

Strongly enhanced negative dispersion from thermal lensing or other focusing effects in femtosecond laser cavities

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We demonstrate a new scheme for dispersion compensation in femtosecond laser cavities, exploiting the observation that the negative dispersion from a Brewster interface can be strongly enhanced by the focusing effect of a curved surface on a prism or of a thermal lens in the gain medium. Based on this scheme, a high-power Nd:glass laser generated femtosecond pulses without a prism pair. We present a detailed analytical, numerical, and experimental analysis of the discovered dispersion effects. Also, we anticipate the application of these effects for compensation of higher-order dispersion in a broad bandwidth, using a specially designed nonspherically curved mirror surface. © 2000 Optical Society of America [S0740-3224(00)01404-1]

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1. INTRODUCTION

Mode-locked lasers for the generation of subpicosecond pulses are usually operated in the regime of negative group-delay dispersion (GDD) of the resonator where soliton or solitonlike pulses can be formed. A source of negative GDD is then required for overcompensation of the usually positive material dispersion. The most common way to generate negative GDD is to use a prism pair.¹ Here a broadband (e.g., pulsed) laser beam experiences angular dispersion between the prisms, the magnitude of which is basically determined by the material dispersion. The obtained negative GDD is governed by the angular dispersion and by the separation of the prisms, apart from the positive material dispersion that is due to the path length in the prisms. If a large value of the negative dispersion is required, the prism separation can become impractically large, even if a strongly dispersive prism material is chosen. The higher-order dispersion generated by the prism pair can be controlled only to some extent by the choice of the prism material.

It has been found that a single prism^{2,3} or a so-called prismatic output coupler⁴ can be sufficient for dispersion compensation in a laser cavity. Here again the generated GDD depends on the angular dispersion (which is basically determined by the material dispersion), whereas the quantity analogous to the prism separation in the case of the prism pair is now the distance between the prism and the so-called X point. The X point is defined as the point where all wavelength components of a beam intersect,² and its position is governed not by the prism but by the other cavity optics. Basically, the limitations of this approach are the same as for the prism pair. A notable difference from the prism pair is that different wavelength components are spatially separated everywhere in the laser cavity, in particular also in the gain medium.

In this paper we demonstrate that the negative dispersion generated by one or several prisms in a laser cavity can be significantly increased by a focusing effect that may occur either within the prism (e.g., by thermal lensing) or in an optical element near the prism. We illustrate this effect in Section 2 for a simple laser cavity for which an analytical treatment is possible. In Section 3 we report the experimental observations that initiated this study. As the laser cavity was more complicated in the latter case (so the analytical theory of Section 2 is not applicable), we developed a numerical computer program for general laser cavities (Section 4). The predictions from this model allowed for another experimental test as described in Section 5. Finally, in Section 6 we discuss the application of the new scheme for the compensation of higher-order dispersion.

2. ANALYTICAL THEORY FOR A SIMPLE CAVITY

An analytical treatment is possible for the simple cavities as shown in Fig. 1. A prism, which has a reflective coating on the left side, acts as an end mirror. The angle of incidence θ on the right side of the prism is usually chosen to be Brewster's angle. In Fig. 1(a) we assumed both surfaces of the prism to be flat; this situation was discussed earlier.^{2,4} In Fig. 1(b) we introduced a curvature with radius R on the left side. The rest of the cavity optics consists only of a lens (or curved mirror) and a plane end mirror, but it will become apparent that the dispersive properties of this cavity depend only on the properties of the prism and the position of the X point. In the given cases, the distance between that point and the lens is given by the focal length of the lens.

In Fig. 1 we have indicated ray paths for a reference wavelength λ_{ref} and for some other wavelength λ . The

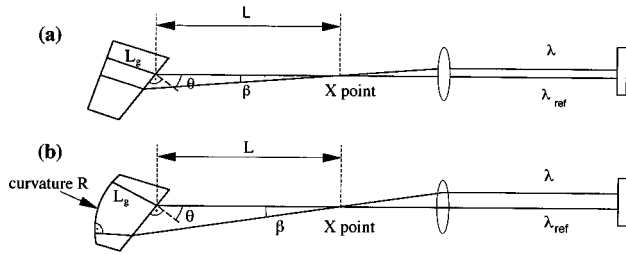


Fig. 1. Simple cavities with prisms that have reflective coatings on the left side: (a) a prism with flat surfaces, (b) a prism with a curved surface on the left side. The beam paths for a wavelength λ and a reference wavelength λ_{ref} are shown.

rays cross at the so-called X point, and the angle between them is $\beta(\lambda)$. First we consider the case $R = \infty$ as in Fig. 1(a). Here it is easy to show that the angular dispersion is

$$\frac{d\beta}{d\omega} = \frac{\tan \theta}{n} \frac{dn}{d\omega}, \quad (1)$$

where $\omega = 2\pi c/\lambda$ is the angular frequency and n is the refractive index of the prism material. The generalization of this result for finite values of R as in Fig. 1(b) requires some trigonometric calculations, which we do not present in detail; the result is

$$\frac{d\beta}{d\omega} = \frac{\tan \theta}{n} \frac{dn}{d\omega} \left[1 - \left(\frac{\cos \theta'}{\cos \theta} \right)^2 \frac{nL}{R - L_g} \right]^{-1}. \quad (2)$$

Here L_g is the path length in the prism and θ' is the angle of incidence within the prism. It is apparent that the angular dispersion diverges as R approaches the critical value

$$R_{\text{crit}} = L_g + nL \left(\frac{\cos \theta'}{\cos \theta} \right)^2. \quad (3)$$

We illustrate this critical behavior with the following explanations: For $R = R_{\text{crit}}$, any value of angle β would lead to a closed beam path in the resonator at the fixed wavelength λ_{ref} , because any beam approaching the prism from the X point would be refracted such that normal incidence on the reflective coating would occur. For a slightly larger value of R there would be a slight deviation from normal incidence at the coating; i.e., the beam path would no longer be closed for nonzero values of β . However, this small deviation could be compensated for with a small change of wavelength that modifies the angle of refraction at the prism interface. Thus only a small change of wavelength corresponds to a given change of β , which means that the angular dispersion is large.

The calculation of the GDD is more involved. The simple type of argument as used by Fork *et al.*¹ for the case of the prism pair does not work here because the curved surface implies that we cannot work with plane waves. Therefore, we use the formalism introduced by Martinez.^{5,6} According to Eqs. (12)–(15) of Ref. 6, the round-trip phase shift φ (and thus the dispersion) in a cavity results from different terms. In our case, the dominant contributions come from Eq. (12) of Ref. 6, which contains the material dispersion, and from Eq. (15) of that reference, which accounts for the effects of angular

dispersion. The latter term, calculated in the given situation and differentiated twice with respect to ω , lead us to the result that

$$\text{GDD}_{\text{ang}} = \frac{d^2\varphi}{d\omega^2} = -2 \frac{kL}{n^2} \tan^2 \theta \left(\frac{dn}{d\omega} \right)^2 \times \left[1 - \left(\frac{\cos \theta'}{\cos \theta} \right)^2 \frac{nL}{R - L_g} \right]^{-1}. \quad (4)$$

Here we have introduced the wave number $k = 2\pi/\lambda$, and L is the distance between the prism and the X point (measured along the reference beam). Note that this term accounts only for the effect of the angular dispersion and does not contain the material dispersion (which is easily calculated). Using Eqs. (2) and (3), we reduce the result to

$$\text{GDD}_{\text{ang}} = -2 \frac{kL}{n} \tan \theta \left(\frac{dn}{d\omega} \right) \left(\frac{d\beta}{d\omega} \right). \quad (5)$$

We see that the curvature of the prism surface, if it is close to the critical value, introduces not only a large angular dispersion but also a large GDD.

Another important observation is that the divergence of angular dispersion and GDD for $R \rightarrow R_{\text{crit}}$ is related to the edge of the stability range of the cavity. It has been shown by Magni⁷ that in general each standing-wave cavity has two stability zones (designated zone I and zone II) with respect to a variable lens (which we identify here with the focusing action of the curved surface). At one edge of zone II, the sensitivity of the resonator against misalignment diverges; this edge is always the one where the mode sizes on both end mirrors diverge. In our case, the wavelength-dependent refraction at the Brewster face of the prism can be seen as causing a wavelength-dependent misalignment of the resonator, and the point of diverging angular dispersion is indeed identical with the mentioned edge of stability zone II. We note that in the particular case when the variable lens is at one end of the cavity, only one stability zone exists, and this is zone II in Magni's notation; i.e., it is always the one with critical alignment sensitivity. [We can show this with Eqs. (9)–(12) of Ref. 7 by assuming a vanishing distance between the variable lens and one end mirror.]

As the diverging negative dispersion from the Brewster interface occurs only when a cavity stability edge is approached where the mode sizes are also diverging, one might believe that this effect would not provide a practical way to generate negative dispersion in a laser cavity. However, Fig. 2 (in addition to our experimental observations) demonstrates the opposite. Here we have plotted the total cavity dispersion and the tangential beam radius w in the prism as functions of the inverse radius of curvature R . We have assumed a prism made from Schott LG-760 glass (operated at Brewster's angle), a path length $L_g = 5$ mm in the glass, and a distance $L = 40$ cm between prism and the X point. The rest of the cavity (lens and mirrors) is not dispersive. Figure 2 shows that a significant negative contribution to the GDD can be obtained at a point that is so far from the stability limit that the mode size is not changed dramatically. Even without the curvature ($1/R = 0$) the wavelength-

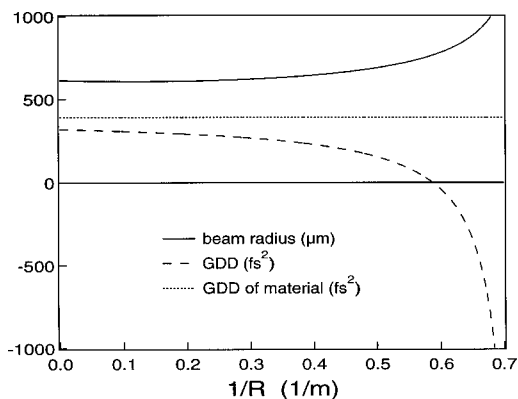


Fig. 2. Beam radius in the prism and GDD of a simple cavity as in Fig. 1, both plotted as a function of the inverse radius of curvature on the left side of the prism. The dotted curve indicates the material dispersion alone. Assumptions: a prism made from Schott LG-760 glass (operated at Brewster's angle), $L_g = 5$ mm, $L = 40$ cm.

dependent refraction at the Brewster interface generates some negative dispersion, but only the focusing effect makes this contribution large enough to get into the regime of negative GDD.

As was mentioned above, the same dispersion effect can be obtained if the focusing action of the curved prism surface is replaced by a thermal lens in the prism (if the prism is actually the gain medium in a laser cavity) or by a curved mirror close to a prism with flat surfaces. It is only then that the simple analytical analysis applied in this section can no longer be applied; in Section 4 we show how such cases can be treated. Another remark is that the dispersion effect does of course occur only in cavities, not in single-pass arrangements as used, e.g., outside a laser cavity for external compression of pulses; it is essential for the principle of operation that each wavelength component correspond to a cavity mode the position of which is determined by the interplay of dispersion and focusing action in the cavity.

3. EXPERIMENTAL OBSERVATIONS ON A Nd:GLASS LASER

Originally, this study was initiated by some experimental observations that appeared strange at the time. We were working on a diode-pumped high-power Nd:glass femtosecond laser similar to the laser discussed in Ref. 8. The laser cavity is shown in Fig. 3. The Nd:glass gain medium (made from Nd-doped Schott LG-760 glass) is pumped from two sides with the radiation from a 20-W diode bar. The pump spot is strongly elliptical, and the laser mode is adapted to this shape by the use of a cylindrical mirror in the cavity (M2 in Fig. 3). As discussed in Ref. 8, this pump geometry is well adapted to the spatial emission pattern of high-power diode bars with widely different beam qualities in the vertical and the horizontal directions; also, it allows for efficient cooling of the thin gain medium and minimizes the effects of thermal lensing and thermally induced birefringence. One arm of the cavity contains a semiconductor saturable absorber mirror⁹ (SESAM) to start and stabilize soliton mode locking.¹⁰ The arm with the output coupler at the end

would normally contain a prism pair¹ for operation in the regime of negative GDD where soliton pulses are formed. However, we observed that even without the prism pair the laser generated soliton pulses with a duration of a few hundred femtoseconds when the pump optics were somewhat misaligned in the axial direction. The pulse duration was highly sensitive to changes of the pump power and also to the pump alignment: A shift of one of the pump lenses by only 0.1 mm was sufficient to change the pulse duration significantly. The minimum pulse duration, measured with an autocorrelator, was 266 fs, and the time-bandwidth product was 0.34, not far from the value of 0.315 for ideal sech^2 soliton pulses. Despite the critical alignment it was possible to obtain stable operation over hours without intervention, and the M^2 factor, indicating the spatial beam quality, was <1.3 in both directions. The output power was typically near 0.9 W, whereas a power of 1.4 W was achieved when the pump alignment was optimized for output power and the dispersion compensation was done in the conventional way with a prism pair near the output coupler. In the latter case, the minimum pulse duration was 275 fs.

As soliton pulses were generated, it was apparent that the cavity operated in the regime of negative GDD, despite the fact that all the cavity mirrors were standard dielectric mirrors with relatively small dispersion and no other dispersive component was used. The dispersion effect from the Brewster interface of the gain medium seemed at first to be much too weak to overcompensate for the significant material dispersion of $+1200 \text{ fs}^2$ per round trip. The explanation that we finally found was that the thermal lens in the gain medium strongly enhanced the negative dispersion generated by the Brewster interface; the mechanism is the same as that which was qualitatively described by the model in Section 2, although the laser cavity is more complicated than in that model. An important point is that we increased the strength of the tangential component of the thermal lens in the gain medium by misaligning the pump optics in the axial direction as described above; only in this way was a focal length of the order of 1.5 m generated, which was enough to bring the cavity close enough to the critical stability edge of zone II. The critical dependence of the soliton pulse duration on pump power and pump alignment is now easily understood, because these effects affect the strength of the thermal lens and thus the proximity to the stability edge. Whereas some misalignment of the pump

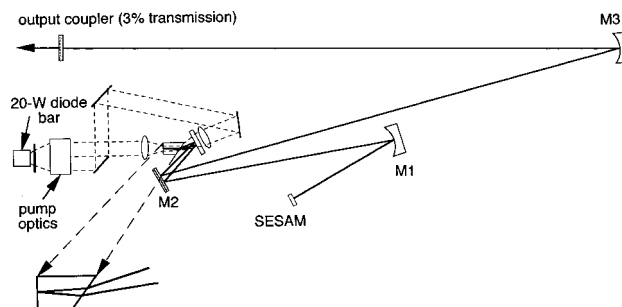


Fig. 3. Nd:glass laser cavity that generated femtosecond pulses without a prism pair: M1, concave mirror with 40-cm radius; M2, cylindrical mirror with 20.3-cm radius in the sagittal direction; M3, concave mirror with 150-cm radius.

beam was required in our laser (and caused some reduction of the output power), of course one could design a cavity so that such a misalignment would not be required.

We also note that in many of the common cavities of mode-locked lasers the X point is located quite close to the Brewster interface of the gain medium, or parallel end faces are used. In such cases, the dispersion effect is not expected to occur.

The analytical treatment of Section 2 is not applicable to this case because the gain element is not located at an end of the laser cavity; therefore, a numerical approach was required. We describe this approach in Section 4, where we also give a quantitative analysis for the case of this laser. In Section 5 we give further support for the claim that the dispersion effect mentioned above is indeed the reason for the peculiar observations.

4. NUMERICAL APPROACH

The analytical treatment in Section 2 leads to a good physical understanding of the dispersion effect and allows for quick calculations. However, its limitations are that the prism with the focusing effect must be at one end of the cavity and that only the effect of the wavelength-dependent refraction at a single interface can be calculated. Also, we can treat only the effect of parabolically curved surfaces. To overcome all these limitations we also made numerical calculations.

There are basically two totally different numerical methods for calculating these dispersion effects. The most direct way is to implement a ray-tracing algorithm. Here we operate with rays that represent the beam axes of Gaussian beams with different wavelengths. The cavity is assumed to be perfectly aligned for the reference wavelength λ_{ref} and is thus misaligned for other wavelength components, which means that these rays propagate along different paths. These rays can be propagated in the cavity by straightforward ray tracing, based on pure geometrical considerations and wavelength-dependent refraction at the Brewster interface. (Note that, although wave effects control the transverse mode sizes, a pure ray analysis is sufficient for calculating the center positions of the cavity modes.) The start position of a ray at one end of the cavity must then be computed numerically so the ray reproduces itself after one round trip in the cavity. In this way the mode position and thus the optical path length and phase change φ for one round trip can be obtained for an arbitrary wavelength. By numerically differentiating the phase change φ with respect to ω we readily obtain the GDD. The advantages of this method are that it is conceptually simple and allows the GDD to be calculated at arbitrary wavelengths or dispersion in higher orders. Also, it is straightforward to treat optical components with nonspherical surfaces. The disadvantage is that the implementation of the ray-tracing algorithm is cumbersome in detail.

A second method is based on the matrix algorithm as presented in Refs. 5 and 6 (unfortunately with a number of errors in the equations). Each optical component is represented by a 3×3 *ABCDEF* matrix, which is an extension of the well-known 2×2 *ABCD* matrices, also accounting for misalignment of components. The

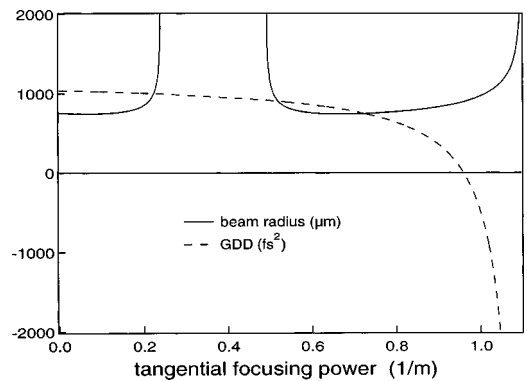


Fig. 4. Beam radius in the gain medium and overall GDD in the Nd:glass laser cavity as shown in Fig. 3, plotted as functions of the focusing power of the tangential thermal lens.

wavelength-dependent misalignment is described by the component F of these matrices (component E is usually zero). As the matrix formalism is based on the paraxial approximation in second order with respect to the ray angles, it allows only for the calculation of the second-order dispersion (GDD) at the reference wavelength, not the GDD at other wavelengths or higher-order dispersion.

We implemented both methods (ray tracing and matrix algorithm) in a computer program to calculate the ray positions and the cavity dispersion for arbitrary standing-wave laser cavities containing optical elements such as mirrors and prisms (with arbitrary types of curved surfaces), Gaussian ducts, and lenses. Comparison of the results from the two methods in various cases was useful for verification of the code. Also, we verified the results with the known analytical results for simple cases such as a prism pair and a prismatic output coupler (without curvature).

Figure 4 shows the calculated beam radius in the gain medium and the overall GDD for the Nd:glass laser as described in Section 3, plotted as functions of the focusing power of the tangential thermal lens. Two stability zones can be recognized. A negative total GDD, suitable for soliton pulse generation, is obtained in zone II (the right-hand zone) for a focal length of the order of 1 m. The experimental results indicate that with the pump alignment optimized for output power a large value of the focal length is obtained, whereas one can approach the critical value by moving the pump focus in the axial direction. A straightforward prediction is that this misalignment of the pump beam could be avoided if a suitable curvature on the surface of the gain medium could be fabricated. In this way, it should be possible to obtain the dispersion effect without any loss of output power, or possibly even with more output power as the losses on the usual prism pair are eliminated.

5. ANOTHER EXPERIMENTAL TEST

Although all the observations described in Section 3 are well explained by the dispersion effect under discussion, we made another experimental test to confirm that this mechanism is indeed at work in our laser. This test was based on the observation that the critical radius of curvature [Eq. (3)]—or in our case the critical strength of the

thermal lens—depends on geometrical factors. This means that moving the position of the mirrors in the cavity can move the singularity of the GDD in Fig. 4 and thus also modify the GDD and consequently the duration of the soliton pulse if the strength of the thermal lens stays constant. Indeed, it was easy to verify both in the computer model and in experiment that the pulse duration changed significantly when the distance between mirrors M1 and M2 (Fig. 3) was modified. Thus our model had correctly predicted a behavior that would normally not be expected. For a more quantitative test we inserted an SF10 prism pair near the output coupler mirror. We then operated the laser at various positions of mirror M1, keeping the pump power and pump alignment unchanged. At each point we adjusted the position of the SESAM to keep the distance between SESAM and mirror M1 constant; in this way, we minimized changes of the mode sizes on the SESAM as well as in the gain medium. Also, we adjusted the insertion of the prisms in the prism pair to obtain a constant pulse duration of 400 fs for each position of mirror M1. From the required changes of prism insertion we calculated the changes of GDD, using the known dispersion of the prism material. Figure 5 shows the excellent agreement between the experimental data and the theoretical expectations from the numerical model. We conclude that the dispersion effect that we are discussing is indeed the explanation for the observed phenomena in our Nd:glass laser. Also, we note that it could be convenient to use the demonstrated dependence of the GDD on a mirror position for fine adjustment of the GDD.

6. HIGHER-ORDER DISPERSION

As discussed in Section 4, the numerical code based on ray tracing allows the dispersion to be calculated in higher than second order. Using the corresponding computer code, we first calculated the GDD as a function of wavelength for the same cavity as discussed in Section 2. The results are plotted in Fig. 6 (solid curves). We used the same parameters as in Fig. 4, except that we plotted the GDD for different values of the radius of curvature R of the left-hand prism surface, which we assumed to be exactly spherical. The GDD is strongly negative for shorter wavelengths, particularly for values of R near the

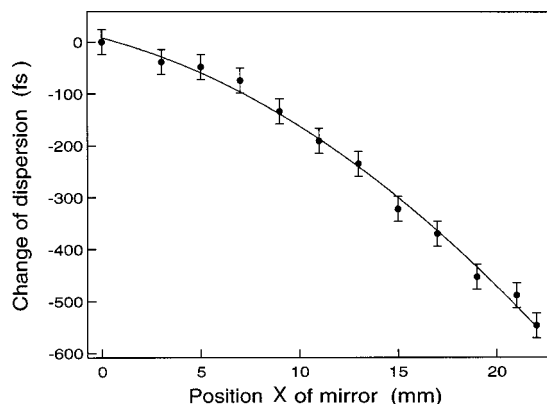


Fig. 5. Changes of the GDD generated by the dispersion effect when mirror M1 (see Fig. 3) is moved away from the original position ($x = 0$).

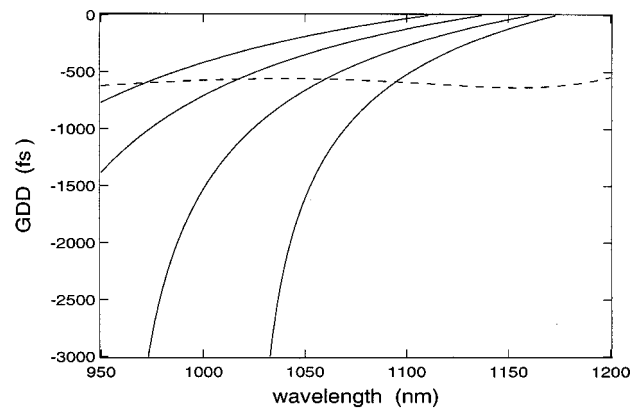


Fig. 6. Solid curves, GDD as a function of wavelength for a simple cavity with spherical surfaces of four radii on the prism: $R = 1.45$ m (lowest curve), $R = 1.50$ m, $R = 1.55$ m, and $R = 1.60$ m (uppermost curve). Dashed curve, GDD for a cavity with an optimized nonspherical surface as described in the text.

critical value, which is 1.39 m in this case. This effect is not caused by the increasing dispersion of the material at shorter wavelengths; using an imaginative material for which the refractive index as a function of wavelength has terms only up to second order in wavelength, we obtained quite similar curves.

Now we consider a more general type of curvature of the prism surface. We define the shape of the surface by the function $y(x) = a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5$, which specifies the height of the surface as a function of the horizontal coordinate x . If we set a fixed value of $R = 1.5$ m of the curvature in the center part, the second-order coefficient is given by $a_2 = 1/(2R)$. The higher-order coefficients can now be used to adjust the shape of the function $GDD(\lambda)$. By numerical optimization we obtain a set of coefficients ($a_3 = 5.48 \text{ m}^{-2}$, $a_4 = -32.8 \text{ m}^{-3}$, $a_5 = 1000 \text{ m}^{-4}$), which gives a nearly flat shape of the GDD; the dashed curve in Fig. 6 shows the resultant GDD.

The potential of such a method for the compensation of higher-order dispersion in a laser cavity—which could be useful for generation of ultrashort pulses—is clearly demonstrated, although we note that quite a high fabrication precision is required. It seems possible to fabricate an optimized surface of the required form or alternatively to use a deformable mirror the shape of which could be automatically optimized by computer control. We also remark that the variable radius of curvature of this mirror introduces an additional wavelength dependence of the mode size; to keep this effect small enough, one should limit the amount of dispersion that has to be compensated for by these means.

7. CONCLUSIONS

We have discovered that the negative dispersion from a Brewster interface of a prism in a laser cavity can be strongly enhanced by a focusing effect of a curved mirror or of a thermal lens in the prism. Our analysis shows that this effect is directly related to a stability edge of the cavity where the sensitivity to misalignment diverges and consequently that a large angular dispersion results from the wavelength-dependent refraction at a Brewster inter-

face. Stable operation is possible because significant negative dispersion is obtained sufficiently far away from the stability edge. We have presented an analytical description for laser cavities with a prism at one end and also discussed results obtained from numerical algorithms. The theoretical results explain well all the observations on a Nd:glass laser that generated femtosecond pulses without any conventional means for dispersion compensation. Possible applications include compact femtosecond laser cavities that do not require a prism pair or special dispersive mirrors, and in particular the compensation of higher-order dispersion in lasers for pulse durations below 10 fs.

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