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Soliton-like pulse-shaping mechanism in passively mode-locked surface-emitting semiconductor lasers

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ABSTRACT We discuss a mechanism that allows the formation of nearly transform-limited soliton-like pulses in passively mode-locked optically pumped external-cavity surface-emitting semiconductor lasers. It involves the interplay of positive dispersion and the nonlinear index changes in gain medium and saturable absorber, while ordinary solitons are based on dispersion and the Kerr effect. The obtained quasi-soliton pulses share some of the properties of ordinary solitons (in particular, their stability and near bandwidth-limited nature), while other properties are different. In particular, the pulse duration scales with the square root of the cavity dispersion, and an excessive drift of the laser wavelength must be avoided by proper design.

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1 Introduction

Recently, we demonstrated the first external-cavity surface-emitting semiconductor laser (VECSEL) which was optically pumped and passively mode-locked with a semiconductor saturable absorber mirror (SESAM) [1]. This concept promises to lead to sources of picosecond or even femtosecond pulses with high average power (potentially several watts) and a high (multi-GHz) repetition rate. The optically pumped gain structure allows for power scaling by increasing the pumped mode area, provided that a design with sufficiently low thermal impedance [2, 3] is used. In this way, one can obtain far higher average powers and pulse energies than from any other type of mode-locked semiconductor laser. Recently, we have demonstrated a device generating 15-ps pulses at 952 nm with nearly 1-W average power and 6-GHz repetition rate [4], and devices with even significantly higher output power appear to be possible. In comparison to passively mode-locked high repetition rate lasers based on Nd:YVO_4-doped crystals [5], passively mode-locked VECSELs promise higher powers in the multi-GHz regime. In addition, we have the possibility to design such devices for different wavelengths, e.g. 950 nm for blue-light generation via frequency doubling or longer wavelengths around 1.3 μm or 1.5 μm, when different semiconductor materials are used. Also, the large gain bandwidth of semiconductors allows for shorter pulses; pulse durations < 0.5 ps have recently been demonstrated [6]. Note that with other broadband gain media such as rare-earth-doped glasses it is difficult to do passive mode locking at multi-GHz repetition rates, because these gain media typically have small laser cross sections which lead to a strong tendency for Q-switching instabilities [7, 8]. Semiconductor lasers have much weaker Q-switching tendencies due to their high differential gain.

Another important consequence of the high differential gain of semiconductors is that significant gain saturation can occur during the amplification of a pulse circulating in a mode-locked semiconductor laser. This leads not only to a reduction of the gain during a pulse, but also to a refractive-index change which causes a nonlinear phase change. Mainly for this reason, mode-locked semiconductor lasers often produce strongly chirped pulses. This was also the case for early mode-locked VECSELs [1]. Bandwidth-limited operation of mode-locked semiconductor lasers has been obtained in some cases, e.g. in monolithic devices with high repetition rate by balancing the chirps from gain medium and saturable absorber, which have opposite signs [9]. This mechanism was effective only at low output powers and with weak saturation of the absorber, because otherwise the phase change from the absorber becomes strongly nonlinear and cannot be compensated in this way. Therefore, this can not be the explanation for the first nearly bandwidth-limited pulses from a passively mode-locked VECSEL that we recently reported [3, 6] but could not explain at that time.

In this paper we describe a soliton-like pulse-shaping mechanism which can generate nearly bandwidth-limited pulses in a wide range of intracavity dispersion values and with strong saturation of the absorber, as is typical for mode-locked VECSELs. We found that this mechanism had been discovered already in 1985 by Martinez et al. [10] in the context of a model for passively mode-locked dye lasers. Such lasers share some properties with mode-locked semiconductor lasers, in particular the strong gain saturation and associated refractive-index changes. We have investigated this pulse-shaping mechanism in detail, using a different kind of model. In Sect. 2 we describe the basic mechanism and in Sect. 3 the model with which we did extensive simulations. In Sect. 4 we discuss effects which can cause a strong

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red or blue shift of the pulses and possibly an instability of the mode-locking process; these effects, which are not relevant for ordinary soliton lasers, have important consequences on the design of mode-locked VECSELs. Section 5 then treats the effect of the cavity dispersion in both the positive- and negative-dispersion regimes. More details on the origin, magnitude, and nonlinearity of the phase changes in semiconductors are given in Sect. 6. In Sect. 7 we apply the model to interpret results from the previously reported nearly bandwidth-limited pulses from a VECSEL, and in Sect. 8 we discuss further design guidelines and basic limitations for lasers with either low or high repetition rate. We finally summarize the most important findings in Sect. 9.

2 The pulse-formation mechanism

Important parameters of a passively mode-locked VECSEL are the saturation parameters S_g and S_a of the gain medium and absorber, respectively, defined as the ratios of intracavity pulse energy to the saturation energies of both components. For the absorber, we typically have $S_a > 3$, so that the phase change from the absorber, which is usually negative, occurs mainly in the leading edge of the pulse, and its magnitude is basically independent of the pulse energy. On the other hand, the gain medium is typically operated with S_g well below 1. It is important to note, however, that the gain-saturation energy of a quantum well depends on the carrier density. The value of S_g at the operating point of a VECSEL is therefore sensitive to cavity loss, and can easily increase by a factor of two in response to e.g. cavity misalignment.

The nonlinear phase changes in the gain medium and absorber are directly related to the corresponding carrier densities and thus to the gain or loss. For pulses of duration > 1 ps, for which the rate-equation model is an adequate approximation, these phase changes may be represented by the phenomenological linewidth enhancement factor α (see [21]). The relation between phase change and gain or loss is linear [11]; we then have

$$\Delta\varphi(t) = -\alpha_g\, g(t)/2 \tag{1}$$

for the phase change (relative to the state with zero gain) in the gain medium, where α_g is the linewidth enhancement factor, and the (dimensionless) gain coefficient $g(t)$ corresponds to the power gain of the whole structure, not the gain per unit length. The corresponding equation for the absorber involves the value α_a. The linewidth enhancement factors for gain and loss are wavelength- and carrier-density-dependent; it is not easy to determine their effective values in a given experimental situation. Both α_g and α_a are assumed to be constant during the passage of the pulse, although we note that for the absorber this may not be a good approximation, as we discuss further in Sect. 6.

The phase change in the gain medium is positive, and rises slowly during the pulse. Its magnitude depends nearly linearly on the pulse energy. We note that α_g may be significantly larger than α_a, because the absorber is typically operated fur-

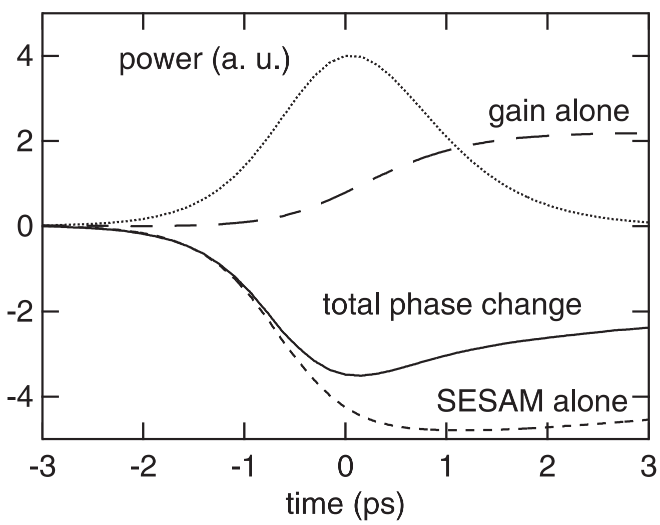

FIGURE 1 Total nonlinear phase change and phase changes from gain medium and absorber (all in mrad), as generated on the quasi-soliton pulse of Fig. 2. The positive cavity dispersion exactly compensates this phase change, so that the total phase change is constant over time. *Dotted curve*: intensity profile of the pulse

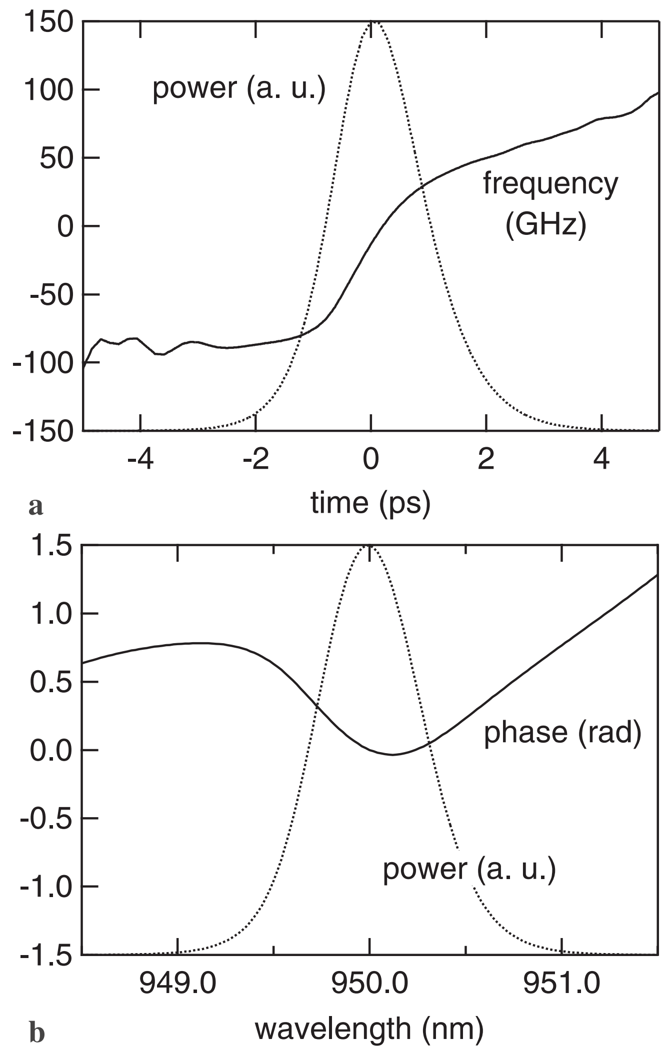

FIGURE 2 Quasi-soliton pulse as obtained from interplay of positive dispersion and nonlinear phase changes. **a** Intensity and instantaneous frequency (in GHz, relative to the gain peak) versus time. **b** Spectral intensity (a.u.) and phase versus wavelength

ther above the band gap and at a lower excitation level than the gain medium (see Sect. 6 for more details). As a consequence of this, the nonlinear phase changes in the absorber can be of similar magnitude to those in the gain medium, although the modulation of the loss (negative gain) in the absorber is typically larger than in the gain medium.

For typical parameters, the sum of the nonlinear phase changes from gain medium and absorber has a pronounced minimum at a time somewhat before the pulse maximum and is reminiscent of the phase change from a Kerr medium with negative Kerr nonlinearity, although the phase profile is not directly linked to the intensity profile. Figure 1 shows a numerically calculated example (with details discussed in Sect. 3). This leads us to expect that with positive intracavity dispersion there could be a stable state with a pulse shape such that the phase change from the dispersion exactly cancels the nonlinear phase changes (apart from a constant phase offset) and nearly bandwidth-limited pulses are obtained in a similar way as in a soliton mode-locked laser [12, 13]. Numerical simulations, described below in detail, have shown that such pulses, which we call quasi-solitons, indeed exist over a wide range of dispersion values. Their intensity profile (Fig. 2a) and spectrum (Fig. 2b) are in general slightly asymmetric, and this asymmetry is linked to the asymmetric nonlinear phase change. (Ordinary solitons, based on the symmetric nonlinear phase change from the Kerr effect, are known to have a symmetric temporal and spectral sech^2 shape.) The pulse has some up-chirp (Fig. 2a), but the time–bandwidth product is 0.375, i.e. not far from the value of 0.315 for bandwidth-limited solitons.

3 The model

As mentioned above, the basic mechanism for soliton-like pulse formation has been previously discovered by Martinez et al. [10] in a model for passively mode-locked dye lasers. He used analytical methods which required a number of relatively crude approximations, including the assumption of symmetric sech^2 pulse shapes and Taylor expansions for various effects including the saturation of the absorber. The strong saturation effects occurring in the absorbers of mode-locked VECSELs are not well approximated in this way.

We use a fully numerical model which requires only a few approximations and is accurate even in the presence of strong saturation effects. Also, it allows us to investigate more subtle pulse properties, such as asymmetric temporal pulse shapes and spectra as well as arbitrary nonlinear chirps. Stability of the solutions, or the nature of eventually occurring instabilities, can also be investigated. The principle of the model is straightforward: a pulse, represented by a series of typically 512 or 1024 complex amplitudes either in the time or in the frequency domain, is propagated through the laser cavity. Step by step we take into account the effects of saturable gain and loss, associated nonlinear phase changes, the limited gain bandwidth, and the cavity dispersion. The saturable gain is taken to be wavelength-independent, and the finite gain bandwidth is accounted for by addition of a fixed wavelength-dependent loss. This procedure makes the model conceptually simpler and numerically more efficient than rigorous modeling of wavelength-dependent saturable gain. We also use a simple saturation model with a constant saturation energy $E_{\mathrm{sat,g}}$. Since the differential gain in a semiconductor is a function of the excitation level, the value of $E_{\mathrm{sat,g}}$ is affected by the operating conditions of the laser, and in particular by the cavity loss. The variation of $E_{\mathrm{sat,g}}$ during the passage of the pulse is a small effect provided $S_{\mathrm{g}} < 1$.

Gain saturation during amplification of a pulse is then described by the differential equation

$$\frac{\mathrm{d}g}{\mathrm{d}t} = -\frac{g-g_0}{\tau_{\mathrm{g}}} - \frac{gP}{E_{\mathrm{sat,g}}} \ , \qquad (2)$$

where $g(t)$ is the (wavelength-independent) power gain, P the intracavity optical power, g_0 the small-signal gain, and τ_{g} the gain relaxation time. In addition to the changes of optical power, a phase change depending on $g(t)$ is applied according to (2). Similar equations describe the absorber dynamics. The effect of dispersion is treated in the frequency domain, with a fast Fourier transform algorithm used for changing between time and frequency domains.

We typically start with a sech^2-shaped pulse of a few picoseconds duration and propagate this through the cavity until either a steady state is reached or we recognize an instability. On a state-of-the art PC this typically takes a minute or less. The program allows us to interactively display temporal shapes and pulse spectra after an arbitrary number of cavity round-trips and anywhere in the laser cavity.

Figure 2 shows the temporal and spectral characteristics of a stable quasi-soliton model pulse formed in a cavity with positive dispersion. This simulation assumes the following laser properties: gain medium with a 10-nm bandwidth around 950 nm, saturation energy $E_{\mathrm{sat,g}} = 50$ nJ (corresponding to a spot radius in the order of 100 μm), recovery time 2 ns, $\alpha_{\mathrm{g}} = 5$, absorber with modulation depth $\Delta R = 1\%$, 0.5% nonsaturable loss, saturation energy $E_{\mathrm{sat,a}} = 0.92$ nJ (corresponding to a spot radius on the absorber in the order of 20 μm), recovery time 20 ps, and $\alpha_{\mathrm{a}} = 1$. (As mentioned above, saturable absorbers typically have a lower linewidth enhancement factor compared to gain media, because they are less strongly excited and often operated higher above the band gap.) For simplicity, gain medium and absorber are assumed to be part of a ring cavity, although we typically use the gain medium as a folding mirror in a standing-wave cavity in order to have more round-trip gain. The cavity dispersion is 3000 fs^2 per round-trip. The pump power was adjusted for an intracavity pulse energy of 4.7 nJ, corresponding to 9.4% of the gain-saturation energy or to 0.14 W average output power. Thus for this laser we have $S_{\mathrm{a}} = 5.1$, $S_{\mathrm{g}} = 0.094$.

This data set allowed us to investigate the principle of the pulse formation. The parameter values are broadly consistent with the operating regime of the laser described in [3]. Note, however, that it is particularly difficult to ascribe reliable values to the linewidth enhancement parameters in a given experimental situation, for the reasons discussed in Sect. 6. Other parameter ranges will be explored in what follows.

4 Tendency for red or blue shift

An important observation from the numerical simulations is that in general the pulse spectrum tends to drift away from the wavelength of maximum gain. This is a result of the remaining chirp, combined with saturable loss and gain. In steady state, we typically observe that the pulse has an up-chirp, i.e. with the lower-frequency components in the leading edge. As the saturable absorber (with its slow recovery) causes more loss for the leading edge, it attenuates the lower-frequency components more and thus applies a blue shift to the pulse spectrum. As the absorber is typically fully saturated, the effect is not sensitive to the exact value of the pulse energy.

A similar effect occurs in the gain medium: due to gain saturation during amplification of a pulse, the trailing edge (containing the higher-frequency components) is slightly less amplified, with the effect of a net red shift of the pulse spectrum. In contrast to the case of the absorber, in the gain medium the pulse energy is well below the saturation energy, so that the strength of this effect increases with increasing pulse energy, as this increases the gain difference between the low-frequency leading part and the high-frequency trailing part of the pulse.

It now becomes clear that depending on the pulse energy and other parameters like the saturation energies of gain and absorber, there can be a balance of the tendencies for red and blue shifts, so that the maximum of the pulse spectrum will be close to the peak of the net gain spectrum. For the pulse shown in Fig. 2, both effects have been balanced by adjusting the saturation energy $E_{\mathrm{sat,a}}$ of the absorber. High values of ΔR and $E_{\mathrm{sat,a}}$ favor a blue shift, while the tendency for a red shift increases with rising intracavity pulse energy. This balance obviously depends on the gain-saturation energy, but also on other parameters like the cavity dispersion and the linewidth enhancement factors. For this reason, it appears that a simple quantitative criterion for this balance can not be given; any concrete cases should be checked with a numerical model as described above.

We now consider cases where the red and blue shifts do not balance. In the case where there is a net tendency for a blue shift, the situation is in general unstable: the blue shift reduces the gain and thus the pulse energy, which increases the blue shift further, so that the pulse can finally fade away, and a new pulse is formed. Experimentally one expects to see a very unstable regime with strong fluctuations of pulse energy and autocorrelation signals.

The other case, with a net red shift, can be obtained, for example, by increasing the pump power or by decreasing the mode area on the absorber. It is very different from the blue-shift regime: if some red shift leads to a reduction of the pulse energy, the red-shift tendency is reduced, and a stable situation is reached, unless the red-shift tendency is very strong. A stable situation with only a slight reduction of pulse energy can be achieved by optimizing the nominal parameters for a weak red shift. It is also more easily obtained when the gain rapidly drops (or some loss rapidly increases) on the long-wavelength side, so that the spectrum is forced to stay near the point of maximum gain. Fortunately, this kind of gain spectrum is typical for quantum well gain structures, where the gain drops rapidly at the band gap.

As the regime with a net blue shift is generally unstable, a laser should ideally be designed so that for the given pump power there is a weak red-shift tendency. (Later we will discuss in more detail how to achieve this.) For lower pump powers (and thus weaker gain saturation), stable mode locking will then usually not be possible, because then there is a net blue shift. Also note that stable mode locking will typically occur at a somewhat longer wavelength than continuous-wave lasing (without the absorber). Indeed, we have occasionally observed this in experiments.

One might expect that soliton lasers based on other solid-state gain media with very high saturation energies (such as rare-earth-doped glasses or crystals) should suffer from an excessive blue-shift tendency, as the balance described here is not possible given the negligibly small gain saturation during a pulse. However, this is usually not the case, because the soliton-shaping effects (from dispersion and the Kerr effect) are strong enough for the pulse not to be significantly disturbed by the additional phase changes in the absorber. (Typical Kerr phase shifts are in the order of 100 mrad, while the absorber contributes only a few mrad.) We then get nearly unchirped pulses and no significant blue or red shift. In the mode-locked VECSEL, the balance is more delicate because the whole pulse formation is done by rather weak phase effects (in the order of a few mrad). This makes the pulse formation more sensitive to other effects such as the gain filter and saturation effects acting on the pulse spectrum. Also, this causes the time–bandwidth product of the pulses to be somewhat larger than from typical soliton mode-locked lasers. We have verified with our model that indeed the time–bandwidth product is reduced e.g. from 0.375 (parameters of Fig. 2) to 0.344 when we do the simulation with increased (perhaps unrealistically high) values of $\alpha_{\mathrm{g}} = 10$ and $\alpha_{\mathrm{a}} = 2.5$. This emphasizes yet again that in this regime of operation the nonlinear phase changes do not act as a disturbing effect but rather as part of the actual pulse-formation mechanism.

5 Effect of cavity dispersion

If the laser is designed as discussed above, the quasi-soliton shaping mechanism works over a wide range of dispersion values. In Fig. 3 we plot the characteristics of model quasi-soliton pulses as a function of positive intracavity dispersion from $5000\,\mathrm{fs}^2$ to $50\,000\,\mathrm{fs}^2$. The other laser parameters, similar to those used to generate the pulse in Fig. 2, are kept constant. Over the whole range of dispersion, the pulses are close to bandwidth-limited (with time–bandwidth products between 0.35 and 0.42). This demonstrates that the quasi-soliton pulses automatically adjust their shape so that the dispersion can compensate the nonlinear phase shift, very much as is the case with ordinary solitons. However, the pulse duration is not proportional to the dispersion, but rather approximately to the square root of the dispersion, as shown by the fit in Fig. 3. This can be understood as follows. The dispersive phase change within the pulse spectrum is proportional to the product of the group-delay dispersion and the square of the pulse bandwidth. (Note that the group delay dispersion term is the quadratic one in the expansion of the frequency-dependent spectral phase change.) This dispersive phase change must compensate the nonlinear phase shift, which is roughly independent of the pulse duration if it is generated not by the Kerr effect but by refractive-index changes due to changes of carrier density in the gain medium and the absorber (assuming

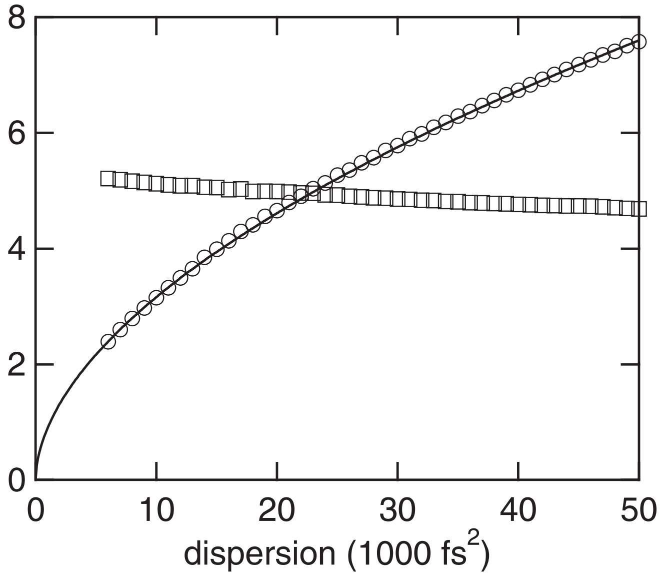

FIGURE 3 Pulse parameters of a mode-locked VECSEL versus cavity dispersion: *circles* = pulse duration (ps), *rectangles* = pulse energy (nJ). For dispersion values below $\approx 5000\,\mathrm{fs}^2$ the pulses are not stable. The fit function for the pulse duration is proportional to the cavity dispersion to the power of 0.545

that the relaxation times are longer than the pulse duration). Therefore, the product of dispersion and the square of the pulse bandwidth must be constant, so that the pulse duration must scale with the square root of the dispersion (assuming a constant time–bandwidth product). On the other hand, the nonlinear phase shift from the gain medium increases with increasing pulse energy, so that the pulse duration typically decreases with increasing pump power. The latter effect is qualitatively similar to the one in soliton lasers.

In the situation of Fig. 3, the pulse energy drops slightly with increasing dispersion. This results from the increased absorber loss due to absorber recovery during the pulse; we used a recovery time of 40 ps. For a longer absorber-recovery time, the pulse energy would be affected less, and the square-root law would be followed even more closely, but the minimum obtainable pulse duration would then be increased. This minimum is set by an instability which appears to be quite similar to that usually observed in soliton lasers. With a faster absorber, shorter pulses can be obtained. A 5-ps absorber would even allow us to operate with zero dispersion. This would lead to sub-picosecond pulses, but with a somewhat increased time–bandwidth product of 0.49.

We also investigate the regime of negative dispersion. Here we obtain longer and strongly down-chirped pulses with larger time–bandwidth products. The chirp is quite nonlinear, particularly for small negative dispersion (e.g. $-500\,\mathrm{fs}^2$), so that bandwidth-limited pulses would not be obtained with a simple external dispersive compressor. Also, the nonlinearity of the chirp causes the spectrum to be strongly asymmetrical, with a much longer tail on the short-wavelength side. This is just the type of spectrum which we have observed in some experiments [1]. In this regime, the temporal shape (without external compression) is also strongly asymmetric, with a fast rise and a slower decay.

6 Nonconstant linewidth enhancement factors

For a simple two-level absorber, the nonlinear phase change is linearly related to the excitation level. Semiconductor media, however, exhibit a more sophisticated behavior: the shape of the gain/loss spectra strongly depends on the excitation level. For low excitation, the differential gain $\mathrm{d}g/\mathrm{d}n_\mathrm{c}$ (with respect to the carrier density n_c) is largest for photon energies somewhere near the band gap. For higher excitation levels, the gain near the band gap saturates, and the maximum of $\mathrm{d}g/\mathrm{d}n_\mathrm{c}$ occurs at higher photon energies. As the differential refractive index change for the photon energy $h\nu$ is given by the nonlinear Kramers–Krönig relation [14]

$$(\mathrm{d}n/\mathrm{d}n_\mathrm{c}) = -\frac{\hbar c}{\pi}\wp\int\frac{(\mathrm{d}g/\mathrm{d}n_\mathrm{c})}{E^2-(h\nu)^2}\mathrm{d}E \qquad (3)$$

(with the derivatives taken for a given carrier density n_c), we can see that the index changes depend nonlinearly on the excitation level. (Note that in this equation g is the gain per unit length.)

To study this effect in more detail, we used a simple numerical model for calculating gain/loss spectra and phase changes for different excitation levels. This model is based on the assumption of simple parabolic conduction and valence bands, although it is known that e.g. in GaAs and InGaAs structures the valence-band structure is actually more complicated. We also disregarded effects like increases of electron or hole temperatures, carrier trapping, exciton formation, or band-gap renormalization. By calculating the effect of band filling alone we can reasonably approximate the gain/loss spectra resulting from more sophisticated models (see e.g. [15, 16]) and obtain estimates of the nonlinearity of the phase changes.

Using this model, we calculated the phase change versus gain (negative loss) for a 10-nm unstrained GaAs quantum well absorber, excited at different wavelengths up to the fully saturated state (see Fig. 4). For excitation close to the band gap (e.g. at 830 nm), the behavior can be reasonably approximated by a constant positive linewidth enhancement factor α_a, while for excitation with photon energies higher above the band gap, this approximation breaks down: with increasing excitation, the phase change is first positive, then reaches a maximum, and decreases again. This can be approximated by a quadratic relation of the form

$$\Delta\varphi(t) = -\left[\alpha_\mathrm{a}(g(t)/2)+\alpha_\mathrm{a}^{(2)}(g(t)/2)^2\right] \quad , \qquad (4)$$

where $g(t)$ is the intensity gain coefficient e.g. for propagation through 1 μm of the absorber material and the phase change is relative to the fully saturated state ($g=0$). α_a determines the slope near $g=0$, and α_a^2 is another constant. For example, at 810 nm ($\approx$ 13 nm above the band gap), we have $\alpha_\mathrm{a}=1.34$ and $\alpha_\mathrm{a}^2=0.508$, and for full saturation the phase change returns approximately to the phase change in the unsaturated state (see Fig. 4). For higher photon energies, α_a becomes smaller and $\alpha_\mathrm{a}^{(2)}$ larger. We see that the nonlinearity of the dependence of $\Delta\varphi$ on the excitation is rather strong.

On the other hand, for the gain medium we can still use a constant value of α_g because the excitation level does not strongly change in a mode-locked VECSEL. However, note

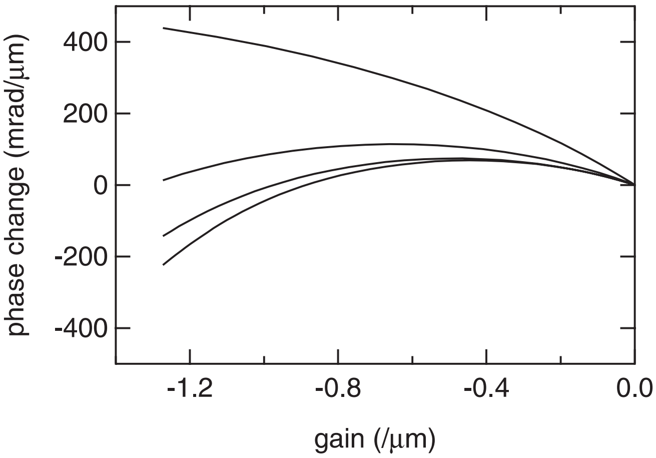

FIGURE 4 Phase change versus gain (negative loss), calculated for a 10-nm GaAs quantum well excited at different wavelengths of 830 nm (*top curve*), 810 nm, 790 nm, and 770 nm (*lowest curve*)

that α_g should be calculated for the average excitation level, not for zero excitation. Typical values of α_g are in the order of 2 to 3, i.e. significantly larger than those of α_a.

Note that, for example, effects of strain in InGaAs quantum wells can significantly modify the valence-band structure (see e.g. [15]). Our model can not easily take account of such effects, but we have tested the influence of modified effective masses in the valence band, which also affect the density of states in the valence band. We found that the resulting changes of the linewidth enhancement factors are noticeable but not large enough to invalidate the qualitative results given above.

We have also extended the pulse-propagation model (Sect. 3) for nonlinear phase changes according to (4). This allowed us to test whether quasi-solitons can still exist in situations where the mentioned nonlinearity is strong. Indeed we found that this is possible, using e.g. the parameters given above for 810 nm. Compared to the case with $\alpha_a^2 = 0$, we get pulses with a somewhat increased time–bandwidth product (0.57 instead of 0.40), which can be interpreted as a consequence of the overall weaker nonlinear phase change. However, the basic properties of the quasi-solitons are maintained.

7 Comparison with experiments

So far, there are few experimental situations which can be compared with the results of simulations. We have done such a comparison for the laser which generated nearly transform-limited 3.2-ps pulses with 213-mW average power [3]. Unfortunately, some of the parameters are difficult to precisely determine, namely the saturation fluence $F_{sat,g}$ of the gain medium, the bandwidth and spectral shape of the gain, the linewidth enhancement factors, the mode area on the absorber (which critically depends on the cavity adjustment), and the cavity dispersion (which can be affected by growth errors and can vary somewhat with the used spot on the gain sample). However, we got satisfactory agreement for both pulse duration and time–bandwidth product (≈ 0.5) with realistic parameters of $\alpha_g = 3$, $\alpha_a = 0.3$, $E_{sat,g} = 50$ nJ ($F_{sat,g} = 160\ \mu J/cm^2$), $E_{sat,a} = 1$ nJ, and $+3000$-fs^2 dispersion per round-trip. These parameters are certainly only approximate, but the results clearly show that the nearly transform-limited nature of the pulses is consistent with the discussed pulse-formation mechanism. Similar agreement was also found for nearly bandwidth-limited pulses with 600-mW average power from a similar device [4].

For the laser of [3], the dispersion results mainly from the gain medium, while the dispersion of mirrors and saturable absorber are negligible. There is an interference effect between the surface reflection and the Bragg mirror of the gain medium, which leads to a calculated dispersion of $+3000\ fs^2$ in the vicinity of the laser wavelength and smaller dispersion at longer or shorter wavelengths.

Future designs will be developed with smaller positive dispersion so as to obtain significantly shorter pulse durations, possibly even below 1 ps. Note that dispersion could also be incorporated in the saturable absorber structure [17, 18]. It is also a possibility to construct cavities with a tunable dispersive element, e.g. a Gires–Tournois interferometer (GTI) [19].

As both the laser wavelength and the spectral position of the dispersion profile vary by a few nanometers across the wafer, different dispersion regimes (even with dispersion values of different signs) can be obtained by using different parts of the same wafer. Indeed, we have observed that the pulse characteristics can be quite different between such parts.

8 Optimization of laser designs

In the following, we discuss the optimization of lasers operating in the positive-dispersion regime, which is clearly most attractive as nearly transform-limited pulses can be generated. We have already mentioned in Sect. 4 that a net blue shift for the pulse, generated by the interplay of the chirp and the saturable gain and absorption, has to be avoided because this can lead to an unstable situation. As an exact balance of the red- and blue-shift tendencies is difficult to achieve experimentally, the laser should be designed for a weak net red shift, for which a stable situation with only a slight reduction of efficiency is achieved. The most important parameters in this respect are ΔR and $E_{sat,a}$ of the absorber on one side and on the other side the parameters governing the amount of gain saturation, in particular the intracavity pulse energy E_p (which is influenced by the output coupler transmission T_{OC} and the repetition rate) and the gain-saturation energy $E_{sat,g}$. A blue shift is generally favored by large values of ΔR and $E_{sat,a}$, where the latter can be adjusted via the mode area on the absorber. A larger red shift can be obtained by increasing E_p and decreasing $E_{sat,g}$.

When we optimize a mode-locked VECSEL for a relatively low repetition rate (e.g. 2 GHz), the gain-saturation effect and thus the red-shift tendency is strong. In this situation, we found that the highest pulse energy is achieved with a relatively high value of ΔR. This may seem surprising because it increases the total cavity losses, but it also decreases the red-shift tendency, so that the pulses are not so strongly pushed out of the wavelength region with maximum gain. One might also increase the output coupler transmission and increase the mode area on the gain medium (so as to increase $E_{sat,g}$). However, all these measures increase the laser threshold and can therefore be applied only to a limited extent. It turns out that repetition rates below about 1 GHz are possible but will usually be obtained only by operation closer to threshold, i.e. with a reduced power efficiency. Clearly, a further reduction of the repetition rate of a mode-locked VECSEL is not a suitable way to increase the obtained pulse energies or peak powers. This is different with lasers based on ion-doped gain media, where gain-saturation effects are much weaker.

In the opposite case of high repetition rates, gain saturation becomes weak, and measures have to be taken to compensate the blue shift generated by the absorber. In particular, low values of ΔR and $E_{\mathrm{sat,a}}$ may have to be used. On the other hand, the gain saturation should be maximized by operating the laser with a high pump intensity (small spot size on the gain medium, small $E_{\mathrm{sat,g}}$) and small output coupler transmission. For very high repetition rates (e.g. 40 GHz), the absorber action has to be so weak that the quasi-soliton pulse-shaping mechanism gets very weak. In this situation, pulse formation needs many thousands of round-trips, and the pulses are quite sensitive to any disturbance. However, another limiting factor for the repetition rate is the difficulty to construct extremely short cavities – e.g. a 40-GHz standing-wave cavity is only 3.75-mm long. Note also that it is then difficult to obtain sufficiently small mode areas on the absorber. So far, we have always operated mode-locked VECSELs at repetition rates of > 1 GHz, where the cavities are relatively easy to build and the pulse formation works well.

One should also consider optimization of the linewidth enhancement factors of gain medium and absorber. Particularly for the absorber, operation relatively close to the band gap seems to be beneficial, because otherwise the nonlinear phase changes get rather small and even depend nonlinearly on the excitation (see Sect. 6). In general, larger values of α_{a} and α_{g} are better for a higher stability and a smaller time–bandwidth product of the pulses, because then the nonlinear phase changes are larger and the pulses are less susceptible to other effects. The steady state is then also reached more quickly. All this is similar to the case of ordinary soliton lasers, where the Kerr phase shift should not be too small [20].

9 Conclusions

In conclusion, we have shown that quasi-soliton pulses in passively mode-locked external-cavity surface-emitting semiconductor lasers can arise from the interplay of positive cavity dispersion with the nonlinear phase changes from gain medium and saturable absorber. Like ordinary solitons, quasi-solitons are close to bandwidth-limited and can automatically adjust their shape to the given dispersion level, which may be varied in a larger range. Some of the properties of quasi-solitons are quite different from those of ordinary solitons. In particular, their duration scales with the square root of the cavity dispersion, and in general the pulse spectrum tends to shift away from the wavelength of maximum gain. For stable operation, a net blue-shift tendency must be avoided, and ideally the laser design should be adjusted so as to obtain a weak red shift. We have discussed the consequences of this for the design of lasers with particularly low or high repetition rates. Optimization for low repetition rates tends to decrease the power efficiency, while the pulse-formation process gets weaker at high repetition rates, making the quasi-solitons more susceptible to various kinds of disturbance. In most cases the operation of the absorber near the band gap will be beneficial because this leads to large nonlinear shaping effects.

Our findings explain important aspects of the operation of previously built mode-locked VECSELs. In particular, they explain why some devices generated strongly chirped pulses [1], while nearly bandwidth-limited pulses were obtained from others [3]. This understanding will allow for very significant further progress. In particular, we can now deliberately optimize VECSEL designs with an appropriate amount of positive dispersion so as to obtain nearly bandwidth-limited pulses and also to achieve shorter pulse durations, even below 0.5 ps [6]. We envisage that mode-locked VECSELs will soon deliver bandwidth-limited pulses with average output powers of well above 1 W and $\approx$ 1-ps pulse duration. Such sources are very interesting for a number of applications, e.g. those involving further nonlinear frequency conversion. Frequency-doubled multi-watt VECSELs may be used to generate red, green, and blue beams for displays, and such lasers are also very interesting pump sources for parametric oscillators with multi-GHz repetition rates and wavelength-tunable output.

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REFERENCES

1 S. Hoogland, S. Dhanjal, A.C. Tropper, S.J. Roberts, R. Häring, R. Paschotta, U. Keller: IEEE Photon. Technol. Lett. **12**, 1135 (2000)
2 F. Kuznetsov, F. Hakimi, R. Sprague, A. Mooradian: IEEE Photon. Technol. Lett. **9**, 1063 (1997)
3 R. Häring, R. Paschotta, E. Gini, F. Morier-Genoud, H. Melchior, D. Martin, U. Keller: Electron. Lett. **37**, 766 (2001)
4 R. Häring, R. Paschotta, A. Aschwanden, E. Gini, F. Morier-Genoud, U. Keller: to appear in IEEE J. Quantum Electron. (2002)
5 L. Krainer, R. Paschotta, M. Moser, U. Keller: Electron. Lett. **36**, 1846 (2000)
6 A. Garnache, S. Hoogland, A.C. Tropper, I. Sagnes, G. Saint-Girons, J.S. Roberts: Appl. Phys. Lett. **80**(21), 3892 (2002)
7 F.X. Kärtner, L.R. Brovelli, D. Kopf, M. Kamp, I. Calasso, U. Keller: Opt. Eng. **34**, 2024 (1995)
8 C. Hönninger, R. Paschotta, F. Morier-Genoud, M. Moser, U. Keller: J. Opt. Soc. Am. B **16**, 46 (1999)
9 R.A. Salvatore, A. Yariv: IEEE Photon. Technol. Lett. **7**, 1151 (1995)
10 O.E. Martinez, R.L. Fork, J.P. Gordon: J. Opt. Soc. Am. B **2**, 753 (1985)
11 W. Jiang, J. Bowers: In *Compact Sources of Ultrashort Pulses,* ed. by I.I.N. Duling (Cambridge University Press, New York 1995) p. 208
12 F.X. Kärtner, U. Keller: Opt. Lett. **20**, 16 (1995)
13 F.X. Kärtner, I.D. Jung, U. Keller: IEEE J. Sel. Top. Quantum Electron. **2**, 540 (1996)
14 D.C. Hutchings, M. Sheik-Bahae, D.J. Hagan, E.W. v. Stryland: Opt. Quantum Electron. **24**, 1 (1992)
15 S.W. Corzine, R.-H. Yan, L.A. Coldren: In *Quantum Well Lasers,* ed. by P.S. Zory (Academic, San Diego 1993)
16 Y.H. Lee, A. Chavez-Pirson, S.W. Koch, H.M. Gibbs, S.H. Park, J. Morhange, A. Jeffery, N. Peyghambarian, L. Banyai, A.C. Gossard, W. Wiegmann: Phys. Rev. Lett. **57**, 2446 (1986)
17 D. Kopf, G. Zhang, R. Fluck, M. Moser, U. Keller: Opt. Lett. **21**, 486 (1996)
18 R. Paschotta, G.J. Spühler, D.H. Sutter, N. Matuschek, U. Keller, M. Moser, R. Hövel, V. Scheuer, G. Angelow, T. Tschudi: Appl. Phys. Lett. **75**, 2166 (1999)
19 F. Gires, P. Tournois: C. R. Acad. Sci. Paris **258**, 6112 (1964)
20 R. Paschotta, U. Keller: Appl. Phys. B **73**, 653 (2001)
21 G.P. Agrawal, C.M. Bowden: IEEE Photon. Technol. Lett. **5**, 640 (1993)