Theory of coupled-cavity mode locking with a resonant nonlinearity

H. A. Haus

Department of Electrical Engineering and Computer Science and Research Laboratory of Electronics, Cambridge, Massachusetts 02139

U. Keller and W. H. Knox

AT&T Bell Laboratories, Crawfords Corner Road, Holmdel, New Jersey 07733

Received November 12, 1990; accepted January 22, 1991

Recently, mode locking of a Ti:sapphire laser with a quantum-well reflector in an external cavity was reported [Opt. Lett. **15**, 1377 (1990)]. This laser was found to operate with a stable mode-locked pulse train without stabilization of the external cavity. We develop a theory that shows that the system operates by small frequency adjustments. The laser continuously seeks the carrier frequency, which ensures coherent superposition of the pulses at the coupling mirror. Coupled-cavity resonant passive mode locking, as this new mode-locking mechanism has been called, is a form of self-stabilized coupled-cavity mode locking with a resonant nonlinearity. We show that self-stabilizing operation cannot be realized with a Kerr medium (reactive nonlinearity) in the cavity.

INTRODUCTION

Recently the successful mode locking of solid-state lasers with a Kerr medium in an external cavity has attracted a great deal of attention.[1–10] The Kerr medium, when interferometrically coupled to the main cavity, acts as a fast saturable absorber, thus producing mode locking without gain saturation. This principle of operation has been called additive pulse mode locking (APM) and also coupled-cavity mode locking. Thus far all these systems have required stabilization of the length of the auxiliary cavity with respect to the length of the main cavity in order to produce self-starting and to sustain oscillation.

Recently a form of mode locking with an external cavity was reported[11] that does not require stabilization of the external cavity in order to sustain oscillation. Clearly such a mode of operation is desirable because it simplifies the operation and is likely to perform better under adverse conditions. The principle, called coupled-cavity resonant passive mode locking (RPM) by its inventors, simply uses a quantum-well saturable absorber in the auxiliary cavity, as illustrated schematically in Fig. 1. Stable pulse trains are obtained only when the auxiliary cavity optical length is different from the optical length of the main cavity, and mode locking persists with length differences as large as 1 mm. Small fluctuations in the external cavity length are found not to affect the mode-locked pulse train or autocorrelation; however, they induce small spectral changes. The pulse width increases with cavity mismatch. At and near exact alignment of the optical lengths the operation is sensitive to length shifts of the order of a fraction of a wavelength.

In this paper we attempt to explain the operation of this new system on the basis of a master equation derived from saturable absorber mode-locking theory[12] and APM theory.[5] Before we enter into the details, we look at certain general characteristics that will help us to construct a model.

The observation that the pulses, while chirped, exhibit stable pulse shapes in the presence of small fluctuations in the optical spectrum can be explained only if the pulses interfere coherently at the coupling mirror. Since the mode-locking mechanism is not sensitive to cavity length changes of a wavelength or less, the relative phases of the pulses that meet at the mirror must be stable. Such a situation can be maintained only if the system is capable of adjusting the phases by changing its frequency. This is the basic postulate of the theory. In the course of the development we show that the system can accomplish this self-stabilization of phase by small adjustments of its carrier frequency.

We proceed as follows. In Section 1 we present a simple physical picture that will explain the difference between coupled-cavity mode locking with a resonant nonlinearity (RPM) and coupled-cavity mode locking with a nonresonant nonlinearity (APM) and investigate how a frequency adjustment can produce the required phase alignment of the two cavities. In Section 2 we look at the reflection from the auxiliary cavity. In Section 3 we set up the master equation. In Section 4 we proceed with the interpretation of the solution of the master equation. In Section 5 we present some experimental results of an RPM Ti:sapphire laser[11] in the context of the theory.

1. PHASE ADJUSTMENT BY FREQUENCY ADJUSTMENT

Following Goodberlet *et al.*,[10] we model our coupled cavity as a mirror in which the reflectivity depends on the intensity [Fig. 2(a)]. For the case of the saturable quantum-well reflector, the reflectivity increases with intensity. Figure 2(b) shows schematically the Fabry–Perot reflec-

0740-3224/91/061252-07$05.00

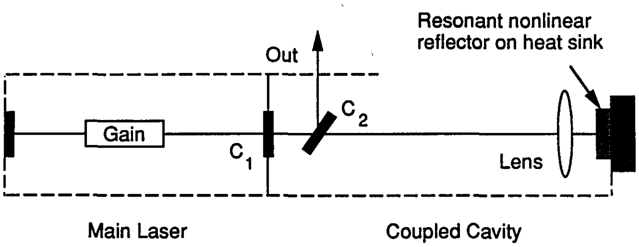

Fig. 1. Schematic of a system that performs RPM.

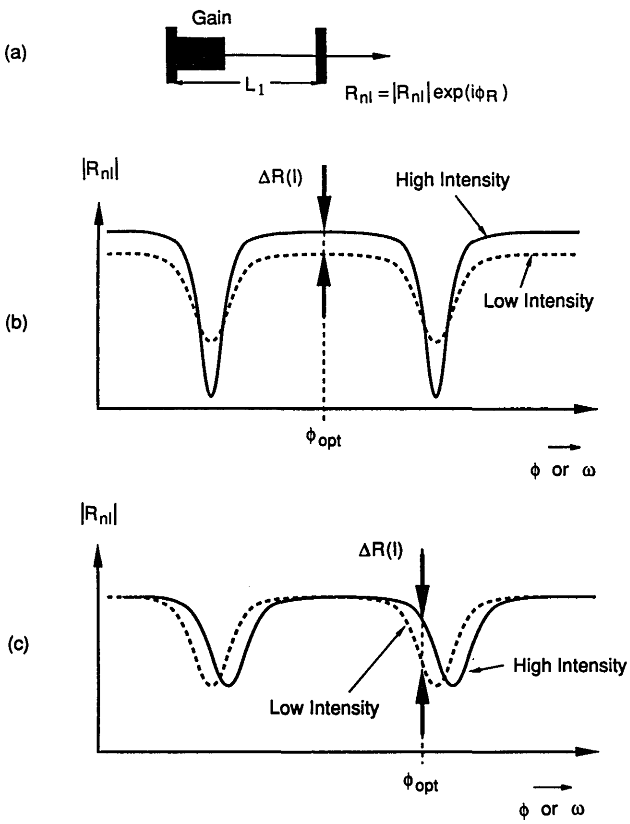

Fig. 2. Reflectivity as a function of phase.

tivity for low- and high-intensity conditions for RPM. Figure 2(c) shows the reflectivity for a pure nonlinear phase shift, as in the APM laser. For cw operation (or self-starting) any coupled-cavity laser will seek the frequency that corresponds to maximum gain. For RPM the phase corresponding to maximum cw gain is the same as that for the peak nonlinear reflectivity change, whereas for APM the optimum phase for cw gain is different from the phase of maximum nonlinear reflectivity change. Therefore the APM laser has to be actively held at a lower reflectivity position, as shown in Fig. 2(c). APM requires active control of cavity length to a fraction of a wavelength.

For the case of the coupled-cavity laser the relation between the operation frequency and the coupled-cavity phase is now further discussed. The cavity modes are defined by

$$(\omega/c)n(\omega)l = m\pi, \tag{1.1}$$

and the mode spacing is given by

$$(\Delta\omega/v_g)l = \pi, \tag{1.2}$$

where v_g is the group velocity:

$$\frac{1}{v_g} = \frac{n}{c}\left(1 + \frac{\omega}{n}\frac{dn}{d\omega}\right). \tag{1.3}$$

If the two cavities are of different length, then the modes are spaced as shown in Fig. 3. At a certain carrier frequency the mode frequencies are locally coincident. As one changes the frequency, the modes separate, but at some number N_J of modes away from the original coincidence there is another coincidence. The difference between the mode spacings is

$$\delta\Delta\omega = -(\delta l/l)\Delta\omega, \tag{1.4}$$

where δl is the difference of the cavity lengths. If there is a coincidence at some frequency ω_0, then the next coincidence is at

$$\omega_0 + N_J|\delta\Delta\omega| = \omega_0 + \Delta\omega$$

or

$$N_J = \left|\frac{\Delta\omega}{\delta\Delta\omega}\right| = \left|\frac{l}{\delta l}\right|. \tag{1.5}$$

2. AUXILIARY CAVITY

We analyze the situation shown in Fig. 4. The mirror of reflectivity r relates the wave amplitudes a_i and b_i:

$$b_1 = ra_1 + (1 - r^2)^{1/2}a_2, \tag{2.1}$$

$$b_2 = (1 - r^2)^{1/2}a_1 - ra_2. \tag{2.2}$$

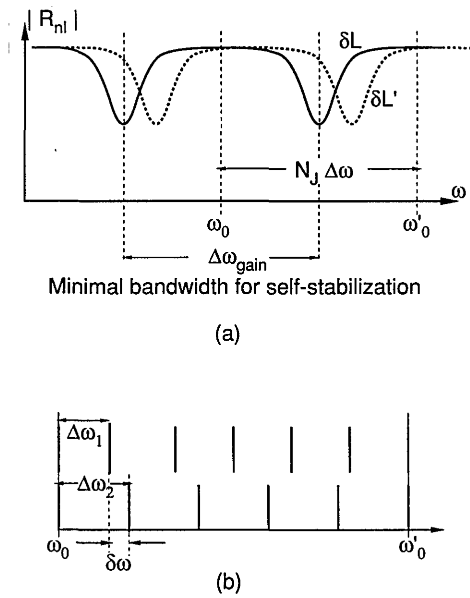

Fig. 3. Mode spacing of the main and auxiliary cavities.

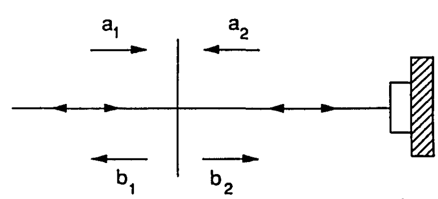

Fig. 4. Schematic of the auxiliary cavity.

As in the APM theory, we consider the waves in the time domain. If the auxiliary cavity is dispersion free, then a_2 is a replica of b_2:

$$a_2 = L\exp(-j\phi)b_2, \tag{2.3}$$

where L is the loss factor ($L < 1$) and ϕ is the phase shift of the carrier that is due to deviation of the cavity length from an integer multiple of half wavelengths. The reflection coefficient is [5]

$$\Gamma = \frac{b_1}{a_1} = \frac{r + L\exp(-j\phi)}{1 + rL\exp(-j\phi)} \simeq r + L(1 - r^2)\exp(-j\phi) \tag{2.4}$$

for small L. Now, the loss L is in fact nonlinear. We make the simplifying assumption that the multiple quantum well has only a fast recovery time (faster than the laser pulse width). It has been shown that longitudinal optical phonon ionization of quantum-well excitons results in a rapid partial recovery of absorption in ~300 fs at room temperature.[13] In the present case we neglect the long time transient, which is governed by carrier recombination. Quantum wells have been used to mode lock a laser diode and color-center laser.[14] Under these conditions we may write L in the form

$$L = L_0\left(1 - \left|\frac{u}{u_s}\right|^2\right), \tag{2.5}$$

where u is the time-dependent normalized field amplitude and $|u_s|^2$ is proportional to the saturation intensity of the saturable absorber. Note that in the simplified notation used here we have not separated the linear and the nonlinear losses, so that $|u_s|^2$ has been, in effect, renormalized:

$$\Gamma \simeq r + L_0(1 - r^2)\exp(-j\phi) + (1 - r^2)L_0\exp(-j\phi)\left|\frac{u}{u_s}\right|^2. \tag{2.6}$$

3. THE MASTER EQUATION

We now develop the master equation for the present problem. It is quite similar to the equation used above in the theory of APM. First, look at the effect of the gain. In the frequency domain

$$\text{Gain} = \frac{g}{1 + [(\omega - \omega_g)/\Delta\omega_g]^2}; \tag{3.1}$$

g is, of course, a function of time average power, ω_g is the frequency with maximum gain, and $\Delta\omega_g$ is the effective cavity gain bandwidth under mode-locked conditions, which may include filters inside the cavity. We assume that the relaxation of the gain medium is slow; the gain is pulled down by a succession of many pulses. Suppose that the carrier frequency is ω_0. Then, for a deviation $\Delta\omega$ from ω_0,

$$\text{Gain} \simeq g\left[1 - \frac{2\Delta\omega}{\Delta\omega_g}\frac{\omega_0 - \omega_g}{\Delta\omega_g} - \left(\frac{\Delta\omega}{\Delta\omega_g}\right)^2 - \left(\frac{\omega_0 - \omega_g}{\Delta\omega_g}\right)^2\right] \tag{3.2a}$$

or, as an operator in the time domain,

$$\text{Gain} \simeq g\left[1 + 2j\frac{\omega_0 - \omega_g}{\Delta\omega_g}\frac{1}{\Delta\omega_g}\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{\Delta\omega_g^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2} - \left(\frac{\omega_0 - \omega_g}{\Delta\omega_g}\right)^2\right]. \tag{3.2b}$$

The group-velocity dispersion in the cavity has the effect $jD(\mathrm{d}^2/\mathrm{d}t^2)$, corresponding to $D = \beta''l/2$ in the frequency domain. Further, a phase factor $\exp(-j\psi)$ may be caused after one passage through the laser resonator. Finally, there may be some loss represented by lu. Therefore the effect of the pulses $u(t)$ on the change Δu of the gain medium, group-velocity dispersion, and loss is, after one passage,

$$\Delta u = \left\{-j\psi + g\left[1 + 2j\frac{\omega_0 - \omega_g}{\Delta\omega_g}\frac{1}{\Delta\omega_g}\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{\Delta\omega_g^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2} - \left(\frac{\omega_0 - \omega_g}{\Delta\omega_g}\right)^2\right] + jD\frac{\mathrm{d}^2}{\mathrm{d}t^2} - l\right\}u. \tag{3.3}$$

The auxiliary cavity effect must be handled with some care when it is detuned, as it must be for RPM to be operative. Γu must be evaluated at a time $t + \delta T_A$, compared with the time reference t applied to the main cavity. When δT_A is positive, the pulse is returned earlier. This means that the auxiliary cavity is foreshortened compared with the laser cavity. We make the appoximation of expanding to first order in $|u/u_s|^2$ and second order in δT_A, ignoring product terms, because otherwise we cannot obtain a closed form solution. The change per pass is

$$\begin{aligned}\Delta u &= (\Gamma - 1)u \\ &= ru + L_0(1 - r^2)\exp(-j\phi)\left[1 + \delta T_A\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{2}\delta {T_A}^2\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right]u + (1 - r^2)L_0\exp(-j\phi)\left|\frac{u}{u_s}\right|^2 u - u.\end{aligned} \tag{3.4}$$

After the action of all the elements, the sum of the Δu's must vanish except for the possibility that the pulse is shifted by δT_L. Indeed, the round-trip time of the pulse in the laser cavity may not coincide with the pulse repetition time because of temporal pulling of the pulse envelope. If δT_L is positive it means that the pulse is advanced in one round trip. This means that the laser cavity round-trip time is longer than the pulse repetition time,

$$\begin{aligned}&\left[-j\psi + g\left(1 + 2j\frac{\omega_0 - \omega_g}{\Delta\omega_g}\frac{1}{\Delta\omega_g}\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{\Delta\omega_g^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right) - g\left(\frac{\omega_0 - \omega_g}{\Delta\omega_g}\right)^2 + jD\frac{\mathrm{d}^2}{\mathrm{d}t^2} - l - 1 + r\right.\\ &\quad + L_0(1 - r^2)\exp(-j\phi)\left(1 + \delta T_A\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{2}\delta {T_A}^2\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right)\\ &\quad \left. + (1 - r^2)L_0\exp(-j\phi)\left|\frac{u}{u_s}\right|^2\right]u = \delta T_L\frac{\mathrm{d}u}{\mathrm{d}t}.\end{aligned} \tag{3.5}$$

4. SOLUTION

We solve the above by the ansatz[15]

$$u = A\,\mathrm{sech}(t/\tau)\exp[j\beta ln\ \mathrm{sech}(t/\tau)]. \tag{4.1}$$

The derivatives are

$$\tau \frac{du}{dt} = -(1 + j\beta)\tanh\left(\frac{t}{\tau}\right)u, \tag{4.2}$$

$$\tau^2 \frac{d^2u}{dt^2} = \left[-(2 + 3j\beta - \beta^2)\operatorname{sech}^2\left(\frac{t}{\tau}\right) + (1 + j\beta)^2\right]u. \tag{4.3}$$

We introduce these expressions into the master equation and equate terms involving $\operatorname{sech}^2(t/\tau)$, $\tanh(t/\tau)$, and a constant multiplier of u. The sech^2 terms are

$$L_0(1 - r^2)\exp(-j\phi)\frac{A^2}{|u_s|^2} - \left[\frac{g}{\Delta\omega_g{}^2\tau^2} + j\frac{D}{\tau^2} + \frac{1}{2}\frac{\delta T_A{}^2}{\tau^2}(1 - r^2)L_0 \exp(-j\phi)\right](2 + 3j\beta - \beta^2) = 0. \tag{4.4}$$

The hyperbolic tangent terms are

$$-2j\frac{\omega_0 - \omega_g}{\Delta\omega_g{}^2}g - L_0(1 - r^2)\exp(-j\phi)\delta T_A = -\delta T_L. \tag{4.5}$$

The constant terms are

$$-j\psi + g - g\left(\frac{\omega_0 - \omega_g}{\Delta\omega_g}\right)^2 - [l + (1 - r)] + L_0(1 - r^2)\exp(-j\phi) + \left[\frac{g}{\Delta\omega_g{}^2\tau^2} + j\frac{D}{\tau^2} + \frac{1}{2}\frac{\delta T_A{}^2}{\tau^2}(1 - r^2)L_0 \exp(-j\phi)\right](1 + j\beta)^2 = 0. \tag{4.6}$$

We have three complex and six real equations for the unknowns of interest. These are the gain g, the phase shift on one pass ψ, the pulse timing deviation δT_L, the frequency deviation, the pulse width τ, and the chirp parameter β. There is an equation for the gain as a function of pulse energy:

$$g = \frac{g_0}{1 + [(2A^2\tau)/(T_R P_s)]}, \tag{4.7}$$

where we assume, as is appropriate for a solid-state laser, that the gain relaxes slowly and responds only to the average power; the saturation power is P_s, and T_R is the cavity round-trip time. The evaluation of the gain from Eq. (4.6) then serves to establish the value of the peak pulse intensity A^2. One may simplify this step by equating the gain to the loss l. In this case one may solve for $A^2\tau$ as a given and proceed with the solution, avoiding the complicated coupling between the pulse width and the gain as contained in Eq. (4.6). We do so in the remainder of the paper, assuming that the pulse energy $W = 2A^2\tau$ is a given.

Consider next Eq. (4.5). This equation determines the frequency shift and pulse timing. Now, the phase ϕ is not independent of the frequency shift. The number of modes by which the center frequency has to shift in order to produce a phase shift of ϕ follows from Eq. (1.5):

$$N_\phi = \frac{\phi}{2\pi}\frac{l}{\delta l}, \tag{4.8}$$

where δl is the difference of lengths between the two cavities. On the other hand, the frequency shift is

$$\omega_0 - \omega_g = N_\phi \Delta\omega = \frac{\phi}{2\pi}\frac{\pi v_g}{\delta l} = \frac{\phi}{2\delta T_A}, \tag{4.9}$$

where we have introduced the detuning time of the auxiliary cavity. We can relate the frequency shift to the detuning time:

$$\phi = (\omega_0 - \omega_g)\delta T_A. \tag{4.10}$$

We can now write

$$-2j\left(\frac{\omega_0 - \omega_g}{\Delta\omega_g}\right)g + \Delta\omega_g\delta T_L - L_0(1 - r^2)\Delta\omega_g\delta T_A \times \exp\left(-j\frac{\omega_0 - \omega_g}{\Delta\omega_g}\Delta\omega_g\delta T_A\right) = 0. \tag{4.11}$$

If the gain bandwidth is broad, the first term is relatively unimportant, and the second and third term must cancel. But this means that the phase factor must be real. It is made real by the frequency shift $\omega_0 - \omega_g$. There are two choices for the phase factor, + and −. From Eq. (3.5) we realize that the choice of + corresponds to saturable absorption, that is, decrease of the loss with increasing intensity. The choice of − simulates the opposite action, which cannot produce stable pulses. Therefore, we pick the +. Let us solve for β and τ from Eq. (4.4). We take the pulse energy as given and normalize the three parameters of interest: the pulse width

$$\tau_n = \frac{\Delta\omega_g{}^2}{2g}L_0(1 - r^2)\frac{W}{|u_s|^2}\tau, \tag{4.12}$$

the dispersion

$$D_n = \frac{\Delta\omega_g{}^2 D}{g}, \tag{4.13}$$

and the detuning parameter

$$\delta T_n{}^2 = \frac{\Delta\omega_g{}^2\delta T_A{}^2}{2g}(1 - r^2)L_0. \tag{4.14}$$

In terms of these normalized parameters we obtain the equation

$$(2 + 3j\beta - \beta^2)\frac{1}{\tau_n} = \frac{1}{1 + jD_n + \delta T_n{}^2}. \tag{4.15}$$

We are now ready to plot pulse width and chirp parameter versus detuning (Figs. 5 and 6). One can further show that the bandwidth is equal to $(1 + \beta^2)^{1/2}/\tau$, and so this quantity is also plotted, in Fig. 7.

Any solution of the master equation has to be tested for stability. Equation (4.6) gives an expression for the difference between the gain and the loss as affected by the mode locking; the contribution of the mode locking is recognizable by the multiplier $1/\tau^2$. If the mode-locked solution is to be stable against a takeover by cw operation, the cw excitation has to see net loss (negative gain). For this purpose we rewrite the real part of Eq. (4.6) in the form

$$g - g\left(\frac{\omega_0 - \omega_g}{\Delta\omega_g}\right)^2 - [l + (1 - r)] + L_0(1 - r^2) = -\frac{g}{\Delta\omega_g{}^2\tau^2}[(1 - \beta^2)(1 + \delta\tau_n{}^2) - 2\beta D_n], \tag{4.16}$$

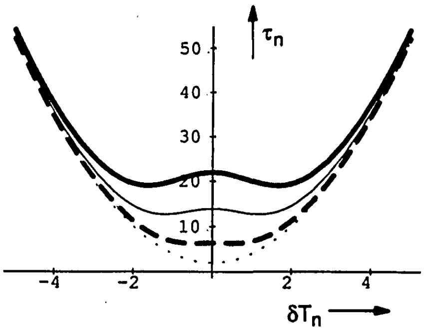

Fig. 5. Normalized pulse width versus δT_n.

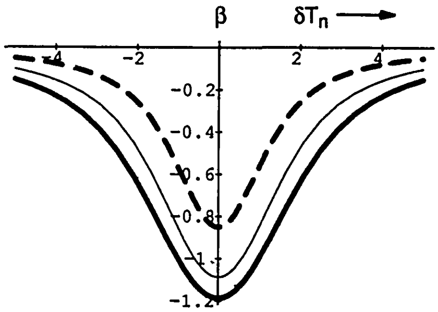

Fig. 6. Chirp parameter β versus δT_n for $D_n > 0$. For $D_n < 0$, β reverses sign.

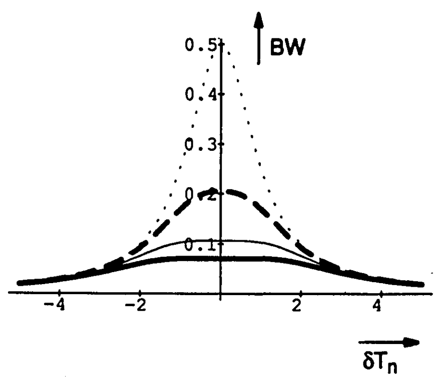

Fig. 7. Bandwidth $(1 + \beta^2)/\tau$ versus δT_n.

where we have used the fact that $\phi = 0$. The terms collected on the left-hand side are the gain experienced by a cw excitation offset by the frequency difference $\omega_0 - \omega_g$, the detuning term. The terms on the right-hand side are the contributions of the pulsed excitation. A cw excitation at the center of the gain line would experience a net gain that is equal to the left-hand side without the detuning term. We assume that this term is small, consistent with the previous assumption of a broad gain line. If cw excitation is to be suppressed, the left-hand side has to be negative, or

$$(1 - \beta^2)(1 + \delta\tau_n^2) - 2\beta D_n > 0. \tag{4.17}$$

This is the stability criterion. We have tested our solutions in Figs. 5–7 for stability and found it to be realized.

5. SOME EXPERIMENTAL RESULTS

The following results are based on an RPM Ti:sapphire laser.[11] The idea of self-stabilized operation by small frequency adjustments was experimentally verified. In the Ti:sapphire laser a two-plate birefringent tuning element was used, for which a FWHM bandwidth of 0.16 nm for cw operation was measured. Under mode-locked conditions a much wider spectrum was observed. Figure 8(a) shows the time-averaged spectrum width of the RPM laser at different cavity length detuning. However, the time-

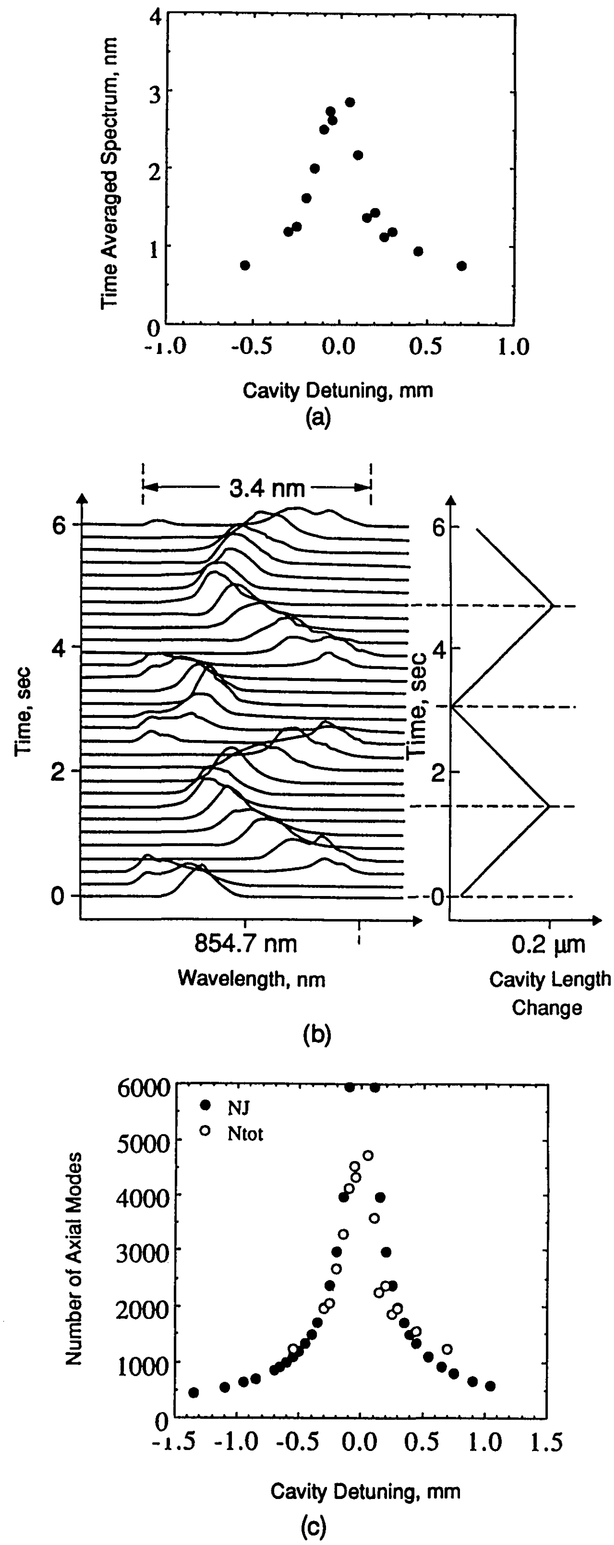

Fig. 8. (a) Time-averaged spectrum versus detuning. (b) Short-term spectrum versus detuning. (c) Number of axial modes versus detuning.

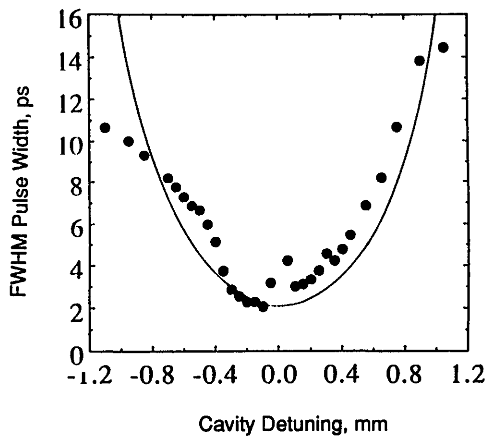

Fig. 9. Pulse width versus detuning. Solid curve, theory.

averaged spectrum includes the self-frequency shifts of the laser in order to maintain stable pulse trains, as discussed earlier, and does not correspond to the bandwidth of the mode-locked pulses. This is demonstrated with Fig. 8(b), where the optical spectrum is displayed as a function of an auxiliary cavity length change of only a fraction of a wavelength. While stable mode-locked pulse trains were observed, the center frequency of the optical spectrum adjusted itself to compensate for the phase shift introduced by the cavity length change and to maintain coherent superposition at the coupling mirror. Figure 8(b) also demonstrates that the actual bandwidth of the pulse is much less than that of the time-averaged spectrum of 3 nm. Figure 8(c) displays N_J, the minimal number of axial modes required in the gain bandwidth for self-frequency adjustments, Eq. (1.5), and N_{tot}, the number of axial modes in the time-averaged spectrum given in Fig. 8(a), as a function of cavity detuning. The fact that N_{tot} and N_J are equal demonstrates that the Ti:sapphire laser produces a time-averaged spectral width that is required for self-stabilization. Close to the matched cavity length, the phase-matched frequencies are too widely spaced compared with the bandwidth $\Delta\omega_g$, and the laser does not produce stable pulse trains without active cavity length stabilization. The optical multichannel analyzer used for this experiment had a measurement bandwidth of ~0.25 nm, and it was not possible to measure the real pulse bandwidth and the chirp parameter calculated in Figs. 6 and 7.

We observe that the repetition rate of the RPM Ti:sapphire laser depends on the cavity length detuning and output coupler. For a cavity length detuning of 11 mm, a pulse repetition rate change of 17.3 kHz was measured, with a repetition rate of 150.6 MHz. This result is in good agreement with the theory; from Eq. (4.11) and $\phi = 0$ it follows that

$$\delta T_L = L_0(1 - r^2)\delta T_A. \tag{5.1}$$

For a 3.5% transmission of C_1 (Fig. 1) r is 0.98 and the average power reflected from the quantum-well saturable absorber is ~25%, which gives an L_0 of 0.45, including the 10% output coupler, and for an 11-mm cavity detuning δT_A is 73.2 ps; with Eq. (5.1) the repetition rate change is 26 kHz, which is close to the measured value of 17.3 kHz.

Figure 9 displays the measured pulse width versus cavity length detuning. The solid line in Fig. 9 is the theoretically predicted pulse width as a function of cavity detuning. In evaluating the curve the following values are used: $L_0 = 0.45(1 - r^2) = 0.035$, $\Delta\omega_g = 3.8 \times 10^{12}$, and $g = 0.45$. The bandwidth is reasonably consistent with the measured time-averaged bandwidth, an indication of the frequency range over which the pulses can still find amplification as their carrier frequency drifts.

There is an asymmetry in the experimental curve that does not show up in the theory. We believe that the asymmetry is due to the slow relaxation process in the saturable absorber.[16] Close to matched cavity length the laser becomes unstable, as discussed above, and this produces longer autocorrelation pulses when averaged many times. Figure 9 shows that the pulses from the main cavity and the auxiliary cavity do overlap in time, because the pulse width is larger than the cavity length detuning. Equations (4.12) and (4.15) predict the pulse width as functions of cavity length detuning. We performed some initial pump–probe experiments[17] on the quantum-well reflector with another reflector piece with an identical structure in order to mode lock the Ti:sapphire laser. A fast reflectivity transient was observed with a time constant of the mode-locked pulse superimposed on a long time transient that was due to carrier recombination. At this point it is still not clear whether the small-gain saturation in the Ti:sapphire laser is enhancing the mode-locking process. However, the present theory suggests that the fast transient is enough to mode lock the laser.

DISCUSSION

The thrust of our theoretical argument is that RPM is self-stabilizing APM. RPM still depends on the coherent superposition of the pulses at the mirror. This self-stabilization is possible because the pulse width becomes longer with cavity detuning, so that the pulses still overlap as they meet at the coupling mirror.

The theory predicts the spectral shift associated with cavity length detuning. It gives the observed change of spectrum and the change of pulse width as a function of cavity length detuning. The theoretically predicted values agree reasonably well with the experimentally observed values, since the theory includes a number of approximations that may not be fully satisfied by the parameters of the physical system. There is no doubt, however, that the theory accounts qualitatively for the observed behavior of the physical system.

Note added in proof. The argument goes as follows. The loss factor L of a slow saturable absorber saturates according to the law

$$L = L_0 \exp - \int_{\infty}^{t} \mathrm{d}t |u|^2/E_s,$$

where E_s is the saturation energy. When $|u|^2$ is proportional to sech^2, the integral is proportional to tanh. Expansion of the exponential to first order gives

$$L = L_0\left\{1 - \frac{\tau}{E_s}A^2\left[\tanh\left(\frac{t}{\tau}\right) + 1\right]\right\}.$$

Addition of a term of the form $-Lu(\tau A^2/E_s)\tanh(t/\tau)$ produces a perturbation of the pulse that delays it by

$\delta\tau_d = L_0\tau A^2/E_s$. Hence the expansion parameter δT_A in Eq. (3.4) must be replaced by $\delta T_A + \delta\tau_d$. When this is done, a set of curves is obtained for different values of $\delta\tau_d$ that is asymmetric and matches better with the experiments.

ACKNOWLEDGMENT

The research of H. A. Haus was supported by Joint Services Electronics Program contract DAAL03-89-C-0001 and U.S. Air Force Office of Scientific Research contract F49620-88-C-0089.

REFERENCES

1. L. F. Mollenauer and R. H. Stolen, "The soliton laser," Opt. Lett. **9,** 13 (1984).
2. P. N. Kean, X. Zhu, D. W. Crust, R. S. Grant, N. Langford, and W. Sibbett, "Enhanced mode locking of color-center lasers," Opt. Lett. **14,** 39 (1989); D. E. Spence, P. N. Kean, and W. Sibbett, "60-fsec pulse generation from a self-mode-locked Ti:sapphire laser," Opt. Lett. **16,** 42 (1991).
3. K. J. Blow and D. Wood, "Mode-locked lasers with nonlinear external cavities," J. Opt. Soc. Am. B **5,** 629 (1988).
4. J. Mark, L. Y. Liu, K. L. Hall, H. A. Haus, and E. P. Ippen, "Femtosecond pulse generation in a laser with a nonlinear external resonator," Opt. Lett. **14,** 48 (1989).
5. E. P. Ippen, H. A. Haus, and L. Y. Liu, "Additive pulse mode locking," J. Opt. Soc. Am. B **6,** 1736 (1989).
6. J. Goodberlet, J. Wang, J. G. Fujimoto, and P. A. Schulz, "Femtosecond passively mode-locked Ti:Al_2O_3 laser with a nonlinear external cavity," Opt. Lett. **14,** 1125 (1989).
7. C. P. Yakymyshun, J. F. Pinto, and C. R. Pollock, "Additive-pulse mode-locked NaCl:OH^- laser," Opt. Lett. **14,** 621 (1989).
8. E. P. Ippen, L. Y. Liu, and H. A. Haus, "Self-starting condition for additive-pulse mode-locked lasers," Opt. Lett. **15,** 183 (1990).
9. L. Y. Liu, J. M. Huxley, E. P. Ippen, and H. A. Haus, "Self-starting additive-pulse mode locking of a Nd:YAG laser," Opt. Lett. **15,** 553 (1990).
10. J. Goodberlet, J. Wang, J. G. Fujimoto, and P. A. Schulz, "Starting dynamics of additive pulse mode locking in the Ti:Al_2O_3 laser," Opt. Lett. (to be published).
11. U. Keller, W. H. Knox, and H. Roskos, "Coupled-cavity resonant passive mode-locked Ti:sapphire laser," in *Digest of Ultrafast Phenomena VII* (Springer-Verlag, Berlin, 1990), p. 69; U. Keller, W. H. Knox, and H. Roskos, "Coupled-cavity resonant passive mode-locked Ti:sapphire laser," Opt. Lett. **15,** 1377 (1990); U. Keller, T. K. Woodward, D. L. Sivco, and A. Y. Cho, "Coupled-cavity resonant passive mode-locked Nd:YLF laser," Opt. Lett. **16,** 390 (1991).
12. H. A. Haus, "Theory of modelocking with a fast saturable absorber," J. Appl. Phys. **46,** 3049 (1975).
13. W. H. Knox, R. L. Fork, M. C. Downer, D. A. B. Miller, D. S. Chemla, C. V. Shank, and W. Wiegmann, "Femtosecond dynamics of resonantly excited excitons in room temperature GaAs quantum wells," Phys. Rev. Lett. **54,** 1306 (1985).
14. Y. Silberberg, P. W. Smith, D. J. Eilenberger, D. A. B. Miller, A. C. Gossard, and W. Wiegmann, "Passive mode locking of a semiconductor diode laser," Opt. Lett. **9,** 507 (1984); M. N. Islam, E. R. Stunderman, C. E. Soccolich, I. Bar-Joseph, N. Sauer, T. Y. Chang, and B. I. Miller, IEEE J. Quantum Electron. **25,** 2454 (1989).
15. O. E. Martinez, R. L. Fork, and J. P. Gordon, "Theory of passively mode-locked lasers including self-phase modulation and group-velocity dispersion," Opt. Lett. **9,** 156 (1984).
16. See the note added in proof.
17. U. Keller, G. W. 't Hooft, W. H. Knox, and T. K. Woodward, "Coupled-cavity modelocking of solid-state lasers using quantum wells," in *Quantum Optoelectronics, 1991,* Vol. 7 of 1991 OSA Technical Digest Series (Optical Society of America, Washington, D.C., 1991), pp. 232–235.