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Simple analytical expressions for the reflectivity and the penetration depth of a Bragg mirror between arbitrary media

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Abstract

Starting with the coupled-mode equations, we derive analytical expressions for the reflectivity and the penetration depth of a dielectric Bragg mirror. In contrast to previous coupled-mode calculations, the interfaces to the adjacent media have been included, and we show that they can have a large effect on the penetration depth. The simple analytical formulas are very accurate, as comparisons to numerical calculations show.

1. Introduction

High reflectivity coatings consisting of a stack of dielectric quarter wave layers, also denoted as Bragg mirrors or Bragg reflectors, have a wide range of applications. For the design of such a highly reflecting dielectric multilayer structure, knowledge of the reflectivity, phase, and dispersion is of great importance. In general, numerical methods based on the transmission matrix formalism [1] or analytical methods based on the coupled-mode equations [2–5] are used for the design of the structure. The latter, however, only yields accurate results if the difference of the refractive indices of the layers is not too large. Although these techniques are well established and documented in many textbooks, the topic is still of current interest, motivated by a wide range of applications in laser optics, integrated optics, and optoelectronics. For example, analytic expressions for the reflectivity and the penetration depth at the Bragg wavelength have recently been derived based on a hyperbolic tangent substitution technique [6,7]. The penetration depth is related to the group delay or the derivative of the phase with respect to the wave number. The effective optical length of a Fabry–Pérot sandwiched between two Bragg mirrors and therefore the free spectral range is given by the sum of the optical thickness of the layer and the penetration depths of the two Bragg mirrors. This parameter is important for small-cavity optoelectronic devices, such as vertical cavity lasers, microcavity lasers, Fabry–Pérot modulators, or Fabry–Pérot saturable absorbers used to start and stabilize passive mode-locking in solid-state lasers [8,9].

In this paper, we derive an analytical expression for the reflectivity and the penetration depth of a Bragg mirror. We use the coupled-mode equations to find a matrix which connects the forward and backward travelling field amplitudes at any point in the Bragg medium. We then derive the boundary conditions at the interface to a medium with constant refractive index and use them to obtain an expression for the reflectivity of the entire structure. These boundaries in general have been neglected in previous analytical treatments based on coupled-mode theory. A comparison to numerical calculations, however, shows that without inclusion of the boundaries, a large error can

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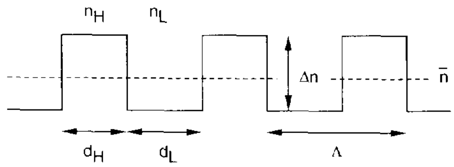

Fig. 1. Schematic view of a medium with a periodic variation of refractive index.

occur, especially in the phase of the reflectivity. In contrast, our analytical expressions lead to very accurate results. Furthermore, we calculated the penetration depth of an infinitely thick Bragg mirror at the Bragg wavelength and found surprisingly simple expressions.

2. Coupled-mode equations and propagation in a periodic dielectric medium

In this section we will derive a matrix that describes the propagation of the field in a period medium as is depicted in Fig. 1 with two alternating indices n_H and n_L, the periodicity Λ, and an average refractive index $\bar{n}$ defined as

$$\Lambda\bar{n}=d_H n_H + d_L n_L\,, \tag{1}$$

which, multiplied by the length of the medium, gives its correct optical thickness.

Consider a wave with the amplitude $u(z)=u_0\exp(\mathrm{i}\beta z)$ with the (complex) propagation constant $\beta=\bar{n}k+\mathrm{i}\alpha$ propagating in this medium, where u denotes one component of the electric field, k the vacuum wave number, and α the amplitude absorption coefficient. It is well known that the periodic perturbation of the medium generates side-bands with the wave numbers $\beta\pm2\pi/\Lambda$ which are in resonance with counter propagating waves if $|\beta|=\pi/\Lambda=\bar{n}k_B$ (i.e. the Bragg condition with the Bragg wave number k_B). We derive now a matrix which connects the amplitudes travelling in positive (u^+) and negative (u^-) z-direction at two different points z_0 and z_1 inside the medium:

$$\begin{pmatrix} u^+(z_1) \\ u^-(z_1) \end{pmatrix} = M(z_1, z_0) \begin{pmatrix} u^+(z_0) \\ u^-(z_0) \end{pmatrix}. \tag{2}$$

In an unperturbed medium, the matrix M simply is given by the propagator

$$M(\Delta z) = \begin{pmatrix} \exp(\mathrm{i}\beta\Delta z) & 0 \\ 0 & \exp(-\mathrm{i}\beta\Delta z) \end{pmatrix}, \tag{3}$$

with $\Delta z=z_1-z_0$. To calculate M in a perturbed medium, we start with the coupled-mode equations which normally are written in the form (see e.g. [3])

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{pmatrix} u^+ \\ u^- \end{pmatrix} = \begin{bmatrix} \mathrm{i}\beta & \kappa\exp\left(\mathrm{i}\dfrac{2\pi}{\Lambda}(z-s)\right) \\ \kappa\exp\left(-\mathrm{i}\dfrac{2\pi}{\Lambda}(z-s)\right) & -\mathrm{i}\beta \end{bmatrix} \begin{pmatrix} u^+ \\ u^- \end{pmatrix}, \tag{4}$$

with the coupling coefficient κ which is, in the case of a rectangular-shaped structure as shown in Fig. 1, given by [5]

$$\kappa=\frac{2\Delta n}{\lambda} \tag{5}$$

with the wavelength λ. The parameter s is introduced to obtain the correct phase of the reflectivity, i.e. the phase relation between u^+ and u^-, at any point in the Bragg medium. It denotes the "shift" of the Bragg grating with

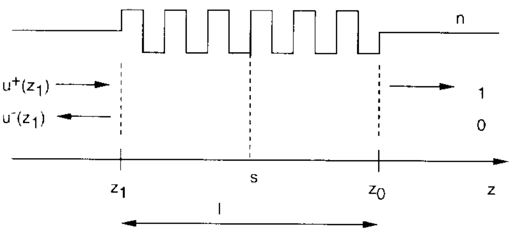

Fig. 2. Schematic view of a distributed Bragg reflector. The parameter s denotes the position of any positive index step. The reflectivity can be calculated by choosing only a wave travelling in the positive z-direction at the right boundary.

respect to the reference point or, in other words, the position z of any positive index step as depicted in Fig. 2.

Eq. (4) can be solved in the well-known manner by splitting off the fast phase factors $\exp[\pm i(2\pi/\Lambda)(z-s)]$ and looking for eigenvectors of the remaining system. We obtain then for the matrix as defined in (2):

$$M(z_1, z_0) = \frac{1}{\Gamma}\begin{pmatrix} \exp\left(i\frac{\pi}{\Lambda}\Delta z\right)[\Gamma\cosh(\Gamma\Delta z)+i\delta\sinh(\Gamma\Delta z)] & \exp\left(i\frac{\pi}{\Lambda}(z_1+z_0+2s)\right)\kappa\sinh(\Gamma\Delta z) \\ \exp\left(-i\frac{\pi}{\Lambda}(z_1+z_0+2s)\right)\kappa\sinh(\Gamma\Delta z) & \exp\left(-i\frac{\pi}{\Lambda}\Delta z\right)[\Gamma\cosh(\Gamma\Delta z)-i\Delta\sinh(\Gamma\Delta z)] \end{pmatrix} \tag{6}$$

with $\Delta z = z_1 - z_0$, the detuning parameter $\delta = \beta - \pi/\Lambda$, and the propagation constant in the Bragg medium $\Gamma = +\sqrt{\kappa^2-\delta^2}$. With (6), the amplitude reflectivity r of a distributed Bragg reflector of length $l = z_0 - z_1 = -\Delta z$ as depicted in Fig. 2 can be calculated by writing

$$\begin{pmatrix} u^+(z_1) \\ u^-(z_1) \end{pmatrix} = M(z_1, z_0)\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad r = \frac{u^-(z_1)}{u^+(z_1)} = \frac{M(z_1, z_0)_{21}}{M(z_1, z_0)_{11}} \tag{7}$$

with the result

$$r = \exp\left(-i\frac{2\pi}{\Lambda}(z_1 - s)\right)\frac{-\kappa\sinh(\Gamma l)}{\Gamma\cosh(\Gamma l) - i\delta\sinh(\Gamma l)}. \tag{8}$$

In the literature, the phase factor in front of the expression for r typically is omitted, It is $+1$ if $z_1 - s = m\Lambda$ for any integer m (the Bragg reflector thus starts with a layer with higher refractive index) and -1 if $z_1 - s = (m+\frac{1}{2})\Lambda$ (i.e. the Bragg reflector starts with a layer with lower refractive index). Eq. (8) is the usually quoted formula for the reflectivity of a Bragg mirror. It is, however, not correct (within the approximations of the coupled-mode theory), since the transmission matrices at the ends of the mirror have not been taken into account. As we will show in the next section, neglecting this matrix may lead to a large error of the phase and the penetration depth into the mirror.

3. Boundary conditions at the end of a periodic medium

The calculation of the reflectivity at an interface between two adjacent dieletric media with transmission matrices is based on the requirement of continuity of the wave amplitudes and their derivatives. In the same manner, we can calculate the transmission matrix of the amplitudes for the case of a Bragg medium described by the parameters $\bar{n}_2$, κ, Λ, and s adjacent to an unperturbed medium with refractive index n_1 (Fig. 3). The derivatives of the waves in the unperturbed medium are

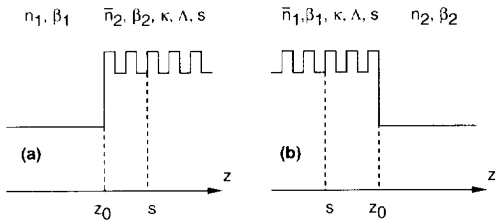

Fig. 3. Schematic view of a Bragg medium adjacent to a medium with constant refractive index.

$$\frac{d}{dz}\begin{pmatrix} u_1^+ \\ u_1^- \end{pmatrix}_{z0} = \begin{pmatrix} i\beta_1 & 0 \\ 0 & -i\beta_1 \end{pmatrix}\begin{pmatrix} u_1^+ \\ u_1^- \end{pmatrix}_{z0}, \tag{9}$$

and in the Bragg medium

$$\frac{d}{dz}\begin{pmatrix} u_2^+ \\ u_2^- \end{pmatrix}_{z0} = \begin{bmatrix} i\beta_2 & \kappa \exp\left(i\frac{2\pi}{\Lambda}(z_0-s)\right) \\ \kappa \exp\left(-i\frac{2\pi}{\Lambda}(z_0-s)\right) & -i\beta_2 \end{bmatrix}\begin{pmatrix} u_2^+ \\ u_2^- \end{pmatrix}_{z0}. \tag{10}$$

Continuity of the amplitudes at $z=z_0$ therefore requires

$$u_1^+ + u_1^- = u_2^+ + u_2^-, \tag{11}$$

and the continuity of the derivatives leads to

$$i\beta_1(u_1^+ - u_1^-) = \left[i\beta_2 + \kappa \exp\left(-i\frac{2\pi}{\Lambda}(z_0-s)\right)\right]u_2^+ + \left[-i\beta_2 + \kappa \exp\left(i\frac{2\pi}{\Lambda}(z_0-s)\right)\right]u_2^-. \tag{12}$$

We can solve (11) and (12) for u_1^+ and u_1^- and obtain the transmission matrix for the transition from the Bragg medium to the unperturbed medium (Fig. 3a)

$$\begin{pmatrix} u_1^+ \\ u_1^- \end{pmatrix} = \frac{1}{2\beta_1}\begin{pmatrix} \beta_1+\beta_2-i\kappa\exp\left(-i\frac{2\pi}{\Lambda}(z_0-s)\right) & \beta_1-\beta_2-i\kappa\exp\left(i\frac{2\pi}{\Lambda}(z_0-s)\right) \\ \beta_1-\beta_2+i\kappa\exp\left(-i\frac{2\pi}{\Lambda}(z_0-s)\right) & \beta_1+\beta_2+i\kappa\exp\left(i\frac{2\pi}{\Lambda}(z_0-s)\right) \end{pmatrix}\begin{pmatrix} u_2^+ \\ u_2^- \end{pmatrix}. \tag{13}$$

For $\kappa=0$, (13) is identical to the well-known Fresnel matrix. In the same way, we obtain for the transmission matrix from the unperturbed to the Bragg medium (Fig. 3b)

$$\begin{pmatrix} u_1^+ \\ u_1^- \end{pmatrix} = \left\{2\left[\beta_1 - \kappa\sin\left(\frac{2\pi}{\Lambda}(z_0-s)\right)\right]\right\}^{-1}$$
$$\times\begin{pmatrix} \beta_1+\beta_2+i\kappa\exp\left(i\frac{2\pi}{\Lambda}(z_0-s)\right) & \beta_1-\beta_2+i\kappa\exp\left(i\frac{2\pi}{\Lambda}(z_0-s)\right) \\ \beta_1-\beta_2-i\kappa\exp\left(-i\frac{2\pi}{\Lambda}(z_0-s)\right) & \beta_1+\beta_2-i\kappa\exp\left(-i\frac{2\pi}{\Lambda}(z_0-s)\right) \end{pmatrix}\begin{pmatrix} u_2^+ \\ u_2^- \end{pmatrix}. \tag{14}$$

Again, (14) becomes the Fresnel matrix for $\kappa=0$.

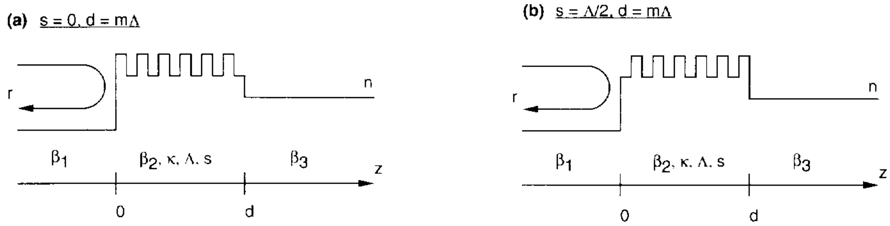

Fig. 4. Schematic view of a Bragg mirror between two media with constant refractive index. Two cases have to be distinguished: either the mirror starts with a layer with higher refractive index (a) or with a layer with lower refractive index (b).

4. Reflectivity and penetration depth of a Bragg mirror

With the matrices as defined in (6), (13), and (14), we are in principle able to calculate the reflectivity of a Bragg mirror taking into account the influence of the two interfaces at the boundaries of the mirror. We consider a Bragg mirror of length d, defined by β_2, κ, Λ, and s. The two adjacent media are described by the propagation constants β_1 and β_3. The Bragg mirror starts at $z=0$, where we can calculate the reflectivity by multiplying the matrice (14), (6), and (13). We consider two important cases (see Fig. 4), where the mirror consists of an integer number of periods (i.e. $d=m\Lambda$) and starts either with a layer with higher index (i.e. $s=0$, Fig. 4a) or with lower index (i.e. $s=\frac{1}{2}\Lambda$, Fig. 4b). After the multiplications of the matrices and using the abbreviations

$$C \equiv \cosh(\Gamma d), \quad S \equiv \sinh(\Gamma d)\,, \tag{15}$$

we obtain in the first case ($s=0$, Fig. 4a) for the amplitude reflectivity and transmission

$$r = -\frac{\Gamma C\beta_2(\beta_3-\beta_1)+\kappa\beta_2 S(\beta_3+\beta_1+2\mathrm{i}\kappa)+\mathrm{i}\delta S[\mathrm{i}\kappa(\beta_3+\beta_1)-\beta_2^2+\beta_1\beta_3-\kappa^2]}{\Gamma C\beta_2(\beta_3+\beta_1)+\kappa\beta_2 S(\beta_3-\beta_1+2\mathrm{i}\kappa)+\mathrm{i}\delta S[\mathrm{i}\kappa(\beta_3-\beta_1)-\beta_2^2-\beta_1\beta_3-\kappa^2]},$$

$$t = \frac{2(-1)^m\beta_1\beta_2\Gamma}{\Gamma C\beta_2(\beta_3+\beta_1)+\kappa\beta_2 S(\beta_3-\beta_1+2\mathrm{i}\kappa)+\mathrm{i}\delta S[\mathrm{i}\kappa(\beta_3-\beta_1)-\beta_2^2-\beta_1\beta_3-\kappa^2]}, \tag{16}$$

and in the second case ($s=\frac{1}{2}\Lambda$, Fig. 4b)

$$r = \frac{\Gamma C\beta_2(\beta_1-\beta_3)+\kappa\beta_2 S(\beta_1+\beta_3-2\mathrm{i}\kappa)+\mathrm{i}\delta S[\mathrm{i}\kappa(\beta_1+\beta_3)+\beta_2^2-\beta_1\beta_3+\kappa^2]}{\Gamma C\beta_2(\beta_1+\beta_3)+\kappa\beta_2 S(\beta_1-\beta_3+2\mathrm{i}\kappa)+\mathrm{i}\delta S[\mathrm{i}\kappa(\beta_1-\beta_3)-\beta_2^2-\beta_1\beta_3-\kappa^2]},$$

$$t = \frac{2(-1)^m\beta_1\beta_2\Gamma}{\Gamma C\beta_2(\beta_1+\beta_3)+\kappa\beta_2 S(\beta_1-\beta_3+2\mathrm{i}\kappa)+\mathrm{i}\delta S[\mathrm{i}\kappa(\beta_1-\beta_3)-\beta_2^2-\beta_1\beta_3-\kappa^2]}. \tag{17}$$

Eqs. (16) and (17) are much more complicated than the usually quoted formula (8) for the reflectivity of a Bragg mirror, but they yield the correct phase and penetration depth as we will show later. We can rewrite (16) for a DBR-structure with $\beta_1=\beta_2=\beta_3=\beta$ and obtain

$$r = \frac{-\kappa S(1+\mathrm{i}\kappa/\beta)+\mathrm{i}\delta S(\mathrm{i}\kappa/\beta-\kappa^2/2\beta^2)}{\Gamma C-\mathrm{i}\delta S(1+\kappa^2/2\beta^2)+\kappa S\,\mathrm{i}\kappa/\beta}. \tag{18}$$

It is interesting to note that (8) is not correct even in this case since the transmission matrices (13) and (14) are not unity for a nonzero coupling coefficient κ. As expected, however, (18) is identical to (8) if κ/β goes to zero.

We consider now a practical example: a dielectric mirror consisting of four pairs of SiO_2 ($n=2.4$) and TiO_2 ($n=1.45$) layers evaporated on a GaAs substrate ($n=3.2$), as was typically used in our antiresonant Fabry–Pérot saturable absorbers for mode-locked Nd-doped lasers [8,9]. The reflectivity has been calculated by (a) using Eq. (16), (b) using Eq. (8) and therefore neglecting the interfaces at the boundaries of the mirror, and (c) by an exact

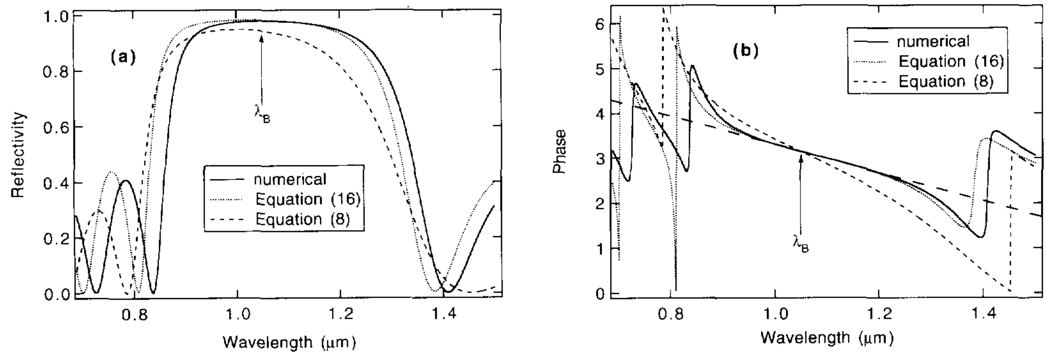

Fig. 5. Calculated reflectivity (a) and phase (b) of a Bragg mirror as depicted in Fig. 4 by using a numerical multiplication of the transmission matrices, Eq. (16) (our result), and Eq. (8) (the usually quoted formula).

numerical multiplication of the transmission matrices at each single interface. The results are depicted in Fig. 5. We see that Eq. (16) gives, at least near the Bragg wavelength (1.05 μm in this case), a very accurate result whereas the commonly used formula (8) leads to a large error in the phase. Note that the error in the derivative of the phase, which is directly related to the group delay or the penetration depth, is almost as high as a factor of two.

As the next step, we derive an analytical expression for the group delay and the penetration depth of an infinitely thick (i.e. $l \to \infty$) Bragg mirror. The reflectivity can be calculated in the same way, but the matrix in (14) is now the unity matrix and $\cosh(\Gamma d) \approx \sinh(\Gamma d)$. We obtain

$$r = \frac{(\beta_1 - \beta_2 + i\kappa)(\Gamma - i\delta) - (\beta_1 + \beta_2 + i\kappa)\kappa}{(\beta_1 + \beta_2 - i\kappa)(\Gamma - i\delta) - (\beta_1 - \beta_2 - i\kappa)\kappa} \tag{19}$$

for $s=0$ (Fig. 4a) and

$$r = \frac{(\beta_1 - \beta_2 - i\kappa)(\Gamma - i\delta) - (\beta_1 + \beta_2 - i\kappa)\kappa}{(\beta_1 + \beta_2 + i\kappa)(\Gamma - i\delta) + (\beta_1 - \beta_2 + i\kappa)\kappa} \tag{20}$$

for $s = \frac{1}{2}\Lambda$ (Fig. 4b). The penetration depth L_{eff} into the mirror is defined by (see e.g. [4])

$$\left.\frac{d\Phi}{dk}\right|_{k=k_B} = \left.\frac{d}{dk}\arg(r)\right|_{k=k_B} = 2\bar{n}_2 L_{\text{eff}} \tag{21}$$

with the phase Φ of the reflectivity, $r(k) = |r| \exp[i\Phi(k)]$, and the average refractive index $\bar{n}_2$ of the mirror as defined in (1). It has the following meaning: the phase nearby the Bragg wave number k_B can be written as the linear approximation (see also Fig. 5b)

$$\Phi(k) \approx \pi + 2\bar{n}_2 L_{\text{eff}}(k - k_B), \quad s=0,$$

$$\Phi(k) \approx 2\bar{n}_2 L_{\text{eff}}(k - k_B), \quad s = \tfrac{1}{2}\Lambda; \tag{22}$$

this is equivalent to the situation with a constant phase of the reflectivity, but with the interface shifted by a distance L_{eff} into the mirror. Indeed, the free spectral range of a Fabry–Pérot consisting of a layer with thickness d and refractive index n sandwiched between two Bragg mirrors is given by

$$\Delta\lambda = \frac{\lambda^2}{2L_{\text{opt}}}, \tag{23}$$

with the optical length

$$L_{\text{opt}} = nd + \bar{n}_t L_{\text{eff}}^t + \bar{n}_b L_{\text{eff}}^b. \tag{24}$$

Table 1
Calculated penetration depth into a TiO_2/SiO_2-quarter wave mirror as depicted in Fig. 4 by using a numerical multiplication of the transmission matrices, Eq. (25) or (26) (our result), and Eq. (27) (the usually quoted formula)

Structure	s	Eq. (27)	Eqs. (25), (26)	numerical
Air–TiO_2SiO_2 4 p.	0	0.395	0.218	0.214
Air–TiO_2SiO_2 4 p.	$\frac{1}{2}\Lambda$	0.395	0.753	0.701
Air–TiO_2SiO_2 8 p.	0	0.395	0.218	0.218
Air–TiO_2SiO_2 8 p.	$\frac{1}{2}\Lambda$	0.395	0.753	0.759

Here, $L_{\rm eff}^{\rm t,b}$ denote the penetration depths of the top and bottom mirror, respectively.

We carry out the derivative in (21), using (5), (19), and (20), and end up with the suprisingly simple expressions

$$L_{\rm eff} = \frac{n_1 \lambda_{\rm B}}{4\bar{n}_2\, \Delta n}, \quad \text{for } s=0 \text{ (Fig. 4a)} \tag{25}$$

and

$$L_{\rm eff} = \frac{\bar{n}_2 \lambda_{\rm B}}{4 n_1\, \Delta n} + \frac{\lambda_{\rm B}\, \Delta n}{2\pi^2 n_1 \bar{n}_2}, \quad \text{for } s = \frac{\Lambda}{2} \text{ (Fig. 4b)}\ . \tag{26}$$

The second term in (26) in general is negligible because it is much smaller than the Bragg wavelength $\lambda_{\rm B}$. We can compare these results to the usualy quoted formula:

$$L_{\rm eff} = \frac{\lambda_{\rm B}}{4\Delta n}\, . \tag{27}$$

Obviously, the correct penetration depth can be obtained simply by multiplying $L_{\rm eff}$ from (27) by either $n_1/\bar{n}_2$ or by $\bar{n}_2/n_1$ depending on s. It is somewhat surprising that the penetration depth depends so strongly on whether the Bragg mirror starts with a layer with higher or lower refractive index. Again, we have compared these results to numerical calculations for the case of a TiO_2/SiO_2 mirror with 4 or 8 layer pairs designed for a wavelength of 830 nm. The results are summarized in Table 1. In all three cases, Eq. (27) of course gives the same value, which is almost a factor of two higher or lower than the exact value. Eq. (25) or Eq. (26), respectively, gives values with errors only in the order of a few percent, even for a mirror consisting of only four layer pairs.

5. Conclusions

We presented a coupled-mode analysis of a Bragg mirror between two media with arbitrary refractive index and derived analytical expressions for the reflectivity and the penetration depth which show an excellent agreement to numerical calculations. The expression for the penetration depth is suprisingly simple and very useful for the design of small-cavity optoelectronic devices relying on Fabry–Pérot effects.

It has often been stated that the coupled-mode theory only leads to exact results for a medium with small periodic variations of the refractive index. This is, in principle, true, but we have shown that the coupled-mode equations lead to very accurate results when the transmission matrices at the boundaries fo the Bragg medium are taken into account in a correct way. At these boundaries, the difference of the refractive indices can be arbitrarily large, corresponding to a situation which is often found in reality.

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