## **Carrier-envelope offset dynamics of mode-locked lasers**

F. W. Helbing, G. Steinmeyer, and U. Keller

Ultrafast Laser Physics Laboratory, Institute of Quantum Electronics, Swiss Federal Institute of Technology, ETH Hönggerberg—HPT, CH-8093 Zürich, Switzerland

**R. S. Windeler** 

Bell Laboratories, Lucent Technologies, 700 Mountain Avenue, Murray Hill, New Jersey

J. Stenger and H. R. Telle

Physikalisch-Technische Bundesanstalt, Braunschweig, Germany

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We investigate coupling mechanisms between the amplitude and the carrier-envelope offset phase in mode-locked lasers. We find that nonlinear beam steering in combination with the intracavity prism compressor is the predominant mechanism that causes amplitude-to-phase conversion in our laser. A second mechanism, induced by self-steepening, is also identified. These mechanisms are important for stabilizing the carrier-envelope offset phase and also explain the extremely low pulse-to-pulse energy fluctuations observed in some lasers with carrier-envelope lock. The coupling mechanisms described have important implications for applications of few-cycle optical pulses. © 2002 Optical Society of America

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The generation of femtosecond pulses has advanced to the generation of pulses with durations of approximately two optical cycles.<sup>1</sup> At this duration, the relative phase between carrier and envelope of a short pulse starts to influence the pulse's conversion efficiency in nonlinear optical processes. Power fluctuations observed in high-harmonic generation have been attributed to fluctuations of the carrierenvelope offset (CEO) phase.<sup>2</sup> Despite the considerable amount of theoretical study in this field, a practical method of measuring the CEO phase was not described until recently.<sup>3</sup> Based on this initial description, measurement and stabilization of a CEO were experimentally reported by several groups of researchers.<sup>4,5</sup> Meanwhile, this method has found widespread use in metrology. Here we address mechanisms for coupling the CEO and the intracavity peak power. Insight into these mechanisms is important for reduction of CEO phase noise in oscillators and also allows for rapid control of the CEO phase.<sup>6,7</sup>

The physical mechanism behind the CEO is the difference between phase velocity  $v_{\varphi}$  and group velocity  $v_g$  of optical materials. For every round trip, this difference induces a phase offset

$$\Delta \varphi_{\rm GPO} = \frac{2\pi}{\lambda} \int_0^L [n_g(z) - n(z)] dz = \frac{\omega^2}{c} \int_0^L \frac{dn(z)}{d\omega} dz$$
(1)

between envelope and carrier of an optical pulse. Here the coordinate z is chosen along the intracavity propagation axis and L is the effective length of the cavity.  $n = c/v_{\varphi}$  is the refractive index,  $n_g = c/v_g$  is the group index, and  $\omega$  is the angular frequency. We call the subcycle part of Eq. (1) the CEO phase,  $\Delta \varphi_{\rm CEO} = \Delta \varphi_{\rm GPO} \mod 2\pi$ . For a laser oscillator we define the CEO frequency as

$$f_{\rm CEO} = \frac{\Delta\varphi_{\rm CEO}}{2\pi T_R} = \frac{\Delta\varphi_{\rm CEO}}{2\pi} f_{\rm rep} \,. \tag{2}$$

The CEO frequency is the rate at which the offset phase of Eq. (1) changes per round trip time  $T_R$ .

As the laser source we employ a Ti:sapphire oscillator similar to that described in Ref. 8. In some of the experiments we also use a prismless variant of the same laser. Inside the 2.3-mm-long Ti:sapphire crystal, effective intracavity intensities of  $4.5 \times 10^{15} \text{ W/m}^2$ for the laser with prisms and  $10^{16}$  W/m<sup>2</sup> for the prismless laser are reached. This average value is defined as in calculation of the soliton phase. For the exploration of amplitude-to-phase coupling effects, we set up the measurement scheme depicted in Fig. 1. This scheme relies on heterodyning two harmonics of different parts of the mode-locked laser spectrum. As in the first demonstration of this method,<sup>4</sup> we additionally broaden the laser spectrum to beyond 1 optical octave by generation of a white-light continuum in a microstructure fiber. The spectral width of the continuum allows the beat of the short-wavelength part of the fundamental and the second harmonic of the long-wavelength part to be used for measuring  $f_{\text{CEO}}$ .

For the beat note we typically achieve a signal-to-noise ratio of ~45 dB in a 300-kHz bandwidth. In the unstabilized laser this beat note may sweep very rapidly, by as much as several megahertz in 1 s. To quantify these frequency fluctuations we converted  $f_{\rm CEO}$  into a proportional voltage signal. The phase noise spectrum of the CEO beat in Fig. 2a shows strong contributions up to several-kilohertz Fourier frequency. The prismless laser clearly shows 1 order of magnitude less noise than the laser with intracavity prisms. Also shown is the CEO phase noise spectrum for a phase lock to a reference oscillator. Integration (10 mHz to 100 kHz) yields a rms phase jitter  $\varphi_{\rm CEO}^{\rm (rms)}$  of ~0.02 rad in the stabilized



Fig. 1. Experimental setup (laser with intracavity prisms): AOM, acousto-optic modulator; Ref, modulation frequency of the pump light; XTAL, Ti:sapphire crystal; PSD, position-sensitive detector; OC, output coupler; MSF, microstructure fiber for continuum generation; SHG, 10-mm lithium triborate crystal for noncritically phase-matched second-harmonic generation of 1100 nm; G, grating used for spectral filtering; APD, avalanche photodiode; f/U, frequency-to-voltage conversion; RF-SA, radio-frequency spectrum analyzer; Lock In, lock-in amplifier.

laser, which compares favorably with the best measurement previously published.<sup>7</sup> To explore the origin of the CEO fluctuations, we use an acoustooptic modulator to deflect a small portion of the pump power into the first diffraction order. The acousto-optical modulator's drive power is periodically modulated with a reference signal, which is also used for phase-synchronous detection of the resultant modulation of  $f_{CEO}$  and beam pointing; see Fig. 1. We monitor the beam position with a position-sensitive detector (Sitek 1L2,5SP), using a spurious reflection off the intracavity Brewster prism.

It is obvious from Eq. (1) that only fluctuations of the first-order dispersion  $dn/d\omega$  have a strong effect on the CEO frequency of a laser cavity. Cavity length fluctuations, however, may contribute only by affecting  $f_{rep}$ in Eq. (2). Typically we observe repetition-rate fluctuations of less than 100 Hz/s in the unstabilized laser, which cannot account for the megahertz sweeps of the CEO frequency. In comparison with the problem of stabilizing the frequency of a continuous single-mode laser, the variations in acoustic and thermal length caused by displacement of the cavity mirrors is clearly insignificant. This leaves dispersion variations of the intracavity elements as the major cause of CEO frequency fluctuations. These variations can be thermally or acoustically induced, or they can be caused by nonlinear refraction.<sup>6</sup> For our laser cavities, we estimate a change of  $\Delta f_{\rm CEO} \approx$  1.4 MHz per 1-K temperature change of the laser crystal.<sup>9</sup> Similarly, we calculate that  $\Delta f_{\text{CEO}} \approx 20 \text{ kHz}$  for an air pressure variation of 1 Pa.<sup>10</sup>

Using a lock-in technique, we carefully mapped out the change in the CEO frequency induced by modulation of the intracavity peak power. We additionally ensured that a change of intracavity pulse energy was accompanied by only a relatively small change of

pulse duration in our laser. For the prismless laser, the frequency dependence of the coupling coefficient is shown in Fig. 2b. We attribute the increased sensitivity at low frequencies to thermally induced index changes. For the laser with prisms at 20-kHz modulation frequency, we measured a CEO frequency change of  $5 \times 10^{-8}$  Hz/(W/m<sup>2</sup>) of intracavity intensity change, i.e., five times stronger than in the prismless case. Similarly, we found an induced beam displacement of the order of  $10^{-19} \text{ m/(W/m^2)}$  of intensity change inside the laser crystal. The beam movement was measured at the location shown in Fig. 1. A 1% modulation of the intracavity intensity translates into a beam movement of 5  $\mu$ m, corresponding to a pointing variation of several microradians. It should be noted that these values have shown some variation with laser adjustment, in particular for a laser with intracavity prisms. However, cavity regimes with strong or weak amplitude-to-phase coupling could not be identified.

Comparing these experimental findings with theoretical estimates, we first explored a direct change in the refractive index by means of the Kerr effect. According to Eq. (1), only the linear dispersion of self-refraction plays a role in the change. This effect is also known as self-steepening.<sup>11</sup> We followed the formalism developed in Ref. 12 to estimate the self-steepening coefficient of sapphire as  $\omega \partial n_2 / \partial \omega = 8 \times 10^{-21} \text{ m}^2/\text{W}$ . The computed self-steepening coefficient leads to a theoretical value of  $\partial f_{\text{CEO}} / \partial I = 4 \times 10^{-9} \text{ Hz}/(\text{W/m}^2)$  for our laser, i.e., slightly less than that observed at a 20-kHz modulation frequency in the prismless laser.

In addition to self-steepening, beam pointing variations also may contribute to the CEO fluctuations in the laser with prisms. Such an effect has already been suspected<sup>13</sup>; however, the laser used in the research reported in Ref. 13 displayed excessive noise, may have prevented the observation of an experimental correlation between CEO frequency and intracavity power. Beam pointing variations can be



Fig. 2. a, Phase noise spectra of the measured CEO signal. Shown are three spectra, those of an unstabilized laser with intracavity prisms and of a prismless laser with and without active phase stabilization. b, Coupling between CEO frequency change (in hertz) and intracavity intensity modulation (in watts per square meter) for a range of modulation frequencies in the prismless laser. Note that the modulation amplitude was kept at a perturbational level of 0.2% and does not fully suppress laser noise at several discrete frequencies.



Fig. 3. Suggested beam pointing variations induced by nonlinear refraction in the gain crystal (XTAL). Nonlinear beam steering induces differential material insertion at intracavity prisms P1 and P2, which leads to changes in the group-phase offset. M1 is an intracavity folding mirror, M2 is an end mirror or output coupler, and PSD is the position-sensitive detector. Experimentally detected beam movements are small compared with the numerical aperture of the beams. The pump acts as a geometric filter and constantly forces the laser mode to maximum overlap with the pumped volume inside the laser crystal.

induced by self-refraction at the interface between the Brewster-cut Ti:sapphire crystal and air, as illustrated in Fig. 3. Assuming that beam angle  $\vartheta_{int}$  inside the Ti:sapphire crystal is fixed, Snell's law demands that an index change inside the crystal be accompanied by a symmetric change of the outside beam angles  $\vartheta_{\text{ext}}$ , yielding an estimated  $\partial \vartheta_{\text{ext}}/\partial I = 3 \times 10^{-20} \text{ rad m}^2/\text{W}$ . Depending on how strongly the pump beam locates the Ti:sapphire laser mode, a translational movement or change in the internal beam angle may also occur and reduce the net beam pointing effect. Therefore the value given above may serve only as an upper estimate for the nonlinear beam steering but is well compatible with measurements. Beam steering inside the intracavity prism compressor is accompanied by a change in the group-phase offset. For the fused-silica prism sequence with 30-cm apex separation that was used, we calculate a beam pointing sensitivity  $\partial f_{\rm CEO}/\partial \vartheta = 2.5 \times 10^{12} \ {\rm Hz/rad.}^{14}$  The beam direction influences chiefly the effective material insertion of the prism compressor. Together, they yield an estimated value of  $\partial f_{\rm CEO}/\partial I = 7 \times 10^{-8}$  Hz (m<sup>2</sup>/W), which agrees well with the measured value of  $5\times 10^{-8}~\text{Hz}~(m^2/W)$  for that laser.

In summary, we have identified two major mechanisms that cause rapid conversion of amplitude fluctuations into changes in cavity group-phase offset. In a laser with prisms the combined action of nonlinear beam steering and the beam pointing sensitivity of the prism compressor is dominant. The self-steepening mechanism prevails in prismless cavities. Thermally induced changes in dispersion are also present and increase the effects at low frequencies. All amplitude-to-phase conversion mechanisms work in two ways: First, they permit one to control the CEO frequency of a cavity by simply modulating intracavity peak power.<sup>6</sup> Second, stabilization of the CEO frequency will also suppress coupled pulse energy fluctuations. This inverse coupling explains the reduction in pulse energy noise found in our laser at frequencies

above 2 kHz and also reported in Ref. 7. This coupling mechanism was initially explained by a powerdependent spectral shift of the laser spectrum.

Our results further suggest the use of prismless dispersion-compensation schemes for CEO-stabilized lasers. A prismless setup rules out CEO frequency changes by means of nonlinear beam steering effects and explains the excellent passive stability reported in Ref. 7. Dynamic changes in pulse shape that lead to modulation of peak power should also be avoided. Fluctuations of the CEO phase are a serious concern in external amplification schemes. A stabilization of the seed may well be swamped by geometrical effects in the compressor-stretcher or by nonlinear optical effects in the amplifier. Equation (1) clearly identifies the group-phase offset as the decisive parameter in such systems. Careful engineering of nonlinear and environmental contributions to this quantity is necessary to prevent excessive fluctuation of the CEO phase. Absence of fluctuation will be of great importance for generation of attosecond pulses and in high-harmonic experiments with few-cycle pulses.

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