In situ small-signal gain of solid-state lasers determined from relaxation oscillation frequency measurements

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We present a simple, *in situ* technique to calculate the small-signal gain of typical solid-state continuous-wave lasers, such as Nd:YAG and Nd:YLF lasers, by measuring the frequency of the relaxation oscillation noise peak. The laser's small-signal gain can be directly calculated from the frequency of the relaxation oscillation with knowledge of the upper-state lifetime, the cavity round-trip time, and total losses, which are typically well-known values. When the laser is pumped many times above threshold the losses do not need to be known accurately. This technique is compared with the traditional method of changing output couplers to establish its accuracy.

The value of a laser's small-signal gain is a fundamental parameter often important for laser design and operation. For example, knowing the small-signal gain value permits calculation of the optimum output coupler in cw lasers, permits predictions of the laser pulse width and the necessary hold-off to prevent lasing in Q-switched lasers, and is an important design parameter affecting the dynamics of passively mode-locked solid-state lasers.[1] For a classical four-level laser it is well established that one can determine both the laser's small-signal gain and internal loss[2,3] by measuring the pump power at threshold and the laser's slope efficiency as a function of pump power, with two or more different output couplers. However, for certain lasers and cavity designs it is impossible accurately to measure the threshold and slope efficiency, e.g., in a lamp-pumped Nd:Yag laser, in which thermal lensing permits only a limited setting of the pump powers, or in a monolithic cavity, such as a nonplanar ring oscillator,[4] in which the output coupler is directly coated onto the laser crystal and cannot be changed or varied. The laser gain can also be measured by probing the gain element with another laser at the same wavelength. However, this is also experimentally more complicated.

We present here a technique to calculate the small-signal gain from the measured relaxation oscillation frequency noise peak of cw solid-state lasers such as Nd:YLF. To verify its reasonableness and accuracy, we compare the calculated small-signal gain obtained with the new technique to that from a conventional technique (slope efficiency versus output coupling). The laser transition's upper-state lifetime, the cavity round-trip time, and the total losses must also be known; however, these values are typically well known or are easy to measure. As the laser is pumped many times above threshold, the value for the internal losses can be neglected while reasonable accuracy is maintained.

The phenomenon of relaxation oscillations in lasers is well established.[5,6] Measurements of relaxation oscillations in solid-state lasers have been used to measure the stimulated emission cross section and upper-state lifetime[7] and internal laser losses.[8] Here we quickly review the fundamental applicable equations necessary for determination of the small-signal gain. For an ideal four-level laser system, the steady-state solutions to the rate equations can be reduced to a small-signal linear form and solved for its natural roots. The result predicts the response of the laser to small perturbations, i.e., pump fluctuations, mechanical noise, etc. The roots are of the form e^{st}, where s is given by[6]

$$s_{1,2} = \frac{-r}{2\tau_2} \pm \left[\left(\frac{r}{2\tau_2}\right)^2 - \frac{(r-1)}{\tau_2\tau_c}\right]^{1/2}, \qquad (1)$$

where $r = \delta_g/\delta_c$ is the normalized inversion ratio, i.e., how many times the laser is pumped above threshold ($r = 1$ at threshold); δ_g is the round-trip unsaturated or small-signal gain coefficient; δ_c is the total loss coefficient (output coupling plus internal losses); τ_2 is the upper-state lifetime; and $\tau_c = T_R/\delta_c$ is the cold cavity decay time, where T_R is the cavity round-trip time. When the second term within the square brackets is larger than the first term, the roots are complex and the response is a damped sinusoid. This condition typically occurs when the upper-state lifetime is large compared with the cavity decay time. The frequency of the damped sinusoid is known as the relaxation oscillation frequency. The relaxation oscillation frequency f_{ro} is then given by

$$f_{ro} = \frac{1}{2\pi}\left[\frac{(r-1)}{\tau_c\tau_2} - \left(\frac{r}{2\tau_2}\right)^2\right]^{1/2}. \qquad (2)$$

For a typical solid-state laser such as Nd:YAG and Nd:YLF the upper-state lifetime τ_2 (hundreds of microseconds) is much longer then the cavity decay time τ_c (tens to hundreds of nanoseconds). We can then neglect the second term in Eq. (2). Substituting the expressions for r and τ_c into Eq. (2), we obtain

$$f_{ro} = \frac{1}{2\pi}\left[(\delta_g - \delta_c)\frac{1}{T\tau_2}\right]^{1/2}. \qquad (3)$$

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Solving then for the small-signal gain coefficient δ_g gives

$$\delta_g = (2\pi f_{ro})^2 T \tau_2 + \delta_c\,. \tag{4}$$

Note that we are using the delta notation as described in Ref. 6. The actual small-signal gain is then $G = I_{out}/I_{in} = \exp(\delta_g)$, and the total round-trip losses are $\exp(-\delta_c)$, where $\delta_c = \delta_{oc} + \delta_{il}$, i.e., output coupling and internal losses. The output coupling should normally be expressed in this delta notation as $\delta_{oc} = \ln[1/(1 - T_{oc}]$, where T_{oc} is the intensity output coupling value. When the output coupling is small, $\delta_{oc} \cong T_{oc}$, e.g., $T_{oc} = 10\%$ gives $\delta_{oc} = 0.105$ (= 10.5%), and we can use these terms interchangeably with negligible error.

After measuring f_{ro}, we can then calculate the small-signal gain from Eq. (4). The upper-state lifetime is reasonably well known for most laser materials, or it can be measured. The round-trip time is determined either from the length of the laser ($T_R = L_{roundtrip}/c$) or by measurement of the frequency of the longitudinal-mode noise peak of the laser. The output coupler value should also be well known and can be verified by direct measurement. The internal losses, however, are an unknown quantity. However, if the laser is pumped many times over threshold (i.e., the small-signal gain is large compared with the internal loss), this term can be ignored with very small error. The maximum error (assuming $\delta_c = 0$) decreases as $1/r$, i.e., when the laser is 10 times over threshold the calculated value is within 10% of the actual value if the losses are known exactly.

It should also be noted that, when r becomes very large, the second term in Eq. (2) can no longer be ignored. When this term dominates, f_{ro} begins to decrease with increasing r. We can calculate this inflection point by setting the derivative of Eq. (2) with respect to r to zero and solving for r, which gives the value $r_{infl} = 2\tau_2/\tau_c$. If we put this value back into Eq. (2) and solve for the inflection frequency, we get $f_{infl} \cong 1/(2\pi\tau_c)$, with the assumption that $\tau_2 >> \tau_c$. Under normal operating conditions for most lasers these values are typically never reached. Additionally, under extreme pumping conditions other nonlinear effects, such as excited-state diffusion, can occur to shift f_{ro}.[9]

To verify the accuracy and reasonableness of this theory, we measured the output power and f_{ro} as a function of pump power for a Ti:sapphire-pumped Nd:YLF laser with several different values of output couplers. Figure 1 is a schematic of the laser cavity. Output coupler with measured values of 0.5%, 2.6%, 5.3%, and 9.5% were used. The longitudinal-mode noise peak was measured to be 250 MHz, giving a round-trip time of $T_R = 4$ ns. We measured the upper-state lifetime of the Nd:YLF of $\tau_2 = 450$ μs (by chopping the pump beam and observing the fluorescence decay time) and found it in good agreement with published values.[10] To measure f_{ro} we monitored the laser intensity with a silicon P-I-N photodiode whose output went into a low-noise, 1-MHz-bandwidth amplifier and then into an rf spectrum analyzer. The output coupler alignment was adjusted to give maximum output power (always with TEM_{00} spatial mode) and maximum f_{ro}.

Figure 2(a) shows the measured output power versus pump power. We fitted a straight line to these data to determine the pump threshold and slope efficiency, as shown in Fig. 2(b). Ideally the threshold would fall on a straight line from which we could extrapolate the internal loss.[2] Because the laser has low losses and high gain, resulting in a very low threshold, however, the threshold value has increasing error for smaller output couplers. Considering the slope efficiency instead of the threshold, we can fit the data to the function $\eta(\delta_{oc}) = \eta_0\delta_{oc}/(\delta_{oc} + \delta_{il})$.[3] This determines that $\eta_0 = 63 \pm 0.1\%$ and $\delta_{il} = 0.8 \pm 0.1\%$. We then fit the threshold assuming the 0.8% losses. From this we determine the threshold power $P_{th}(\text{mW}) = 3.8 + 4.8\delta_{oc}(\%)$. The inverse of the slope of $P_{th}(\delta_{oc})$ determines the small-signal gain, i.e., $\delta_g = 0.21 \pm 0.01\%$ per milliwatt of pump power.

Figure 3 shows the measured relaxation oscillation frequencies versus pump power for the different output couplers. The inset shows the typical spectrum analyzer data used to determine the noise peak of f_{ro}. This value is used to calculate the gain coefficient according to Eq. (4) with $\delta_c = \delta_{oc} + 0.8\%$ (internal loss). Figure 4 shows the calculated small-signal gain versus pump power for the different output couplers. As expected, we see that the small-signal gain coeffi-

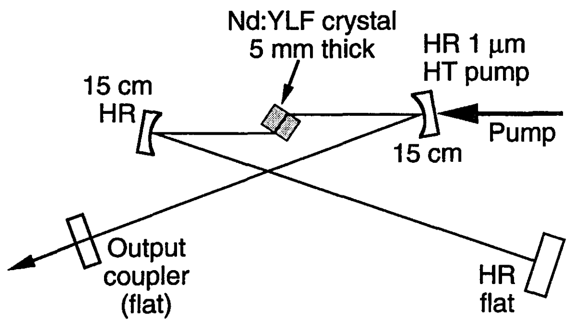

Fig. 1. Schematic of the cavity. The pump laser at 793 nm is focused to a radius at the crystal of ≈20 μm. The laser mode radius in the crystal is calculated as 90 μm × 130 μm by the *ABCD* matrix method. HR, highly reflecting; HT, highly transmitting.

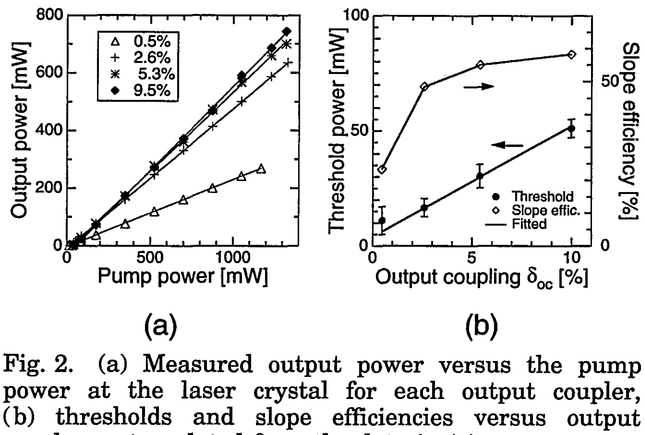

Fig. 2. (a) Measured output power versus the pump power at the laser crystal for each output coupler, (b) thresholds and slope efficiencies versus output couplers extrapolated from the data in (a).

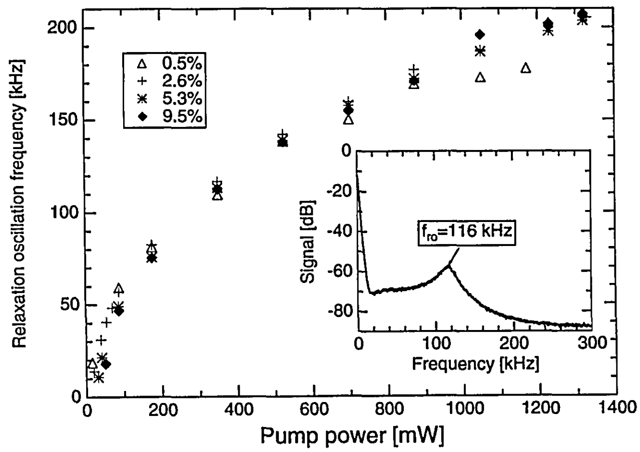

Fig. 3. Measured relaxation oscillation frequency versus pump power. The inset is typical spectrum analyzer data showing the relaxation oscillation noise peak (resolution bandwidth 3 kHz). The total amplitude noise of the laser is less than 1%.

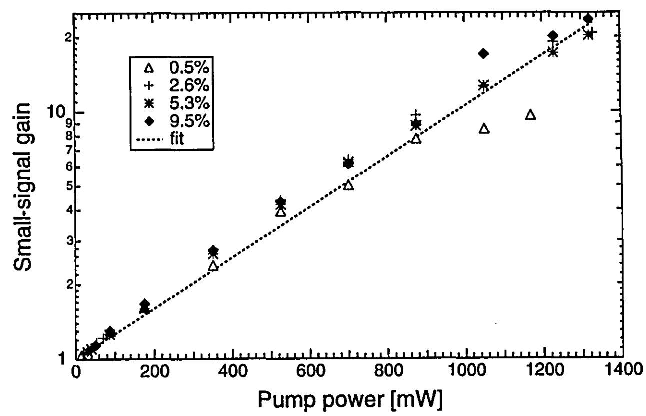

Fig. 4. Calculated small-signal gains from the measured f_{ro} for each value of the output coupler. The small-signal gain is $G = I_{out}/I_{in} = \exp(\delta_g)$, where δ_g is calculated from Eq. (4). The slope of the fit is the small-signal gain, 0.23 ± 0.01% per milliwatt of pump power.

cient is relatively linear with pump power but independent of output coupler. Note that for the higher output couplers or when we are far above threshold we can ignore the internal loss value with very small change in the calculated small-signal gain.

We then compare the values of δ_g calculated by the two methods. Using the average of the best-fit curves for the gain versus pump power, we obtain a value of $\delta_g = 0.23 \pm 0.01\%/\text{mW}$, which shows good agreement with the fit to the slope data of 0.21 ± 0.01%/mW.

We can also check the validity of dropping the second term of Eq. (2). For the 0.5% output coupler and the additional 0.8% estimated internal loss, we calculate an $r_{infl} = 2950$ and a corresponding f_{infl} = 500 kHz. The highest value for r here is approximately 230. This results in a 2% difference between Eq. (2) and (3). For the larger-value output couplers r_{infl} and f_{infl} are much higher, and this error in Eq. (4) can be neglected.

In conclusion, we have demonstrated a useful and simple method to measure the small-signal gain for solid-state lasers by measuring the relaxation oscillation frequency. This new technique gave a small-signal gain value similar to that from the conventional technique of changing output couplers and measuring slope efficiency. The relaxation oscillation frequency method allows for *in situ* measurement of the small-signal gain at any particular operating condition by measuring f_{ro} of the laser and then applying Eq. (4). This technique also takes advantage of the sensitivity, dynamic range, and frequency accuracy of typical spectrum analyzers to measure the frequency of the relatively small relaxation oscillation noise peak. This approach can be particularly useful for characterizing a laser when changing output couplers is not easy or is impossible, such as for a monolithic laser.

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