Stabilization of solitonlike pulses with a slow saturable absorber

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We show that a soliton of the nonlinear Schrödinger equation perturbed by filter losses and/or the finite gain bandwidth of amplifiers can be kept stable by saturable absorbers with a relaxation time much longer than the width of the soliton. This provides for ultrashort pulse generation with a slow saturable absorber only and may have possible applications in the stabilization of soliton storage rings.

The traditional concepts of ultrashort pulse generation rely either on a fast saturable absorber[1] [Fig. 1(a)], as is the case for additive-pulse or Kerr-lens mode-locking systems,[2] or on the interplay between a slow saturable absorber and gain saturation, as is the case with dye lasers[3] [Fig. 1(b)]. The two mechanisms open a net gain window in time so that only the pulse itself experiences gain per round trip. This permits the system to discriminate against noise that may grow outside the net gain interval, and therefore the pulse is kept stable against perturbations or noise.

Many experiments in the past have shown that soliton pulse shaping may lead to additional pulse shortening and stabilization in actively and passively mode-locked lasers.[4–6] Recently we showed both theoretically and experimentally that solitonlike pulse formation in actively mode-locked lasers[7] provides for considerable pulse shortening beyond the usual active mode-locking results. In this Letter we extend this analysis and show that, in the presence of solitonlike pulse shaping, even a slow saturable absorber with a recovery time much longer than the pulse width can stabilize the pulse [see Fig. 1(c)]. This is in contrast to the traditional picture in which the gain window has to close immediately before and after the passage of the pulse. This is possible in the soliton regime because, for the soliton, the nonlinear effects owing to self-phase modulation (SPM) and the linear effects owing to the negative group-velocity dispersion (GVD) are in balance. In contrast, the noise or instabilities that would like to grow are not intense enough to experience the nonlinearity and are therefore spread in time. However, when they are spread in time they experience the higher absorption that is due to the slowly recovering absorber after passage of the solitonlike pulse. Thus the instability modes see less gain per round trip than the soliton and will decay with time. In the following we give a theoretical justification of this simple picture, and we derive the limits on the pulse width set by the finite recovery time of the absorber and confirm this by numerical simulations.

We describe the laser mode locked by a slow saturable absorber by Haus's master equation of mode locking[1]:

$$T_R \frac{\partial}{\partial T} A(T, t) = \left[-iD \frac{\partial^2}{\partial t^2} + i\delta |A(T,t)|^2 \right] A(T, t) + \left[g - l + D_g \frac{\partial^2}{\partial t^2} - q(T, t) \right] A(T, t), \quad (1)$$

where $A(T, t)$ is the slowly varying field envelope, T_R is the cavity round-trip time, D is the intracavity GVD, $D_g = g/\Omega_g{}^2$ is the gain dispersion, δ is the nonlinear coefficient owing to SPM, l is the round-trip losses, g is the saturated gain, and Ω_g is the HWHM gain bandwidth. We assume that the gain has a large gain cross section so that the saturation of the gain medium within one pulse can be neglected. The saturable absorption $q(t)$ obeys the equation[8]

$$\frac{\partial q(T, t)}{\partial t} = -\frac{q - q_0}{T_A} - \frac{|A(T, t)|^2}{E_A}, \quad (2)$$

where we have assumed that the recovery time T_A of the saturable absorber is much faster than the round-trip time of the pulse so that the absorber recovers completely between two consecutive pulses. E_A is the saturation energy of the absorber, and q_0 is the nonsaturated loss of the absorber.

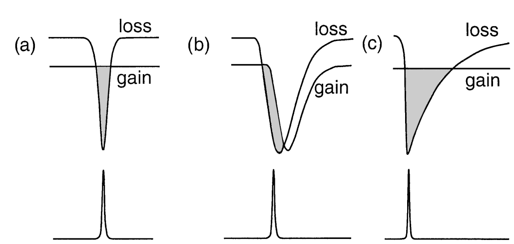

Fig. 1. Pulse-shaping mechanism owing to gain and loss dynamics in a mode-locked laser using (a) a fast saturable absorber only, (b) a slow saturable absorber plus slow gain saturation, (c) a slow saturable absorber plus soliton formation.

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The first part of Eq. (1) is the nonlinear Schrödinger equation, which has the well-known fundamental soliton solution given by

$$A_s(T,\, t) = \left(\frac{W}{2\tau}\right)^{1/2} \operatorname{sech}\left(\frac{t}{\tau}\right) \exp\left(i\Phi_0 \frac{T}{T_R}\right),$$

$$\Phi_0 = \frac{\delta W}{4\tau} = \frac{|D|}{\tau^2}, \tag{3}$$

where W is the pulse energy, Φ_0 is the phase shift of the soliton per round trip in the cavity, and the FWHM pulse width is given by $\tau_{\mathrm{FWHM}} = 1.76\tau$. We further assume that the saturation energy of the saturable absorber is much smaller than the energy of the soliton, i.e., $W \gg E_A$. Then the soliton saturates the absorber completely, and we obtain for the energy balance from Eqs. (2) and (3) in steady state $g = l + D_g/(3\tau^2)$, since the absorber is completely saturated. During passage of the soliton through the filter and the saturable absorber, a continuum is generated that will couple back to the soliton. However, as we showed for the case of active mode locking,[7] if the modulation function varies slowly on a time scale of the width of the soliton, the backcoupling of the continuum to the soliton can be neglected. This is also true for passive mode locking if the absorber has a recovery time much longer than the width of the soliton. Then one can show in first-order perturbation theory, analogous to the case of active mode locking,[7] that the decay of the continuum is described by the following equation:

$$T_R \frac{\partial}{\partial T} G(T,\, t) = \left[g - l + (D_g - iD)\frac{\partial^2}{\partial t^2} - q(t) \right] \times G(T,\, t). \tag{4}$$

We did not include the excitation of the continuum by the soliton when it passes through the perturbing bandwidth limitation and the saturable absorber because it is not important for stability considerations. $G(T, t)$ denotes the associated function of the continuum, as introduced by Gordon,[9] that uniquely determines the continuum. The saturable absorption recovering from full saturation, $q(t) = q_0[1 - \exp(-t/T_A)]$ is shown in Fig. 2(a). The solitonlike pulse is stable if all the continuum solutions of Eq. (4) decay in time; i.e., all the eigenvalues of the differential operator on the right-hand side of Eq. (4) must have a negative real part, i.e., $\mathrm{Re}(\lambda_n) < 0$, with

$$\lambda_n = \frac{D_g}{3\tau^2} - E_n\,,$$

$$\left[-\overline{D}\, \frac{\partial^2}{\partial t^2} + q(t) \right] G_n(t) = E_n G_n(t)\,, \tag{5}$$

where $\overline{D} = D_g - iD$. Equations (5) are an eigenvalue problem of a Schrödinger operator for a one-sided exponential potential. GVD transforms this Schrödinger operator to a non-Hermitian operator. Nevertheless, we can compute the eigenvalues of Eqs. (5) by analytic continuation. Figure 2(b) shows the real part of the ground-state eigenvalue for the limit $-D \gg D_g$, which then depends only on the normalized potential width $w = T_A\sqrt{q_0/\Phi_0}/\tau$, where we have used the relation between the width of the soliton, the GVD, and the phase shift per round trip [Eq. (3)]. As expected, the real part of the ground state goes to zero with increasing potential width. However, increasing the amount of negative GVD (and therefore increasing the nonlinear phase shift per round trip) while maintaining constant soliton width lowers the normalized potential width and therefore increases the range of stability. Increased stability, i.e., increased $\mathrm{Re}(E_0)$, can be used to compensate for the filter losses of the soliton equations (5) and therefore provides for a shorter solitonlike pulse.

So far we have demonstrated that spreading of the continuum by dispersion reduces the round-trip gain of the continuum below threshold. In principle this result shows that arbitrarily short pulses can be stabilized if one is permitted to increase SPM, i.e., the nonlinear phase shift, without limits. This is possible if Eq. (1) describes pulse propagation in a continuous medium. However, in the case in which Eq. (1) is the master equation of a mode-locked laser

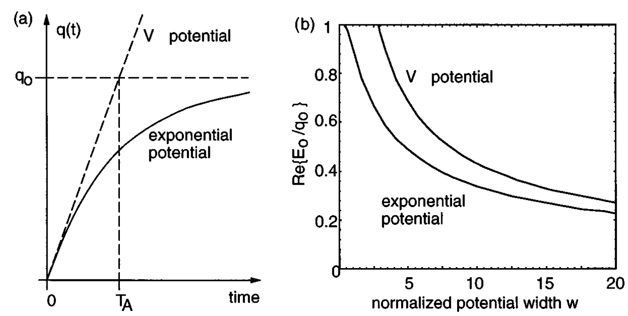

Fig. 2. (a) Potential $q(t)$ if the saturable absorber is immediately bleached after passage of the short intense soliton. The absorber recovers on a time scale much longer than the width of the soliton. (b) Real part of the ground state of the eigenvalue problem equations (5), assuming large negative GVD for the exponential potential and the V potential.

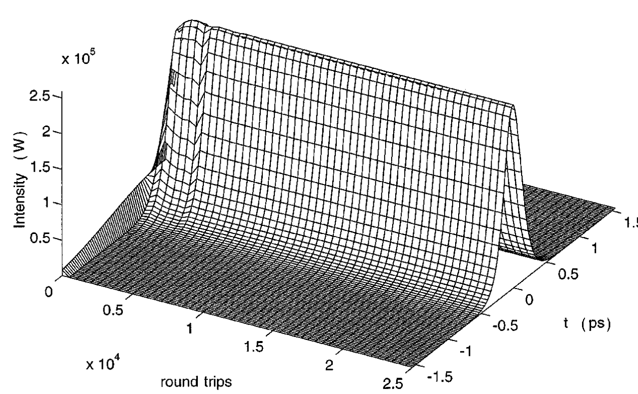

Fig. 3. Numerical simulation of the pulse evolution in a laser in the time domain modeled by the master equation (1); however, the simulation is performed with the discrete action of SPM and GVD per round trip by use of the split-step Fourier-transform method. We start with a 2-ps-long pulse and end up with a 300-fs-long transform-limited sech pulse after 25,000 round trips.

the nonlinear phase shift per round trip must be small, i.e., $\Phi_0 = 10\%$, so that the laser dynamics is well approximated by Eq. (1).

For a large potential width we can derive an approximate formula for the real part of the lowest eigenvalue by approximating the exponential potential in Fig. 2(a) by an infinite V potential. The lowest eigenvalue[10] follows by analytic continuation, $E_0{}^V = 2.338 q_0[(D_g - iD)/(q_0 T_A{}^2)]^{1/3}$ or $\mathrm{Re}(E_0{}^V) \approx 2q_0 w^{-2/3}$ for $-D \gg D_g$, which is shown in Fig. 2(b) as a function of w. For $w > 10$ the V-potential approximation for the ground-state eigenvalue is a useful approximation. We then obtain from Eqs. (5) for the minimum stable pulse width, i.e., $\mathrm{Re}(\lambda_0) = 0$:

$$\tau = \left(\frac{1}{\sqrt{6}\,\Omega_g}\right)^{3/4} \left(\frac{T_A g^{3/2}}{q_0}\right)^{1/4} (\Phi_0)^{-1/8}. \qquad (6)$$

This analytic result permits us to compare this new regime of mode locking, in which soliton formation and stabilization by a slow absorber is used, with the case of fast-saturable-absorber mode locking with respect to the pulse width achievable. In the fast-saturable-absorber case with SPM we obtain[2] $\tau_{\mathrm{FSA}} = \sqrt{g/2q_0}/\Omega_g$, where in this case q_0 denotes the peak change in absorption. The additional factor one half is for the additional reduction in pulse width owing to SPM if the ratio between SPM and the saturable-absorber strength is large. The ratio $\tau/\tau_{\mathrm{FSA}} \approx (T_A \Omega_g)^{1/4}$ indicates that the pulse widths achievable with the same amount of saturable absorption are comparable for fast and slow absorbers and that the ratio decreases only slowly with increasing recovery time of the absorber, a rather surprising result.

As an example, we consider the case of a Nd:glass laser with a gain HWHM bandwidth of $\Omega_g = 2\pi \times 4 \times 10^{-12}\ \mathrm{s}^{-1}$, and with the following other laser parameters: $g = 5\%$, $W = 100$ nJ, $q_0 = 2\%$, $T_A = 10$ ps, $E_A = 10$ nJ, and $\Phi_0 = 10\%$. Then from Eq. (6) we obtain $\tau_{\mathrm{FSA,FWHM}} = 78$ fs or $\tau_{\mathrm{FWHM}} = 164$ fs. Thus in this case the slow absorber is almost as good as the fast absorber. We set the amount of negative dispersion somewhat larger than the minimum required for the shortest pulse, to stay away from instability, and we choose the SPM coefficient so that we achieve a nonlinear phase shift of roughly 10% per round trip, i.e., $-D = 2000\ \mathrm{fs}^2$, $\delta = 0.5 \times 10^{-6}$/W. Figure 3 shows the results of the computer simulation of the laser, which results in a 300-fs short pulse at FWHM in steady state, still more than a factor-of-30 shorter than the recovery time of the absorber.

In conclusion, the results presented here have several implications. In contrast to the passive mode-locking techniques commonly used (and known as fast- or slow-absorber mode locking), in which the essential mechanism is the generation of a net gain window that closes immediately before and after the pulse, we have shown that a slow saturable absorber alone plus soliton formation can lead to the generation of ultrashort pulses much shorter than the recovery time of the absorber. A factor of $T_A/\tau = 20$ is easily reached without too much degradation in the pulse length achievable when compared with an ideal fast saturable absorber. Semiconductor absorbers with a fast recovery time owing to intraband carrier–carrier scattering and with thermalization processes of 100 fs or even below are in principle fast enough to provide for 10-fs pulse generation without the need for Kerr-lens mode locking. Kerr-lens mode locking leads to strong beam deformations owing to self-focusing, especially in the sub-30-fs regime, which requires strong restrictions on the cavity design. In contrast, the slow-saturable-absorber mode locking plus soliton formation presented here does not interrelate the properties of the laser mode with the laser dynamics, and the parameters of the absorber can be chosen in such a way that the mode locking is self-starting and that the laser is stable against Q switching.[11] This will become extremely important in diode-pumped lasers in which Kerr-lens mode locking is weak because of the large pump area in the laser crystal. Recent experimental results with Nd:glass and diode-pumped Cr:LiSAF lasers[12,13] have already shown that semiconductor absorbers can be used to generate sub-100-fs pulses. Replacing T in Eq. (1) by propagation distance, we see that our analysis also shows that a distributed slow saturable absorber can stabilize a propagating solitonlike pulse for which the loss is compensated by a bandwidth-limited gain. Thus a slow absorber can also be used to stabilize propagating solitons or solitons in a storage ring similar to that described in Ref. 14.

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