ln 2}$	0.4413
2. Hyperbolic secant (soliton pulse) $I(t) = \mathrm{sech}^2 x$	1.7627	0.3148
3. Rectangle $I(t) = \begin{cases} 1, & \lvert t\rvert \le \tau/2 \\ 0, & \lvert t\rvert > \tau/2 \end{cases}$	1	0.8859
4. Parabolic $I(t) = \begin{cases} 1-x^2, & \lvert t\rvert \le \tau/2 \\ 0, & \lvert t\rvert > \tau/2 \end{cases}$	1	0.7276
5. Lorentzian $I(t) = \frac{1}{1+x^2}$	2	0.2206
6. Symmetric two-sided exponent $I(t) = e^{-2\lvert x\rvert}$	ln2	0.1420

Frequency comb



Pulse train: Spectrum is a frequency comb

Single pulse: Spectrum is continuous

$$E_{train}(t) = E(t) * \sum_{m=-\infty}^{\infty} \delta(t - mT_R)$$

$$f_{rep} = 1/T_R$$

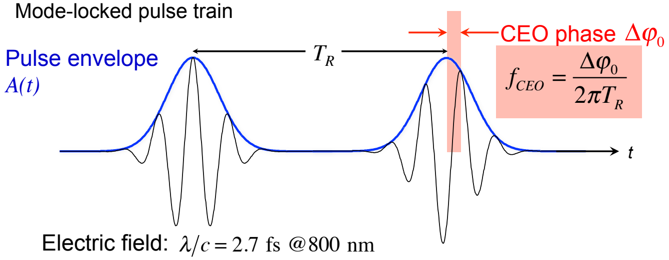
$$\tilde{E}_{train}(f) = \tilde{E}(f) \sum_{m=-\infty}^{\infty} \delta(f - mf_{rep})$$

Carrier-Envelope Offset (CEO) Phase

H.R. Telle, G. Steinmeyer, A. E. Dunlop, J. Stenger, D. H. Sutter, U. Keller
Appl. Phys. B **69**, 327 (1999)

F. W. Helbing, G. Steinmeyer, U. Keller
IEEE J. of Sel. Top. In Quantum Electron. **9**, 1030, 2003

Mode-locked pulse train

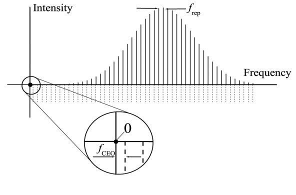


Electric field: $\lambda/c = 2.7$ fs @800 nm

$E(t) = A(t)\exp(i\omega_c t + i\varphi_0(t))$

CEO phase controlled in laser oscillator

Frequency comb



Frequency comb:

$f_n = f_{CEO} + nf_{rep}$

f_{rep} : pulse repetition rate frequency

f_{CEO} : carrier envelope offset frequency

$$\tilde{E}_{train}(f) = \tilde{E}(f)\sum_{m=-\infty}^{\infty}\delta(f - mf_{rep} - f_{CEO})$$

$$E(t) = A(t)\exp(i\omega_c t + i\varphi_0(t))$$

Shift theorem of Fourier transformation

$$\tilde{f}(\omega) \equiv F\{f(t)\} \Leftrightarrow F\{e^{i\omega_0 t} f(t)\} = \tilde{f}(\omega - \omega_0)$$

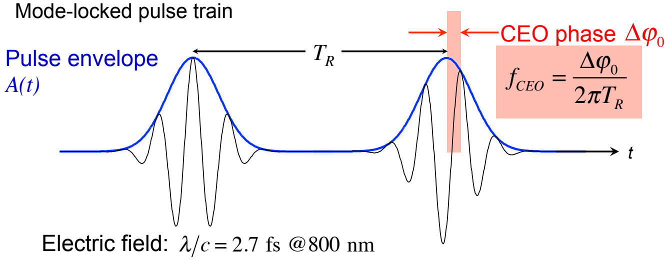
$$E_{train}(t) = A(t)\exp(2\pi i f_c t - 2\pi i f_{ceo} t) * \sum_{m=-\infty}^{\infty}\delta(t - mT_R)$$

H.R. Telle, G. Steinmeyer, A. E. Dunlop, J. Stenger, D. H. Sutter, U. Keller, *Appl. Phys. B* **69**, 327 (1999) and
U. Keller, *Appl Phys. B* **100**, 15-28, 2010

Carrier-Envelope Offset (CEO) Phase

$$E_{train}(t) = A(t)\exp(2\pi i f_c t - 2\pi i f_{ceo} t) * \sum_{m=-\infty}^{\infty}\delta(t - mT_R)$$

Mode-locked pulse train

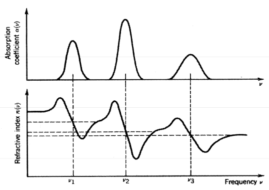


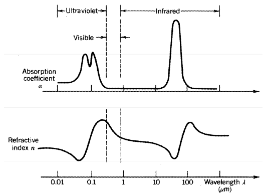
Electric field: $\lambda/c = 2.7$ fs @800 nm

$E(t) = A(t)\exp(i\omega_c t + i\varphi_0(t))$

CEO phase controlled in laser oscillator

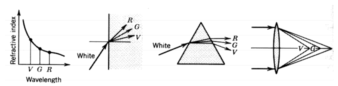
Optical Dispersion





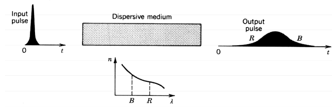
Positive dispersion:

$$\frac{\partial^2 n}{\partial\omega^2} > 0$$

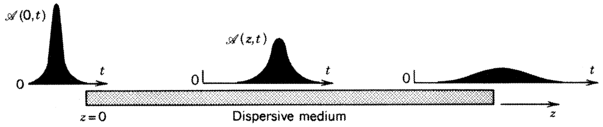


Negative dispersion:

$$\frac{\partial^2 n}{\partial\omega^2} < 0$$

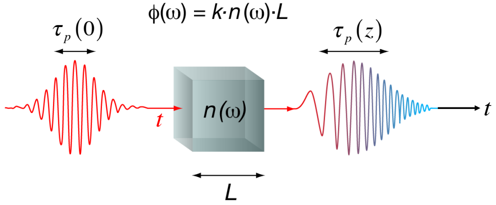


Dispersive pulse broadening



Dispersive medium

$\phi(\omega) = k \cdot n(\omega) \cdot L$



First and second order dispersion

Taylor expansion around the center frequency ω_0: $\Delta\omega = \omega - \omega_0$

$$k_n(\omega) \cong k_n(\omega_0) + k_n' \Delta\omega + \frac{1}{2} k_n'' \Delta\omega^2 + \ldots$$

First order dispersion: $k_n' = dk_n / d\omega$

Second order dispersion: $k_n'' = d^2 k_n / d\omega^2$

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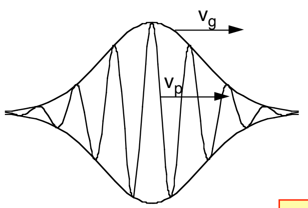
Phase and group velocity

$$E(z,t) \propto \exp\left[i\omega_0 \left(t - \frac{z}{\upsilon_p(\omega_0)} \right) \right] \times \exp\left[-\Gamma(L_d) \times \left(t - \frac{z}{\upsilon_g(\omega_0)} \right)^2 \right]$$



$$\upsilon_p(\omega_0) \equiv c_n = \left. \frac{\omega}{k_n} \right|_{\omega=\omega_0}$$

$$\upsilon_g(\omega_0) \equiv \frac{1}{k_n'(\omega_0)} = \frac{1}{\left(\frac{dk_n}{d\omega} \right)_{\omega=\omega_0}} = \left(\frac{d\omega}{dk_n} \right)_{\omega=\omega_0}$$

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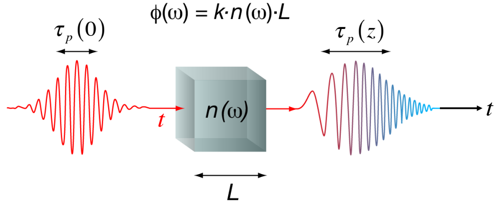
Dispersive pulse broadening

Dispersive medium

$\phi(\omega) = k \cdot n(\omega) \cdot L$



Gaussian pulse:
(initially unchirped pulse)

$$\frac{\tau_p(z)}{\tau_p(0)} = \sqrt{1 + \left(\frac{4 \ln 2 \; d^2\phi / d\omega^2}{\tau_p^2(0)} \right)^2}$$

Approximation for
(strong pulse broadening)

$$\frac{d^2\phi}{d\omega^2} >> \tau_p^2(0) \qquad \tau_p(z) \approx \frac{d^2\phi}{d\omega^2} \Delta\omega_p$$

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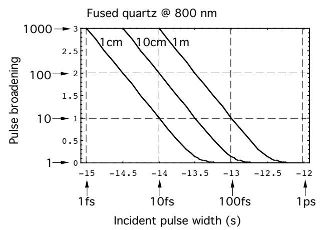
Example: fused quartz

$n(0.8\ \mu m) = 1.45332$	
$\left.\frac{\partial n}{\partial \lambda}\right\|_{800\ nm} = -0.017\ \frac{1}{\mu m}$	$\left.\frac{\partial k_n}{\partial \omega}\right\|_{800\ nm} = 4.84\ \frac{ns}{m}$
$\left.\frac{\partial^2 n}{\partial \lambda^2}\right\|_{800\ nm} = 0.04\ \frac{1}{\mu m^2}$	$\left.\frac{\partial^2 k_n}{\partial \omega^2}\right\|_{800\ nm} = 36.1\ \frac{fs^2}{mm}$



Phase Velocity υ_p	$\frac{\omega}{k_n}$	$\frac{c}{n}$
Group Velocity υ_g	$\frac{d\omega}{dk_n}$	$\frac{c}{n}\frac{1}{1-\frac{dn}{d\lambda}\frac{\lambda}{n}}$
Group Delay T_g	$T_g = \frac{z}{\upsilon_g} = \frac{d\phi}{d\omega},\ \phi \equiv k_n z$	$\frac{nz}{c}\left(1-\frac{dn}{d\lambda}\frac{\lambda}{n}\right)$
Dispersion 1. Order	$\frac{d\phi}{d\omega}$	$\frac{nz}{c}\left(1-\frac{dn}{d\lambda}\frac{\lambda}{n}\right)$
Dispersion 2. Order	$\frac{d^2\phi}{d\omega^2}$	$\frac{\lambda^3 z}{2\pi c^2}\frac{d^2 n}{d\lambda^2}$
Dispersion 3. Order	$\frac{d^3\phi}{d\omega^3}$	$\frac{-\lambda^4 z}{4\pi^2 c^3}\left(3\frac{d^2 n}{d\lambda^2}+\lambda\frac{d^3 n}{d\lambda^3}\right)$

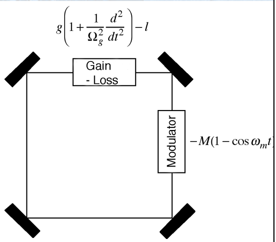
Haus master equation: active modelocking

$$T_R \frac{\partial A(T,t)}{\partial T} = \sum_i \Delta A_i = 0$$

Gain: $\Delta A_1 = g\left(1+\frac{1}{\Omega_g^2}\frac{d^2}{dt^2}\right)A(T,t)$

Loss modulator: $\Delta A_2 \approx -M\left(1-\cos\omega_m t\right)A(T,t)$

Constant loss: $\Delta A_3 \approx -lA(T,t)$



parabolic approximation:

$$M\left(1-\cos\omega_m t\right) \approx M\frac{\omega_m^2 t^2}{2}$$

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g\left(1+\frac{1}{\Omega_g^2}\frac{d^2}{dt^2}\right)-l-M\left(1-\cos\omega_m t\right)\right]A(T,t)$$

Schrödinger equation for harmonic oscillator

H. A. Haus, "Short pulse generation," in *Compact Sources of Ultrashort Pulses*, I. I. N. Duling, Eds. (Cambridge University Press 1995, New York, 1995) pp. 1-56.

Linearized operators: Gain

Gain dispersion:

$$g(\omega) = \frac{g(z)L_g}{1+\left[2(\omega-\omega_0)/\Delta\omega_g\right]^2} = \frac{g}{1+\left[(\omega-\omega_0)/\Omega_g\right]^2} \approx g\left(1-\frac{(\omega-\omega_0)^2}{\Omega_g^2}\right)$$

$$\Omega_g \equiv \frac{\Delta\omega_g}{2}$$

$$\exp\left[g(\omega)\right]\tilde{A}(\omega)$$

$$\exp\left[g(\omega)\right]\tilde{A}(\omega) \approx \left[1+g\left(1-\frac{\Delta\omega^2}{\Omega_g^2}\right)\right]\tilde{A}(\omega) = \left[1+g-\frac{g}{\Omega_g^2}\Delta\omega^2\right]\tilde{A}(\omega)$$

$$\Delta\omega = \omega - \omega_0$$

$$\Delta A_{Gain} = g\left(1+\frac{1}{\Omega_g^2}\frac{d^2}{dt^2}\right)A(T,t)$$

Linearized operators: Modulator

$$A_{out}(t) = \exp\left[-M\left(1-\cos\omega_m t\right)\right]A_{in}(t)$$

$$\downarrow \quad e^x \approx 1+x$$

$$A_{out}(t) \approx \left[1 - M\left(1-\cos\omega_m t\right)\right]A_{in}(t)$$

$$\Rightarrow \; \Delta A_{Mod} = A_{out}(t) - A_{in}(t) \approx -M\left(1-\cos\omega_m t\right)A_{in}(t)$$

$$\boxed{\Delta A_{Mod} \approx -M\left(1-\cos\omega_m t\right)A(T,t)}$$

Linearized operators: Loss

$$A_{out}(t) = e^{-l}A_{in}(t)$$

$$\downarrow \quad e^x \approx 1+x$$

$$A_{out}(t) = e^{-l}A_{in}(t) \approx (1-l)A_{in}(t)$$

$$\Rightarrow \; \Delta A_{Loss} = A_{out}(t) - A_{in}(t) \approx -lA_{in}(T,t)$$

$$\boxed{\Delta A_{Loss} \approx -lA(T,t)}$$

Slow saturable absorber and dynamic gain saturation

Master equation:

$$T_R \frac{\partial A(T,t)}{\partial T} = \left(g(t) - q(t) + \frac{g_0}{\Omega_g^2}\frac{d^2}{dt^2} + t_D\frac{d}{dt}\right)A(T,t) = 0$$

SAM (self-amplitude modulation)

$$A_{out}(t) = \exp\left[-q(t)\right]A_{in}(t)$$

$$\Delta A_{SAM} \approx -q(t)A(T,t)$$

$$q(t) = q_A \exp\left(-\frac{A_L}{A_A}\frac{\sigma_A}{h\nu}\int_{-\infty}^{t}\left|A(t')\right|^2 dt'\right)$$

q_A : unsaturated loss of the absorber

time shift of pulse

$$\Delta A_3 = t_D \frac{d}{dt}A(T,t)$$

Solution: $A(t) = A_0 \operatorname{sech}\left(\frac{t}{\tau}\right)$ $\quad \tau \le \frac{4}{\pi}\frac{1}{\Delta\nu_g}$

Rhodamin 6G:

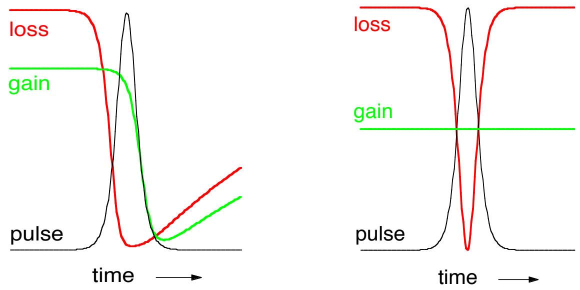
$\Delta\nu_g \approx 4\cdot 10^{13}$ Hz

$\tau_p = 1.76\cdot\tau \le 56$ fs

With SPM: factor of 2 shorter (Martinez numerically)

H. A. Haus, "Theory of modelocking with a slow saturable absorber," *IEEE J. Quantum Electron.* **11**, 736, 1975

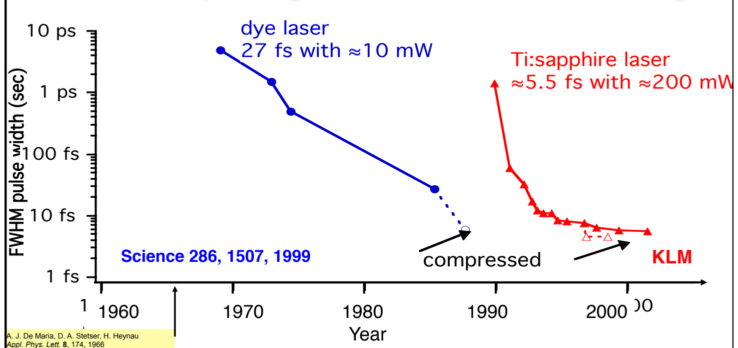
Passive mode locking with an ideally fast saturable absorbers



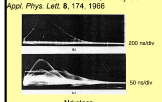
Semiconductor and dye lasers:
Dynamic gain saturation
G.H.C. New,
Opt. Com. **6**, 188, 1974

Solid-state lasers (e.g. Ti:sapphire)
Kerr lens modelocking (KLM)
Ideally fast saturable absorber
Opt. Lett. **16**, 42, 1991

Ultrashort pulse generation with modelocking



A. J. De Maria, D. A. Stetser, H. Heynau *Appl. Phys. Lett.* **8**, 174, 1966

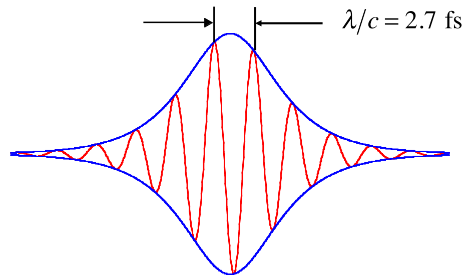


Nd:glass first passively modelocked laser
Q-switched modelocked

Kerr lens modelocking (KLM):
discovered - initially not understood ("magic modelocking")
D. E. Spence, P. N. Kean, W. Sibbett, *Opt. Lett.* 16, 42, 1991
KLM mechanism explained for the first time:
U. Keller et al., *Opt. Lett.* 16, 1022, 1991

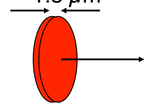
Ultrafast Laser Physics — ETH Zurich ETH

How does such a short pulse look like?



"The shortest pulses (≈5 fs at 800 nm) in the VIS-IR spectral region are still generated with KLM Ti:sapphire lasers"

"under laser lab conditions KLM is stable"



spatial extent of a 5-fs pulse

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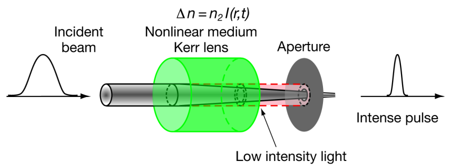
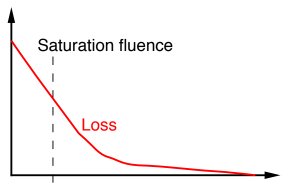
Kerr Lens Modelocking (KLM)

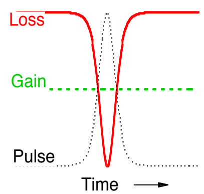
D. E. Spence, P. N. Kean, W. Sibbett, *Optics Lett.* **16**, 42, 1991



Effective Saturable Absorber



Fast Self-Amp. Modulation



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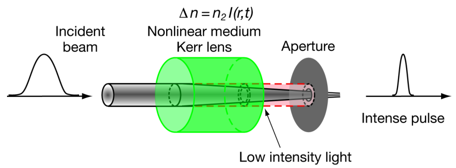
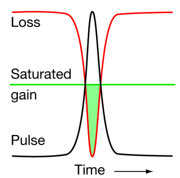
Kerr Lens Modelocking (KLM)

First Demonstration: D. E. Spence, P. N. Kean, W. Sibbett, *Optics Lett.* **16**, 42, 1991
Explanation: U. Keller et al., *Optics Lett.* **16**, 1022, 1991





Advantages of KLM
- very fast thus shortest pulses
- very broadband thus broader tunability

Disadvantages of KLM
- not self-starting
- critical cavity adjustments (operated close to the stability limit)
- saturable absorber coupled to cavity design (limited application)

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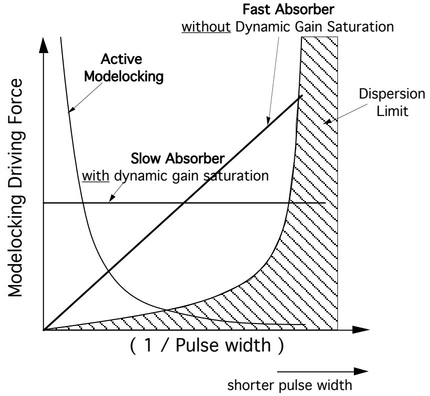
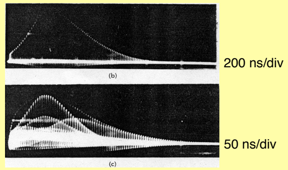
Self-starting modelocking?



E. P. Ippen, "Principles of passive modelocking," *Appl. Phys. B* **58**, 159, 1994

Ultrashort pulse generation with modelocking

A. J. De Maria, D. A. Stetser, H. Heynau
Appl. Phys. Lett. **8**, 174, 1966



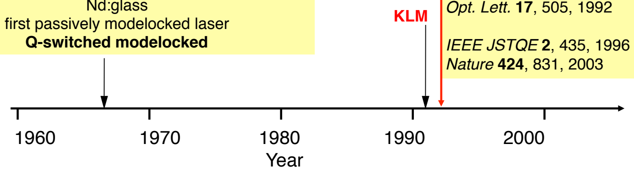
Nd:glass
first passively modelocked laser
Q-switched modelocked

Q-switching instabilities continued to be a problem until 1992

SESAM
First passively modelocked (diode-pumped) solid-state laser without Q-switching

U. Keller et al.
Opt. Lett. **17**, 505, 1992

IEEE JSTQE **2**, 435, 1996
Nature **424**, 831, 2003



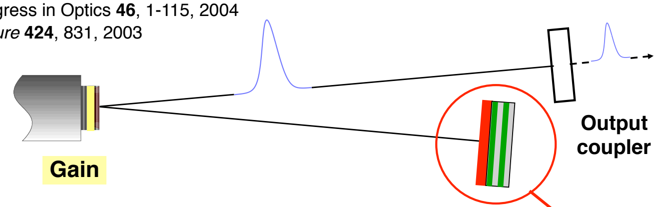
Flashlamp-pumped solid-state lasers

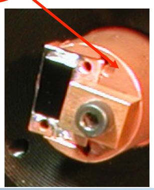
Diode-pumped solid-state lasers
(first demonstration 1963)

SESAM modelocking

U. Keller et al. *Opt. Lett.* **17**, 505, 1992
IEEE JSTQE **2**, 435, 1996
Progress in Optics **46**, 1-115, 2004
Nature **424**, 831, 2003

SESAM solved Q-switching problem for diode-pumped solid-state lasers

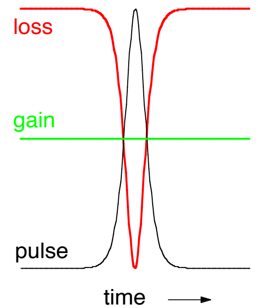




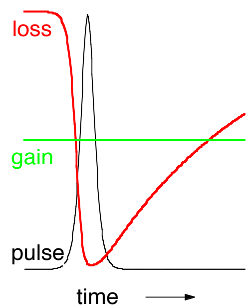
SESAM
SEmiconductor **S**aturable **A**bsorber **M**irror

self-starting, stable, and reliable modelocking of diode-pumped ultrafast solid-state lasers

KLM vs. Soliton modelocking

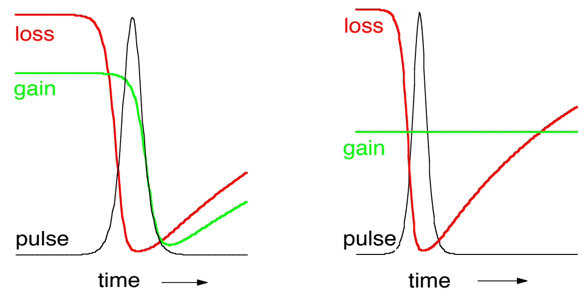


Kerr lens modelocking (KLM)
Fast saturable absorber
D. E. Spence, P. N. Kean, W. Sibbett
Opt. Lett. **16**, 42, 1991



Soliton modelocking
"not so fast" saturable absorber
F. X. Kärtner, U. Keller,
Opt. Lett. **20**, 16, 1995

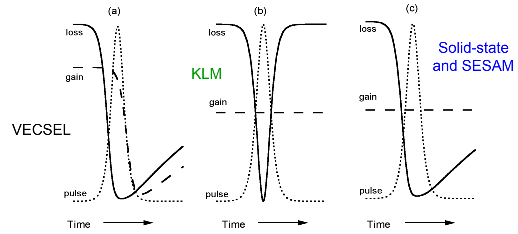
Passive mode locking with slow saturable absorbers



Semiconductor lasers:
Dynamic gain saturation
G.H.C. New,
Opt. Com. **6**, 188, 1974

Ion-doped solid-state lasers:
Constant gain saturation: soliton modelocking
F. X. Kärtner, U. Keller,
Opt. Lett. **20**, 16, 1995

Pulse-shaping



Gain window can be up to 20 times longer than the pulse before mode locking becomes unstable

- fast/broadband sat. abs.
- critical cavity adjustment: KLM better at cavity stability limit
- typically not self-starting

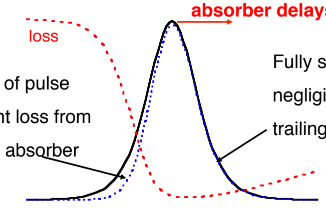
- "not so fast" sat. abs
- absorber independent of cavity design
- self-starting

For stable pulse generation it was initially assumed that we need $g_T < 0$ at both beginning and end of the pulse. **This is however not the case!**
See SESAM modelocking with slow saturable absorber and soliton modelocking

Slow saturable absorber modelocking

R. Paschotta, U. Keller, *Appl. Phys. B* **73**, 653, 2001

absorber delays pulse

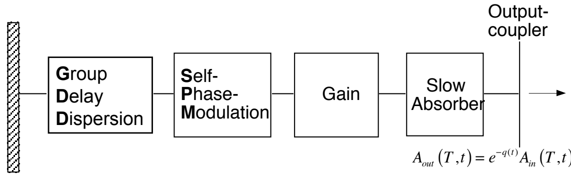


leading edge of pulse has significant loss from the saturable absorber

Fully saturated absorber: negligible loss for trailing edge of pulse

Dominant stabilization process:
Picosecond domain: absorber delays pulse
The pulse is constantly moving backward and can swallow any noise growing behind itself
Femtosecond domain: dispersion in soliton modelocking

Soliton modelocking: GDD negative, $n_2 > 0$



$$A_{out}(T,t) = e^{-q(t)} A_{in}(T,t)$$
$$A_{out}(T,t) = [1 - q(t)] A_{in}(T,t)$$
$$\Rightarrow \Delta A(T,t) = -q(t) A(T,t)$$

Master equation:

$$T_R \frac{\partial}{\partial T} A(T,t) = \left(iD \frac{\partial^2}{\partial t^2} - i\delta |A(T,t)|^2 \right) A(T,t) + \left(g - l + D_g \frac{\partial^2}{\partial t^2} - q(t) \right) A(T,t) = 0$$

Kerr effect and self-phase modulation (SPM)

$n(I) = n + n_2 I$

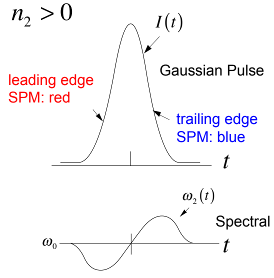
$$n_2\left[\frac{\text{cm}^2}{\text{W}}\right] = 4.19\times10^{-3}\,\frac{n_2[\text{esu}]}{n}$$

Material	Refractive index n	$n_2[\text{esu}]$	$n_2[\text{cm}^2/\text{W}]$
Sapphire (Al_2O_3)	1.76 @ 850 nm	1.25×10^{-13} [89Ada]	3×10^{-16}
Fused quartz	1.45 @ 1.06 µm	0.85×10^{-13} [89Ada]	2.46×10^{-16}
Glass (Schott LG-760)	1.5 @ 1.06 µm	1.04×10^{-13} [93Aza]	2.9×10^{-16}
YAG ($Y_3Al_5O_{12}$)	1.82 @ 1.064 µm	3.47×10^{-13} [93Aza]	6.2×10^{-16}
YLF ($LiYF_4$)	$n_e = 1.47$ @ 1.047 µm		1.72×10^{-16} [93Aza]

Typical order of magnitude for the nonlinear index coefficient: $n_2 \approx 10^{-16}$ cm²/W

Linearized operators: self-phase modulation (SPM)

$n_2 > 0$



Spectral broadening

$$\phi_2(t) = -kn_2 I(t) L_K = -kn_2 L_K |A(t)|^2 \equiv -\delta |A(t)|^2$$

$$\delta \equiv kn_2 L_K$$

$$\omega_2(t) = \frac{d\phi_2(t)}{dt} = -\delta\frac{dI(t)}{dt}$$

$$E(L_K,t) = A(0,t)\exp[i\omega_0 t + i\phi(t)] = A(0,t)\exp\left[i\omega_0 t - ik_n(\omega_0)L_K - i\delta|A(t)|^2\right]$$

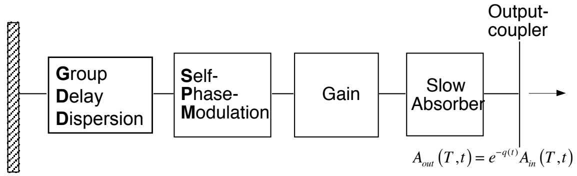
$$A(L_K,t) = e^{-i\delta|A|^2}A(0,t)e^{-ik_n(\omega_0)L_K} \xrightarrow{\delta|A|^2 \ll 1} \approx \left(1 - i\delta|A(t)|^2\right)A(0,t)e^{-ik_n(\omega_0)L_K}$$

$$\Delta A_{SPM} \approx -i\delta|A(T,t)|^2$$

Linearized operators: group delay dispersion (GDD)

Please derive this on your own

Soliton modelocking: GDD negative, $n_2 > 0$



$$A_{out}(T,t) = e^{-q(t)}A_{in}(T,t)$$

$$A_{out}(T,t) = [1 - q(t)]A_{in}(T,t)$$

$$\Rightarrow \Delta A(T,t) = -q(t)A(T,t)$$

Master equation:

$$T_R\frac{\partial}{\partial T}A(T,t) = \left(iD\frac{\partial^2}{\partial t^2} - i\delta|A(T,t)|^2\right)A(T,t) + \left(g - l + D_g\frac{\partial^2}{\partial t^2} - q(t)\right)A(T,t) = 0$$

$$A(T,t) = \left(A_0\ \text{sech}\left(\frac{t}{\tau}\right)\right)\exp\left[i\phi_0\frac{T}{T_R}\right] + \text{continuum}$$

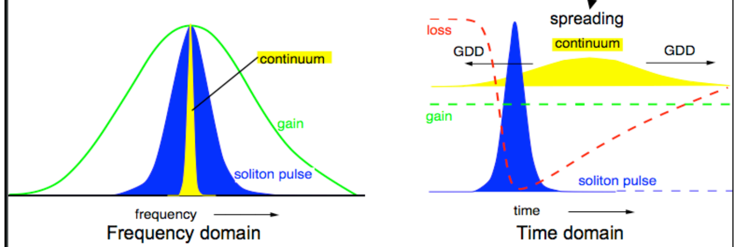
$$\tau = \frac{4|D|}{\delta\cdot F_{p,L}}$$

Soliton modelocking: GDD negative, $n_2 > 0$

F. X. Kärtner, U. Keller, Optics Lett. 20, 16, 1995
Invited Paper: F. X. Kärtner, I. D. Jung, U. Keller, IEEE JSTQE, 2, 540, 1996

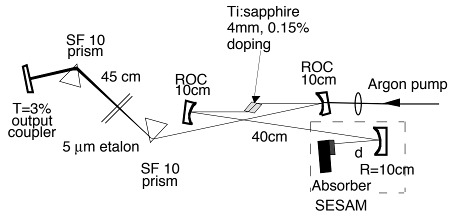
Soliton Perturbation Theory:

$$A(T,t) = \underbrace{\left(A\,\mathrm{sech}\left(\frac{t}{\tau}\right)\right)\exp\left[i\Phi_0\frac{T}{T_R}\right]}_{\text{soliton}} + \underbrace{\textbf{\textit{small perturbations}}}_{\text{"continuum" only GDD \& SAM (no SPM)}}$$



Stabilization: Dispersion spreads continuum out where it sees more loss

Experimental confirmation: example Ti:sapphire laser

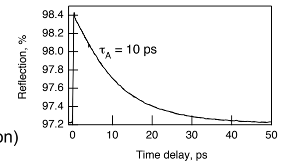


SESAM: LT-GaAs

Impulse response measured with 300 fs pulses

clearly a slow saturable absorber

No KLM: cavity operated in the middle of cavity stability regime (and use etalon for bandwidth limitation)



I. D. Jung, F. X. Kärtner, L. R. Brovelli, M. Kamp, U. Keller, "Experimental verification of soliton modelocking using only a slow saturable absorber," *Opt. Lett.*, vol. 20, pp. 1892-1894, 1995

Experimental confirmation: example Ti:sapphire laser

Solution: soliton pulse $A_s(T,t) = \sqrt{\frac{F_{p,L}}{2\tau}}\,\mathrm{sech}\left(\frac{t}{\tau}\right)e^{i\phi_0\frac{T}{T_R}}$

soliton phase per resonator roundtrip

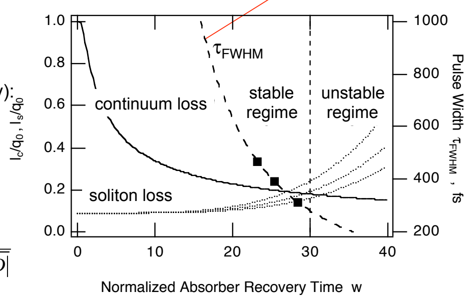
$$\phi_0 = \frac{|D|}{\tau^2} = \frac{\delta\cdot F_{p,L}}{4\tau}$$

$$\tau_{FWHM} = 1.76\cdot\tau = 1.76\frac{4|D|}{\delta\cdot F_{p,L}}$$

Stability (soliton perturbation theory): continuum loss l_c is larger than soliton loss l_s

Normalized abs. recovery

$$w = \frac{\tau_A}{\tau}\sqrt{\frac{q_0}{\phi_0}} = \tau_A\sqrt{\frac{q_0}{|D|}} \propto \frac{1}{\sqrt{|D|}}$$



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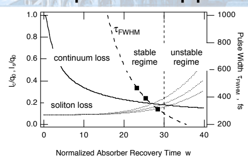

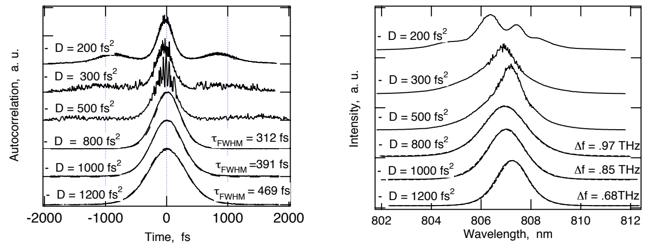

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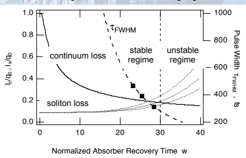
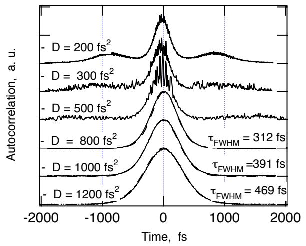
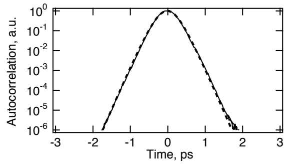




High dynamic range autocorrelation: 330 fs
Opt. Lett. **20**, 1889, 1995



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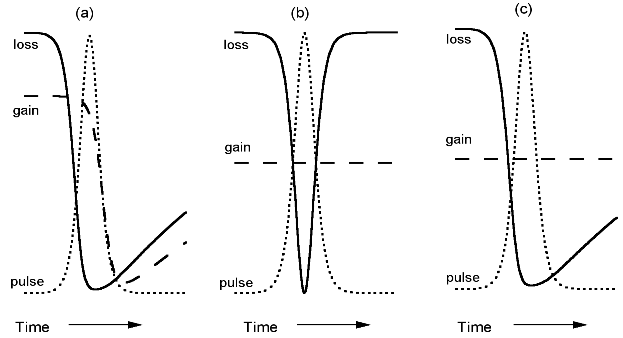
Pulse-shaping in passive modelocking



U. Keller, Ultrafast solid-state lasers, Landolt-Börnstein, Group VIII/1B1, edited by G. Herziger, H. Weber, R. Proprawe, pp. 33-167, 2007, ISBN 978-3-540-26033-2