Appl. Phys. B 73, 653–662 (2001)
DOI: 10.1007/s003400100726

Applied Physics B
Lasers and Optics

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Passive mode locking with slow saturable absorbers

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Received: 20 July 2001/Revised version: 28 September 2001
Published online: 7 November 2001 •

ABSTRACT We give a comprehensive overview on passive mode locking of solid-state lasers with slow saturable absorbers, based on analytical and numerical calculations. For picosecond lasers, we present a simple equation to estimate the obtained pulse duration and compare the results to those for mode locking with fast saturable absorbers. We also discuss how much shorter the pulse duration can be compared to the absorber recovery time and present a simple rule. The effect of self-phase modulation is found to be qualitatively different compared to the case of a fast saturable absorber, and the effect of phase changes in the absorber is also discussed. Finally, we discuss various issues concerning soliton mode-locked lasers.

PACS 42.60.Fc; 42.55.Rz; 42.65.Tg; 42.65.Re

1 Introduction

Passive mode locking with saturable absorbers has been proven to be a very powerful method for the generation of picosecond and femtosecond pulses from different kinds of lasers. Of particular interest are diode-pumped all-solid-state lasers, which can be passively mode-locked with semiconductor saturable absorber mirrors (SESAMs). Pulses with typically between 3-ps and 30-ps duration are routinely obtained with a narrow-band gain medium like Nd:YAG or Nd:YVO_4 and a SESAM, while much shorter pulses can be obtained with broad-band gain media such as Ti:sapphire or rare-earth-doped glasses, using a SESAM and employing soliton-shaping effects in addition.

Particularly in the femtosecond domain, but also in the picosecond domain, the saturable absorbers used are often slow, i.e. their recovery takes a time which can be significantly longer than the pulse duration. A number of publications have treated the physical details of the mode-locking process with slow absorbers. Much of the work [1–6] is based on purely analytical techniques. These usually involve serious approximations that partly are not valid in typical cases. In particular, it is usually assumed that the absorber is only weakly saturated, while typical lasers are operated under conditions of complete saturation. Obviously this can strongly affect the results. We therefore use a fully numerical model that does not require such approximations and is described in Sect. 3.1. In some cases, we also use analytical considerations that allow us to obtain more physical insight and practical design guidelines. These results can be tested with the numerical model.

Using these techniques, we try to give a comprehensive overview on mode locking with slow absorbers. We partially review previously known results but also present some considerations that give new insight into the physical mechanisms of mode locking. In Sect. 2, we review the basic equations to describe the action of slow saturable absorbers. In Sect. 3 we address the question of how short the generated pulses can be (without soliton effects) and find a limit which depends on the absorber recovery time. It turns out that even without soliton effects the pulse duration can be at least 20 times shorter than the absorber recovery time. We discuss the nature of the instability occurring for a too-long recovery time and describe the optimum conditions for short pulses. Sections 4 and 5 are devoted to the influence of self-phase modulation (SPM) and carrier-induced phase changes in the absorber, respectively, considering lasers without soliton pulse shaping. Finally, we review and expand the knowledge on the regime of soliton mode locking in Sect. 6, concentrating on practical design issues.

In all cases we do not consider the effect of spatial hole burning in the gain medium, which can strongly influence the mode-locking behavior and has been addressed in specialized papers [7–9]. Also, we do not discuss Q-switching instabilities [10, 11] here.

2 Equations for slow saturable absorbers

The behavior of a slow saturable absorber is described by a time-dependent power-loss coefficient $q(t)$ which depends on the parameters of the absorber and on the pulse exciting the absorber. (Note that some publications use $q(t)$ for the amplitude loss instead of the power loss.) Although saturable absorber devices typically also have some unsaturable loss, we ignore this here because it can be easily incorporated into a model through an additional constant-loss term.

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The evolution of $q(t)$ is governed by the differential equation

$$\frac{dq}{dt} = -\frac{q - q_0}{\tau} - \frac{I}{F_{sat}} q \quad (1)$$

with the recovery time τ, the unsaturated loss q_0, and the saturation fluence F_{sat}. A spatially constant intensity on the absorber (on some mode area) is assumed, although real pulses typically have a Gaussian transverse intensity distribution. In the case of a slow absorber, where the recovery is so slow that we can ignore the first term, we can simply integrate (1) and find that the value of q after a pulse with fluence F_p is

$$q_{ap} = q_0 \exp(-F_p/F_{sat}) \quad (2)$$

if the pulse hits an initially unsaturated absorber.

Note that in this paper we usually assume that the absorber can fully recover within the round-trip time of the laser resonator. This assumption is well fulfilled in most passively mode-locked solid-state lasers. However, it is not valid in some lasers with extremely high repetition rates of tens of GHz (see e.g. [12]) where the round-trip time can be significantly shorter than the absorber recovery time.

The maximum reflectivity change ΔR, also called the modulation depth, is nearly identical to q_0 provided that $q_0 \ll 1$, which is usually the case for absorbers used in lasers.

The absorbed fluence is

$$F_{abs} = F_{sat} \cdot (q_0 - q_{ap}) = F_{sat} q_0 [1 - \exp(-F_p/F_{sat})] \,. \quad (3)$$

The effective energy loss for the pulse (averaged over the temporal profile) is then (independent of the pulse form)

$$q_p(F_p) = F_{abs}/F_p = q_0[1 - \exp(-S)]/S \quad (4)$$

with the saturation parameter $S := F_p/F_{sat}$. For strong saturation ($S > 3$), the absorbed pulse fluence is $F_{abs} \approx F_{sat} . \Delta R$, and we have

$$q_p(S) \approx q_0/S \approx \Delta R/S \quad (5)$$

Figure 1 shows a plot of (4), compared to the loss after the pulse. It is important to observe that the loss after the pulse

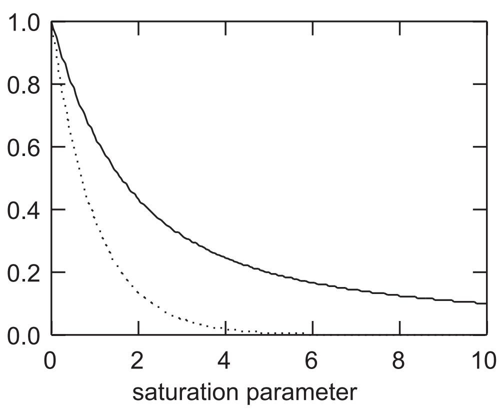

FIGURE 1 *Solid curve*: normalized loss q_p/q_0 for a pulse on a slow saturable absorber versus saturation parameter (pulse energy divided by saturation energy). *Dotted curve*: normalized loss q_{ap}/q_0 after the pulse

gets very small for $S > 3$, while the average loss q_p for the pulse is still significant then. This means that doubling of the saturation parameter still reduces the effective loss by a factor of two (see (5)), even if the absorber was already fully saturated after the pulse in the initial case (2). In other words, even for a strongly saturated absorber there is still a significant loss penalty of $\approx \Delta R/S$ for the break-up into multiple pulses.

Note that these equations may not be precise for real absorbers. For example, semiconductor absorbers exhibit a complicated saturation behavior, depending on the wavelength of excitation relative to the band gap. In the usual cases the behavior is reasonably approximated by the equations discussed above. However, under conditions of strong saturation, semiconductor absorbers can exhibit additional effects such as two-photon absorption [13], free-carrier absorption, thermal effects, or various damage effects. In SESAMs, these effects are normally rather weak for saturation parameters not much larger than 10, which is the usual situation in mode-locked lasers. However, two-photon absorption can be significant for femtosecond pulses or in specially designed SESAMs potentially even in the picosecond domain, and can cause a roll-off of the nonlinear reflectivity curve at high pulse fluences. This can be used as a power-limiting effect, acting against Q-switching instabilities [13, 14].

The equations discussed above describe only the power attenuation of the absorber, but not possible phase changes caused by its excitation. The latter effects are discussed in Sect. 5. Also note that the absorption has been assumed not to depend on the wavelength. For cases with significant wavelength dependence, a significantly more sophisticated model would be required.

3 Obtainable pulse duration without soliton pulse shaping

3.1 *Estimate for the pulse duration*

Analytical results for the pulse durations of passively mode-locked lasers without soliton pulse shaping have been derived [2, 4, 6, 15], but only in the case of weak absorber saturation. In most experimental cases, however, the absorber is operated with saturation parameters of three or more, i.e. at least at three times its saturation fluence, where the assumption of weak saturation is not valid. In this situation, we can still numerically simulate the pulse formation, using a model that repeatedly propagates the pulse through the laser cavity, taking into account the effects of gain, linear cavity loss, saturable absorption, Kerr nonlinearity, and dispersion. The gain is assumed to have a Gaussian spectral shape (not affected e.g. by spatial hole burning), and it saturates according to the average power. (Gain saturation during a pulse is neglected, because this is typically a very weak effect in solid-state lasers.) We also introduce some noise in each round-trip, which is important (see Sect. 3.2). The saturable loss evolves according to (1) and may be accompanied by a phase change proportional to the excitation (see Sect. 5). For the first investigations we ignored effects of Kerr nonlinearity and dispersion in the cavity, as well as phase changes on the absorber. Typically we are interested in the steady-state situation which may be reached after a large number of cavity round-trips. To find this steady state, the program applies more and more cavity round-trips until a number of pulse parameters (energy, duration, spectral width, and center wavelength) no longer change significantly during a round-trip.

Using this model, we found that a useful guideline is to use the equation

$$\tau_{\mathrm{p}} \approx \frac{1.07}{\Delta f_{\mathrm{g}}} \sqrt{\frac{g}{\Delta R}} \tag{6}$$

as an estimate for the obtained steady-state pulse duration. Here, Δf_{g} is the FWHM gain bandwidth (assuming a Gaussian-shaped gain spectrum) and g is the power-gain coefficient. In steady state, the latter has to balance the overall cavity losses, which consist of some (usually dominating) linear (nonsaturable) loss l (output coupler transmission and parasitic losses) and the saturable absorber loss, which is $\approx \Delta R/S$ according to (5), provided that the saturation parameter S is at least ≈ 3.

We found that (6) reasonably matches the results from numerical simulations if the absorber is operated at roughly 3–5 times the saturation fluence (i.e. the saturation parameter S is about 3–5). For significantly weaker or stronger absorber saturation, the pulse duration becomes somewhat longer (see Fig. 2). The absorber recovery time has been assumed to be 100 ps, i.e. much longer than the obtained pulse durations, and the exact value of the recovery time has little influence on the pulse duration. For a discussion of the limits to the recovery time, see Sect. 3.2.

Note that (6) has the same form as an equation derived in [15] for a weakly saturated fast absorber; only the constant factor (adapted to our notation) has been changed from 0.66 to 1.07. For comparison, we did similar simulations for a fast saturable absorber and found that under optimum saturation conditions the obtained pulse duration is only $\approx$15% shorter (for the same modulation depth), compared to the slow absorber. Taking into account that the cavity losses caused by the fast absorber (of equal modulation depth) are somewhat larger, one could make the modulation depth of the slow absorber somewhat larger, which would further reduce the difference in achievable pulse durations.

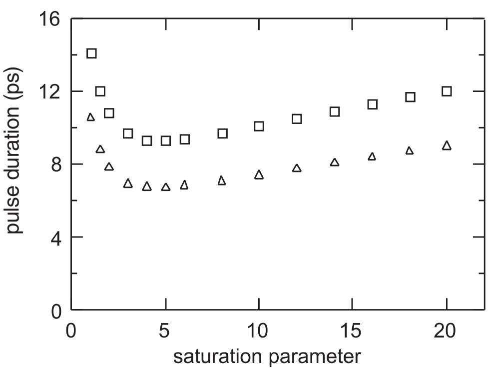

FIGURE 2 Pulse durations from numerical simulations of a laser with 1-nm gain bandwidth, 5% linear cavity losses, and a SESAM modulation depth of 1% (*rectangles*) and 2% (*triangles*), where the saturation parameter of the SESAM is varied along the horizontal axes. Values around 3 to 5 result in the shortest pulses, and the obtained durations agree with (6)

The shortest pulses with a slow absorber are obtained by making the nonsaturable cavity loss (output coupler transmission and parasitic linear losses) smaller than the saturable loss. This somewhat compromises the power efficiency of the laser. Assuming $S = 4$, we obtain the minimum pulse duration

$$\tau_{\mathrm{p,\,min}} \approx \frac{1.07}{\Delta f_{\mathrm{g}}} \sqrt{\frac{\Delta R/4}{\Delta R}} \approx \frac{0.54}{\Delta f_{\mathrm{g}}} \,. \tag{7}$$

The pulse bandwidth is then roughly 60% of the gain bandwidth, assuming a sech^2 shape of the pulse envelope. This shows that even a slow saturable absorber can be used to utilize nearly the full available gain bandwidth.

3.2 *Limit for the absorber recovery time*

Numerical simulations (as e.g. those discussed above) show that the obtained pulse duration τ_{p} can be much shorter than the recovery time τ_{a} of the absorber, even if soliton effects (Sect. 6) are absent. This has been previously recognized [16] but not yet physically explained. The finding may seem quite surprising because on the trailing edge of the pulse there is no shaping action of the absorber. There is even net gain, because the loss caused by the absorber is very small for the trailing edge (always assuming a fully saturated absorber), while the total loss for the pulse (5) is larger and is balanced by the gain in the steady state. Thus one might expect that this net gain would either prevent the pulse from getting so short or destabilize it by amplifying its trailing wing more and more. Indeed there are a number of publications (e.g. [3, 4]) where it was assumed that stable pulse generation is not possible under such circumstances. However, we will show in the following that this is not necessarily the case, so that previously often used stability criteria are not correct.

To understand why stable pulses with a duration far below the absorber recovery time are possible, we have to consider that the action of the absorber steadily delays the pulse: it attenuates mostly its leading wing, thus shifting the pulse center backwards in each cavity round-trip. Note that this shift does not apply to any structures well behind the pulse maximum, because then the absorber is fully saturated already. In effect, the pulse is constantly moving backward and can swallow any noise growing behind itself. This noise has only a limited time in which it can experience gain before it merges with the pulse itself. The same mechanism can also prevent the trailing edge of the pulse from growing. Such a stabilizing mechanism has been discussed in the context of soliton mode locking [17] but has not been applied to the situation of a passively mode-locked laser without soliton effects. A similar temporal shift and its stabilizing influence on the pulses have been discussed for actively mode-locked lasers [18].

To get a quantitative picture, we first estimate the net gain behind the pulse. We always assume a strongly saturated absorber ($S > 3$) and that the absorber recovery time is significantly longer than the pulse duration. As the average loss for the pulse caused by the absorber is $\Delta R/S$ (5), while the loss after passage of the pulse is close to zero (2), there is a net gain of $\approx \Delta R/S$ directly after the pulse. After recovery of the SESAM the net gain will be $\Delta R/S - \Delta R$, i.e. negative. The absorber recovery follows an exponential function, which we can approximate by a linear function between the time directly after the pulse and the point where the net gain becomes zero. The slope of this function is $-\Delta R/\tau_a$, and zero net gain is obtained at a time

$$\frac{\Delta R/S}{\Delta R/\tau_a} = \tau_a/S \tag{8}$$

after the pulse. As an example, the numerically simulated temporal evolution of the net gain is shown in Fig. 3 for a typical case.

Now we need to calculate the temporal shift caused by the absorber in each cavity round-trip (i.e. for one reflection at the absorber). This shift is proportional to the modulation depth and to the pulse duration, while it has a more complicated dependence on the saturation parameter. Numerical simulations (Fig. 4) show that the temporal shift is approximately

$$\Delta t = 0.12 \cdot \Delta R \cdot \tau_p \tag{9}$$

if the pulse has a sech^2 shape and the saturation parameter S is about 3 (a typical value). The shift is smaller for weaker or stronger saturation. Also, it is slightly weaker for Gaussian pulses.

In the following, we assume sech^2-shaped pulses and $S \approx 3$. We consider some noise which starts growing at the point of zero net gain, which occurs at $t_0 = \tau_a/S$ as shown above. This noise will now experience a gain that rises approximately linearly with time because the pulse is steadily shifted towards the noise, until both merge after approximately

$$N = \frac{t_0}{\Delta t} \approx 9\frac{\tau_a}{S \cdot \Delta R \cdot \tau_p} \approx \frac{3\tau_a}{\Delta R \cdot \tau_p} \tag{10}$$

cavity round-trips. Just before this, the gain reaches $\approx \Delta R/S$. The total gain experienced by the noise is

$$g_{tot} \approx \frac{1}{2} N \frac{\Delta R}{S} \approx \frac{\tau_a}{2\tau_p} . \tag{11}$$

We can expect that stable mode locking requires this gain to be so small that any noise can not acquire an energy which is comparable to the pulse energy. While the exact amount of tolerable gain for the noise may vary between different situations, it seems reasonable to use 60 dB as an estimated critical level, because it is well known e.g. from the context of amplified spontaneous emission that a gain of this order of magnitude can amplify quantum fluctuations to significant power levels. This means that

$$g_{tot} < \ln 10^6 \approx 14 , \tag{12}$$

which leads to the remarkably simple stability condition $\frac{\tau_a}{2\tau_p} < 14$ or

$$\tau_a < 28\tau_p . \tag{13}$$

Of course, this limit may vary somewhere roughly between $20\tau_p$ (for 43-dB critical gain) and $40\tau_p$ (for 86 dB), depending on the situation, because the 60-dB critical gain is only a rough estimate. Note that even a change of the actual power-amplification factor by a factor of e.g. 100 changes the gain by 20 dB and thus does not strongly alter the obtained limit for the absorber recovery time.

For somewhat stronger absorber saturation, i.e. $S > 3$, the net gain after the pulse is smaller, and also the time window with gain is shorter. This is only to a small degree compensated by a smaller temporal shift of the pulse (according to Fig. 4). Thus an even significantly longer recovery time could be tolerated for stronger saturation of the absorber.

Equation (13) represents a remarkably simple criterion. It does not depend on the modulation depth of the absorber, because an absorber with e.g. twice the modulation depth would cause twice the net gain after the pulse, but also twice the temporal shift per round-trip and thus only half the number of round-trips for amplification of the noise.

Numerical simulations show that (13) is a reasonable estimate. Note that it is important to include some noise source in the numerical model, which represents e.g. the influence of spontaneous emission in the gain medium. It turns out that the exact amount of noise is not very important, as expected; even a tenfold increase of the noise power would not strongly reduce the limit for the absorber recovery time. We modeled a $Nd:YVO_4$ laser with a typical semiconductor saturable absorber, generating pulses with durations around 7 ps. From (13) we expected an instability for $\tau_a > 200$ ps. Indeed the numerical model exhibited marginal stability for $\tau_a = 200$ ps and

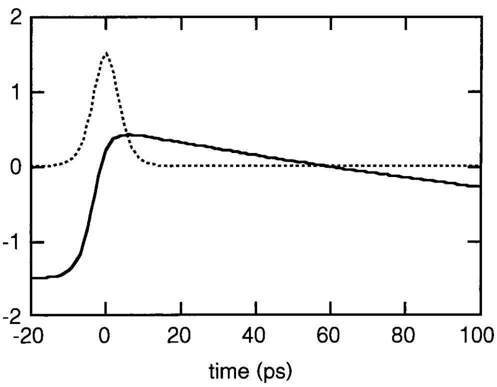

FIGURE 3 Temporal evolution of the net round-trip gain (*solid curve*, in percent) versus time. The absorber with 2% modulation depth is saturated by an 8-ps pulse (*dotted curve*) with four times the saturation fluence. The absorber recovery time is 200 ps. There is a positive net gain between the pulse maximum and ≈ 60 ps after this time

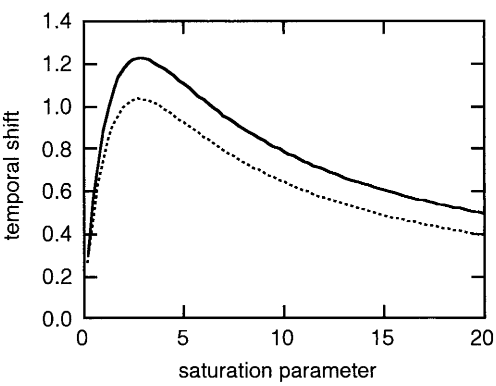

FIGURE 4 Temporal shift of a sech^2-shaped pulse (*solid curve*) and a Gaussian pulse (*dotted curve*), caused by a saturable absorber with 1% modulation depth, as functions of the saturation parameter, in units of $1/1000$ of the pulse duration

instability for $\tau_a = 250$ ps. It also allowed us to investigate the nature of the instability. As expected, an initially very weak background noise starts to grow far behind the pulse and then comes closer and closer to the pulse maximum. In the case with $\tau_a = 250$ ps, the background becomes strong just before it merges with the pulse. One sees that the trailing wing of the pulse becomes quite strong for some time, with the effective half-width of the pulse being significantly increased. Then the pulse shortens to its initial width of around 7 ps, and only after many round-trips does a new noise background emerge.

In all simulated cases with a too-long recovery time, the result was an unstable behavior (as described above, see also [16]) and not simply an increase of the obtained pulse duration. The latter is not substantially influenced by the absorber recovery time, once this is much longer than the pulse duration. Therefore, there could be situations with large gain bandwidth and a rather slow absorber where stability is achieved only when the pulse duration is kept long enough e.g. by restricting the bandwidth with an additional filter.

Note that weak reflections in the laser cavity could generate a weak satellite pulse behind the main pulse, and this satellite could be stronger than the noise level and thus significantly reduce the maximum tolerable recovery time of the absorber.

To conclude, even without employing soliton effects, slow absorbers can be used for the generation of mode-locked pulses which are more than 30 times shorter than the recovery time, particularly if the absorber is strongly saturated. The reason for this is the temporal shift of the pulses caused by the absorber, which limits the time in which noise behind the pulse can be amplified. Another limiting effect usually comes into play in soliton mode-locked lasers [17, 19, 20] (Sect. 6), which allows for even further reduced pulse durations.

We also note that our considerations have to be modified when applied to cases where the absorber can not fully recover between two consecutive pulses. This can occur e.g. in lasers with multi-GHz repetition rates (see e.g. [12]). Here, the width of the time window with gain after the pulse may be limited by the pulse-to-pulse spacing. This effect can further increase the allowed ratio of absorber recovery time and pulse duration.

4 Influence of self-phase modulation

In this section we investigate the effect of self-phase modulation on the mode-locking performance of lasers without significant dispersion effects. Cases with soliton formation are discussed later (Sect. 6).

It is known for passively mode-locked lasers with fast saturable absorbers that some additional influence of SPM can somewhat decrease the pulse duration [21], although the pulses become unstable when there is too much SPM. The decrease of pulse duration is usually explained by the fact that SPM tends to spectrally broaden the pulse.

One might expect that for lasers mode-locked with slow saturable absorbers the influence of SPM is similar. However, our numerical simulations show that this is not the case. We considered a Nd:YVO_4 laser with the following properties: standing-wave cavity with 100-MHz repetition rate, gain bandwidth 1 nm, no spatial hole burning, output-coupler transmission 5%, operation ≈ 10 times above threshold with ≈ 0.14-W average output power, SESAM with 2% modulation depth, 50-ps recovery time, saturation parameter $S = 4$, no phase changes in the absorber, no dispersion in the cavity, and a variable amount of SPM, quantified as γ_{SPM}, the phase shift per watt of optical power. For a medium with thickness d and Kerr coefficient n_2 through which a Gaussian beam with radius w propagates, γ_{SPM} is given by

$$\gamma_{SPM} = \frac{2\pi}{\lambda} n_2 \frac{d}{\pi w^2/2} = \frac{4 n_2 d}{\lambda w^2} \,. \tag{14}$$

Note that in the case of freely propagating beams the nonlinear phase shift must be calculated for the beam axis, rather than for some averaged intensity.

The simulations (Fig. 5) show that the pulse duration without SPM is ≈ 6.6 ps, and a moderate amount of SPM makes the pulses somewhat longer (not shorter!), while the pulse bandwidth is decreased and the time–bandwidth product is somewhat increased. Also, the center wavelength gets somewhat shorter. This results from an interplay of SPM and the temporal delay caused by the absorber (see Sect. 3.2). SPM (with positive n_2, as is the usual case) decreases the instantaneous frequency in the leading wing and increases the frequency in the trailing wing. The absorber always attenuates the leading wing, thus removing the lower-frequency components, with the effect that the center frequency increases and the pulse bandwidth decreases. This also explains the longer pulses. It becomes apparent that again the temporal asymmetry caused by a slow saturable absorber (but not by a fast absorber) plays a crucial role in the mode-locking process.

For stronger SPM action (≈ 3 rad/MW, corresponding to ≈ 11-mrad nonlinear phase shift for the peak), the pulses become unstable. The nature of the instability is that a second pulse grows behind the initial pulse, and then the two pulses merge. This instability actually is similar to the one caused by a too-long recovery time of the absorber (Sect. 3.2). One might therefore expect that more SPM can be tolerated for a faster absorber. However, simulations with one-half the absorber recovery time (25 ps) showed that then the instability occurs already for weaker SPM (≈ 1.4 rad/MW) – at a point where the pulse duration is hardly affected. Con-

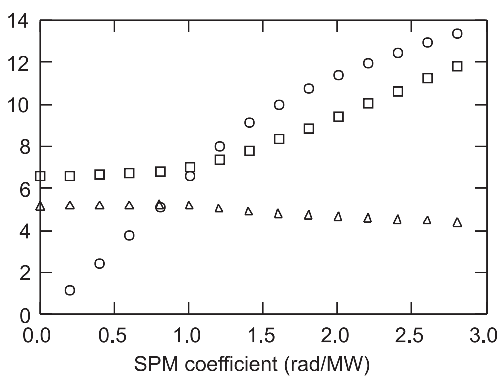

FIGURE 5 Effect of a variable degree of SPM on the obtained pulse parameters from a Nd:YVO_4 laser without dispersion (see text for details): *rectangles*, pulse duration/ps; *triangles*, pulse bandwidth/10 GHz; *circles*, spectral shift/0.01 nm. For higher SPM coefficients, stable pulses are not obtained

versely, for 100-ps recovery time, the critical value is as high as $\approx 5\,\mathrm{rad/MW}$.

The tolerance for SPM also depends on the modulation depth of the absorber. With 1% modulation depth (instead of 2%), the instability occurs at $\approx 1.5\,\mathrm{rad/MW}$ already, although the somewhat longer pulses have a smaller peak power. This means that with twice the modulation depth we can tolerate more than twice the nonlinear phase shift.

Next we increase the saturation parameter of the absorber from $S = 4$ to $S = 8$. This results in only slightly longer pulses, but the instability occurs at $\approx 1.5\,\mathrm{rad/MW}$ already.

We also consider a laser with $S = 4$ and one-half the linear cavity losses (2.5%), operating with about twice the intracavity power. The modulation depth is reduced to 1% in order to obtain pulse durations around 7 ps. The mode area on the absorber is doubled to keep the saturation parameter unchanged. As is to be expected, the instability occurs for $\approx 0.75\,\mathrm{rad/MW}$, i.e. for four times weaker SPM than in the original case, because we have twice the intracavity power and half the modulation depth. However, this does not mean that one should always operate a laser with high cavity losses and large modulation depth, because both favor Q-switching instabilities [10, 11].

If the laser corresponding to the described model contains a 5-mm-long laser crystal, the actual SPM coefficient is only $\approx 0.35\,\mathrm{rad/MW}$ (taking the n_2 coefficient for YAG instead of Nd:YVO_4, for which we have no data). This has little influence on the mode-locking performance, as is apparent from Fig. 5. We can indeed expect that many passively mode-locked picosecond lasers are not significantly influenced by SPM. However, a stronger influence of SPM must be expected in the following situations:

- for gain media with low laser cross-sections, because these require lower cavity losses (higher intracavity powers) to operate above the threshold for stable passive mode locking (without Q-switching instabilities [10, 11]),
- for lasers with a long laser crystal,
- and for lasers generating relatively short pulses, certainly for sub-picosecond pulse durations.

It is important to note that we can generalize the results obtained for one particular gain medium to other cases, because we have essentially the same situation if e.g. we generate two times shorter pulses by using a two times larger gain bandwidth, a two times smaller SPM coefficient (to preserve the nonlinear phase shift), a four times lower group delay dispersion, and a two times shorter absorber recovery time. Thus the results obtained for the Nd:YVO_4 laser can be directly applied to femtosecond lasers, as long as additional effects like higher-order dispersion are not important.

In conclusion, the effect of SPM on passively mode-locked picosecond lasers can be important in cases as described above. For a slow absorber, it is never beneficial as it always tends to make the pulses longer and may also destabilize them, particularly for absorbers with small modulation depth. Thus one should always keep the effect of SPM small, particularly by using short laser crystals. A rule of thumb is that the nonlinear phase shift for the peak should be at most a few mrad per 1% of modulation depth. It is clear that SPM could hardly be made weak enough in the sub-picosecond domain – for this reason (and not only due to the effects of dispersion or a too long absorber recovery time), soliton mode locking [17, 19, 20] is usually required in the sub-picosecond domain, because there the nonlinear phase changes can be much larger (see Sect. 6).

5 Influence of phase changes in the absorber

When an absorber is saturated, its refractive index usually changes somewhat. This behavior can be described by a factor α that relates the phase change (for passage through the absorber) to the change of the power-absorption coefficient q according to

$$\Delta\varphi(t) = \frac{\alpha}{2} q(t) \tag{15}$$

where $q(t)$ is evolving according to (1). Positive values of $\Delta\varphi$ indicate phase delays. α is commonly called the linewidth enhancement factor because in a continuous-wave semiconductor laser it increases the linewidth by a factor of $1+\alpha^2$ [22]. For semiconductor saturable absorber mirrors, α is usually positive, and because of $\Delta q < 0$ we typically have a decrease of the phase delay during the pulse, which causes some increase of the instantaneous frequency in the leading wing of the pulse.

Experimental data on α values for SESAMs are not available. They would be difficult to measure: a phase change in the order of a few tens of mrad or less would have to be monitored during passage of a short pulse. However, we can expect that typical SESAMs should have α values in the same order of magnitude as those of semiconductor lasers based on GaAs, where α can be obtained from the cw laser line width [22] and is typically found to be in the range 1 to 10, with the higher values occurring for excitation closer to the band gap. (Close to the band gap, the absorption vanishes while the phase effect stays finite, so that α diverges.) For absorbers we actually expect somewhat smaller values than for lasers, because they are often excited somewhat higher above the band gap and also operated at a lower excitation level than gain structures. Values between 0 and 3 seem realistic for typical cases. Although we do not know precise values of α for SESAMs, it is important to check theoretically whether any significant effects are to be expected for estimated values, and which lasers would be most susceptible to such effects.

In this section we focus on lasers without dispersion. (We will discuss the effect of the linewidth enhancement factor in the soliton mode-locked case in Sect. 6.) We start with numerical simulations on Nd:YVO_4 lasers with similar parameters as in Sect. 4: standing-wave cavity with 100-MHz repetition rate, gain bandwidth 1 nm, no spatial hole burning, output-coupler transmission 5%, operation ≈ 40 times above threshold with ≈ 0.91-W average output power, SESAM with 2% modulation depth, 50-ps recovery time, saturation parameter $S = 4$, no dispersion in the cavity, and no SPM. In Fig. 6 we varied the α parameter along the horizontal axis. It is apparent that large α values could significantly shorten the pulses, in this case from 6.6 ps for $\alpha = 0$ down to 4.1 ps for $\alpha = 6$. At the same time, the pulse bandwidth is increased, and the pulse spectrum acquires a strong wing towards shorter wavelengths (see Fig. 7), resulting from the above-mentioned frequency

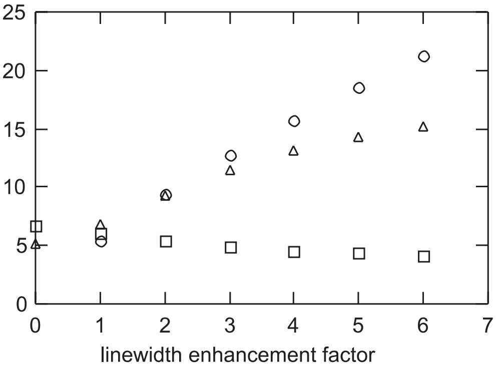

FIGURE 6 Pulse parameters of a Nd:YVO_4 laser versus line-width enhancement factor α of the saturable absorber: *rectangles*, pulse duration/ps; *triangles*, pulse bandwidth/10 GHz; *circles*, spectral shift/0.01 nm

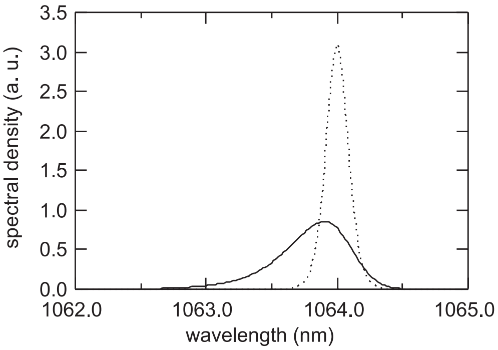

FIGURE 7 Spectral power density (in arbitrary units) of the pulses as obtained for the parameters in Fig. 6 with $\alpha = 6$ (*solid line*) and $\alpha = 0$ (*dotted line*). The gain maximum is at 1064 nm. The phase changes in the absorber cause a stronger wing towards short wavelengths

rise in the leading wing of the pulse. The time–bandwidth product is roughly doubled, and the pulse energy is reduced by more than 10% because of the reduced gain for a broader bandwidth.

For $\alpha > 6$, the pulses become unstable. The strong nonlinear phase shift causes the pulse to break up. The formation of strong shorter-wavelength components, which experience less gain, also excites relaxation oscillations. Thus, this type of instability may appear similar to Q-switching instabilities in an experiment.

With a reduced modulation depth of 1% (instead of 2%), the behavior is quite similar, and the instability again occurs for $\alpha > 7$. For stronger absorber saturation ($S = 8$ instead of 4, and 2% modulation depth), the instability occurs only for significantly higher values of $\alpha > 10$, because then the phase shift varies only in the front part of the temporal profile. Only slightly more stability is achieved for a faster absorber (25 ps instead of 50 ps).

It is interesting to note that a moderate value $\alpha = 3$ allows us to have about twice the amount of SPM before the pulses become unstable.

An important question is whether phase changes in the absorber affect shorter pulses more than longer pulses. A simulation with five times higher gain bandwidth shows that this is not the case – the instability occurs at the same value of α, and the same amount of pulse shortening is achieved. This is essentially because the magnitude of the phase change is independent of the pulse duration. (A different situation is when spectral phase changes are caused by dispersion – here, the phase changes are larger for shorter pulses, which have larger bandwidths.) However, the situation is different in soliton mode-locked lasers (see Sect. 6).

In conclusion, we have seen that the linewidth enhancement factor of a saturable absorber could have a significant influence on the mode-locking performance, if α is large (e.g. around 5). For a semiconductor absorber this is the case if it is operated close to the band gap, while under the usual conditions smaller α values are expected. These lead to accordingly weaker effects such as some reduction of the pulse duration, which may not be noticed in experiments. A signature for effects of large α values would be an asymmetric optical spectrum with an enhanced wing towards shorter wavelengths. We are not aware of experimental cases showing this signature. However, note that such a signature may be masked e.g. if the absorption increases for shorter wavelengths or when the gain spectrum is asymmetric.

6 Soliton mode locking

We have seen in Sect. 4 that the effect of self-phase modulation can become rather strong for sub-picosecond pulses circulating in a laser cavity. Without additional dispersion, the nonlinear phase shifts can easily become strong enough to destabilize the pulses. However, it has been recognized that SPM in combination with an appropriate amount of (typically negative) dispersion can even help to generate shorter pulses. This mechanism is called soliton mode locking [17, 19, 20], because soliton-like pulses can be formed if the effects of SPM and dispersion on the circulating pulse approximately cancel each other (apart from a constant phase shift per round-trip) and other effects are comparatively weak.

Considering only SPM and dispersion, we can have pure soliton pulses with

$$\tau_p \approx 1.76 \cdot \frac{2|D|}{|\gamma_{SPM}| E_p} \tag{16}$$

where τ_p is the FWHM pulse duration, d is the group delay dispersion per cavity round-trip, γ_{SPM} is the SPM coefficient (in rad/W) per round-trip, and E_p is the pulse energy. The quantities γ_{SPM} and d must have opposite signs. (For γ_{SPM}, see Sect. 4 and (14) in particular.) In a laser cavity, (16) is a good approximation if other effects like the limited gain bandwidth, the action of the absorber, and the effects of the discreteness of dispersion and SPM are weak (so that the pulses effectively see only the average dispersion and nonlinearity). If one tries to reduce the pulse duration by reducing $|D|$, these effects will become stronger and eventually destabilize the pulses; therefore, such effects set a limit to the achievable pulse duration.

If a slow saturable absorber is used for mode locking, there remains a time window with net round-trip gain behind the pulse where the loss of the still saturated absorber is smaller than the average loss for the pulse (see Sect. 3.2). This may cause a cw background (often called the continuum) to grow and destabilize the pulse. This effect is further supported by the higher gain which a narrow-band background (compared to the broad-band soliton) can experience. At a first glance, it may seem that the analysis of Sect. 3.2 would apply, where the time for growth of the background is limited by the temporal shift of the pulse caused by the absorber. However, there is another limiting effect which usually becomes more effective in soliton mode-locked lasers: the dispersion causes the background to temporally broaden and thus permanently lose the energy in those parts which drift into the time regions with net loss [19]. (Note that SPM can not compensate the effect of dispersion for the weak background.) Therefore, a larger amount of net gain behind the pulse is acceptable, and the criterion of (13) does not have to be fulfilled. The lower limit for the pulse duration that is set by dispersive broadening of the continuum has been roughly estimated using an analytical model with a number of approximations [19]. The result is

$$\tau_p \approx 0.2 \left(\frac{1}{\Delta f_g}\right)^{3/4} \left(\frac{\tau_a}{\Delta R}\right)^{1/4} \frac{g^{3/8}}{\Phi_0^{1/8}} \quad (17)$$

in our notation with the FWHM gain bandwidth Δf_g, the round-trip power gain g, and the soliton phase shift Φ_0. Comparing this with (6) (valid for the case without SPM and dispersion) we see that the dependence on the gain, its bandwidth, and the modulation depth of the absorber is weaker. We also have the additional influence of the absorber recovery time τ_a (because a larger value of τ_a would result in a longer gain window for the continuum) and of the nonlinear phase shift (because a larger phase shift means stronger dispersion, which causes more broadening and thus a higher loss for the continuum). However, the equation does not tell us what value of Φ_0 is ideal; in fact, it would suggest that very high values are preferred, which actually lead to instabilities. The saturation parameter of the absorber also needs to be optimized with other means. Finally, the equation also does not take into account that the soliton pulse may break up into two solitons with e.g. half the energy, thus having half the bandwidth and consequently a higher gain. This pulse break-up is favored particularly if the absorber is too strongly saturated [16, 23].

We see that numerical simulations are still required to answer the question of how much shorter pulse durations are possible due to the influence of soliton pulse formation, and which values for the absorber parameters, the magnitude of SPM, and the dispersion are ideal. In the following, we study this for Nd:YVO_4 lasers as in the previous sections. (Note again that the obtained results can also be applied to femtosecond lasers, using the scaling rules outlined in Sect. 4, as long as additional effects like higher-order dispersion are not important.) We start with a standing-wave cavity with 100-MHz repetition rate, gain bandwidth 1 nm, no spatial hole burning, output-coupler transmission 5%, operation ≈ 40 times above threshold with ≈ 0.91-W average output power, SESAM with 1% modulation depth, 50-ps recovery time, no phases changes in the absorber, and a saturation parameter $S = 4$. Without SPM and dispersion, the obtained pulse duration is 9.3 ps. Now we assume the amount of SPM to be 0.21 μrad/W, as obtained for a 3-mm-long laser crystal with 80-μm mode radius, and vary the amount of dispersion. We readily obtain stable soliton-like pulses with 10-ps duration, but reducing the dispersion for shorter pulses we find that the pulses become increasingly unstable. The minimum pulse duration with (marginally) stable behavior is ≈ 7 ps. (According to (16), the soliton pulse duration should then be closer to 6 ps; we see that the soliton pulses are significantly modified by the gain filter and the absorber action.) For less-negative dispersion, the soliton is not stable against breakthrough of the continuum, which is seen from oscillations on the pulse spectrum. With the modulation depth increased to 2%, the minimum pulse duration is only slightly shorter, ≈ 6 ps. Without SPM and dispersion, this would result in 6.6-ps pulses.

Note that the soliton-shaping effects are actually quite weak in this case: the nonlinear phase shift from SPM for the peak is only ≈ 5 mrad for the 7-ps pulses. We therefore increase the amount of SPM by ≈ 5 times to 1 μrad/W. (In experiments, this could be achieved with a smaller mode size or a longer crystal, or possibly with an additional undoped crystal.) Even with a modulation depth of 1%, this allows for 5.5-ps pulses. A more detailed investigation of the influence of the amount of SPM is presented below.

Equation (17) should be applicable to the discussed cases because the dispersive broadening can be shown to dominate the effect of the temporal shift of the pulse. Nevertheless, the shortest pulses are still about twice as long as predicted. A similar discrepancy was found in [19]. Apparently the equation gives only a rough estimate.

For gain media with broader bandwidth for correspondingly shorter pulses, the SPM effect can become rather large due to the high peak powers. A minimum length of the laser crystal may then become necessary to avoid instabilities from too strong SPM, apart from the minimization of higher-order dispersion effects which also become relevant for pulses with durations below ≈ 30 fs but are not discussed here. Another measure could be to distribute the negative dispersion on both sides of the laser medium (where SPM originates). With twice the modulation depth (2%), we get 3.6-ps pulse duration; as expected from the analytical theory, this is not much less than with 1% modulation depth (4.2 ps).

The saturation parameter S of the absorber turns out to be important. The simulations (with 50-ps absorber recovery time) show that increasing S from 4 (as above) to 10 decreases the minimum stable pulse duration from 4.2 ps to 3.6 ps (Fig. 8). This is because the net gain behind the pulse and the width of the time window with gain are reduced. Even for $S = 100$ (an unrealistically high value for many absorbers), the soliton pulses stay stable against small distur-

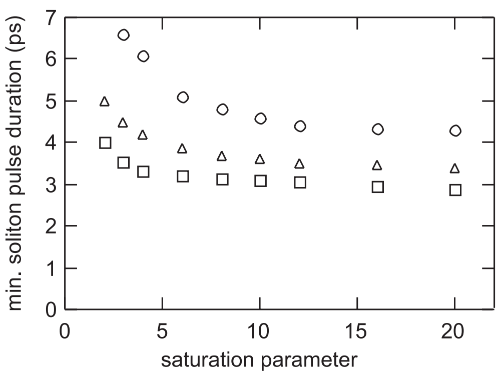

FIGURE 8 Minimum achievable soliton-pulse duration versus saturation parameter of the absorber, for absorber recovery times of 25 ps (*rectangles*), 50 ps (*triangles*), and 100 ps (*circles*)

bances for pulse durations down to 3.4 ps. However, it turns out that for strong absorber saturation and pulse durations close to the given limit the pulses may be unstable against large disturbances. This is revealed e.g. by starting the simulation with double pulses, which may then not merge into a single pulse. For slightly longer pulse durations (e.g. 3.8 ps instead of 3.4 ps for $S = 20$), this bistability and hysteresis does not occur.

It has been proposed [16] that pulse break-up occurs as soon as the loss penalty for double pulses is smaller than the gain advantage which results from the narrower bandwidth of a double pulse. However, this criterion would suggest that e.g. for 1% modulation depth and $S = 8$ the minimum pulse duration is 5.7 ps instead of 3.7 ps. Apparently the nonlinear dynamics do not necessarily lead to the mode of operation which is ideal in terms of utilizing the gain. A detailed and very instructive experimental and theoretical study of pulse break-up is given in [24].

With a faster recovery time (25 ps instead of 50 ps, 1% modulation depth, and $S = 8$), the stability is somewhat increased, and we get a minimum pulse duration of 3.05 ps (instead of 3.7 ps) (see Fig. 8). The improvement of $\approx 18\%$ is roughly as expected from the $\tau_a^{1/4}$ dependence in (17). On the other hand, a longer recovery time of 200 ps raises the minimum pulse duration to 7.6 ps. With weaker absorber saturation ($S = 4$), the situation gets even worse (≈ 11 ps). We see that soliton mode locking does not necessarily mitigate the problem caused by a slow absorber recovery time – in this situation, the pulse duration is not that much reduced compared to the case without SPM and dispersion.

We now investigate further the dependence of the minimum pulse duration on the amount of SPM, assuming $S = 8$ and an absorber recovery time of 50 ps. Figure 9 shows that there is a broad range in which the achievable pulse duration does not vary a lot. However, for very weak SPM, the minimum pulse duration rises sharply, because the soliton-shaping effects become too weak. On the other hand, the stability is compromised by too strong SPM, where the discreteness of the Kerr nonlinearity and the dispersion become relevant. Note that in the numerical simulation we have applied the full amount of SPM and dispersion once per round-trip; by distributing the negative dispersion over both sides of the laser crystal one could somewhat reduce the discreteness and thus tolerate more SPM. The optimum amount for 2% modulation depth, ≈ 10 µrad/W, corresponds to a Kerr phase shift of ≈ 0.4 rad. For 1% modulation depth, the optimum is ≈ 0.2 rad. Note that this is much more than acceptable without dispersion (Sect. 4). Also note that (16) for the soliton pulse duration tends to underestimate the actually obtained pulse duration, particularly in cases with either rather weak or with strong Kerr nonlinearity.

We also consider the effect of phase changes in the absorber (see Sect. 5), which has previously been discussed in the context of soliton mode-locked lasers [17]. We assume an absorber with 1% modulation depth, $S = 8$, 50-ps recovery time, and the other parameters as above. The results are shown in Fig. 10. For $\alpha = 3$, we obtain slightly shorter pulses (3.5 ps instead of 3.7 ps). The center wavelength is somewhat reduced, but there is no significant distortion of the spectrum, which is always reshaped by the soliton effects. With a high value of $\alpha = 10$, we get down to 2.5 ps, and with $\alpha = 15$ we even obtain 1.9 ps. This may appear very surprising because the spectral peak is shifted from 1064 nm to 1063.6 nm, which increases the gain advantage of the continuum. However, it also significantly increases the group velocity of the pulse because of the negative dispersion. This effect can be much stronger than the temporal shift caused by the absorber (see Sect. 3.2). Thus, the pulse 'runs away' from the continuum growing behind it, and the continuum soon gets into the region where the absorber recovery leads to increasing loss. On the other hand, large values of α make the soliton pulses unstable when the pulse duration gets too long (see Fig. 10). Also we note that the soliton pulse duration is always somewhat longer than calculated from (16); this is due to the nonlinear phase change, which acts similarly to some amount of additional dispersion.

To conclude, soliton-shaping effects are very important for the generation of femtosecond pulses. The main reason for this is not the lack of fast saturable absorbers in this pulse-duration regime, as we have seen that pulse durations much shorter than the absorber recovery time are possible without soliton pulse shaping. It is rather the difficulty to avoid excessive nonlinear phase shifts from self-phase modulation (mainly in the gain medium), which occur due to the high peak intensities and destabilize lasers operating without dispersion. With soliton mode locking, much higher nonlinear phase shifts are acceptable and even desir-

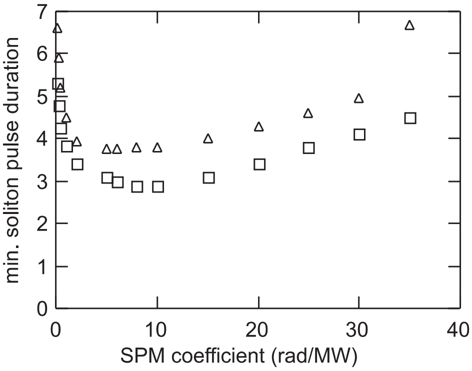

FIGURE 9 Minimum pulse duration (in ps) versus the amount of SPM for modulation depths of 1% (*triangles*) and 2% (*rectangles*)

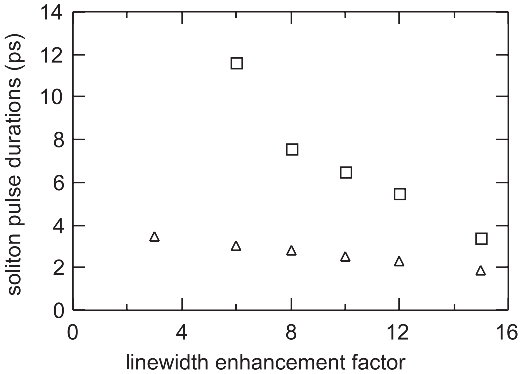

FIGURE 10 Minimum (*triangles*) and maximum (*rectangles*) achievable soliton-pulse durations versus line-width enhancement factor α

able. The pulse shaping is then mainly done by the soliton effects, and the absorber is only needed to stabilize the solitons against growth of the continuum. The absorber parameters are generally not very critical in this regime. The optimum saturation parameter of the absorber is in the order of 10, which is more than in cases without SPM and dispersion. It is important not only to adjust the ratio of dispersion and SPM to obtain the desired soliton pulse duration, but also to keep their absolute values in a reasonable range where the nonlinear phase change is in the order of a few hundred mrad per round-trip. With too-low nonlinear phase changes, the solitons are strongly disturbed by other effects, while excessive nonlinear phase shifts can cause pulse break-up. The latter can also occur for too-strong absorber saturation, although this tendency is found to be significantly weaker than expected from a simple gain/loss argument. Phase changes in the absorber could help to further reduce the pulse duration and would also be recognized through a reduction of the center wavelength. However, typical semiconductor absorbers are expected to have relatively small linewidth enhancement factors, which should lead to only weak effects and are easily masked by other influences. Indeed we are not aware of experimental evidence for such effects.

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