

ETH Eidgenössische Technische Hochschule Zürich
 Swiss Federal Institute of Technology Zürich

SESAM modelocked solid-state lasers

Lecture 1

Passive modelocked solid-state lasers

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 Ultrafast Optics 2013
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Outline of Lectures

- Lecture 1:** Passive modelocked solid-state lasers
- Lecture 2:** SESAMs

Useful references:
 Best review with extended tables for different lasers
 U. Keller, **Ultrafast solid-state lasers**, Landolt-Boernstein, Group VIII/1B1, Laser Physics and Applications. Subvolume B: Laser Systems. Part 1. Edited by G. Herziger, H. Weber, R. Proprawe, Springer-Verlag, Berlin, Heidelberg, New York, October, pp. 33-167, 2007
 ISBN 978-3-540-26033-2
 free download from <http://www.ulp.ethz.ch/research/UltrafastSolidStateLasers>

20 years of ultrafast solid-state lasers: a personal review
 U. Keller, "Ultrafast solid-state laser oscillators: a success story for the last 20 years with no end in sight,"
Appl. Phys. B, vol. 100, pp. 15-28, 2010
 free download Ref. 300, <http://www.ulp.ethz.ch/publications/paper/2010>

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Modelocking

by forcing all modes in a laser to operate phase-locked, "noise" is turned into ideal ultrashort pulses

- axial modes in laser **not** phase-locked
- noise
- axial modes in laser **phase-locked**
- ultrashort pulse
- inverse proportional to phase-locked spectrum

$\tau_p \sim \frac{1}{\Delta\nu}$

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Different mode of operation

Single-mode operation: cw, Q-Switched

Multi-mode operation: Q-Switched Modelocked, cw Modelocked

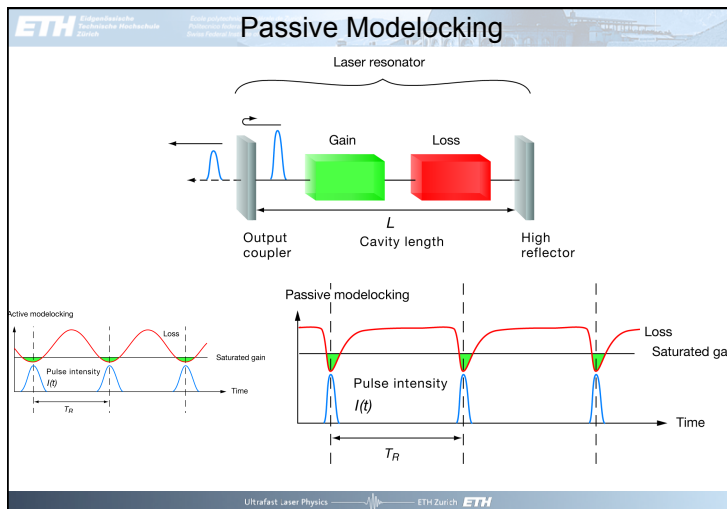
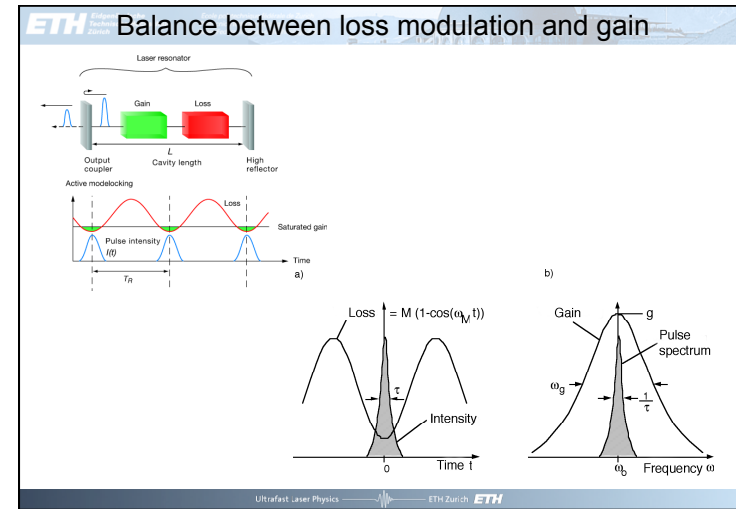
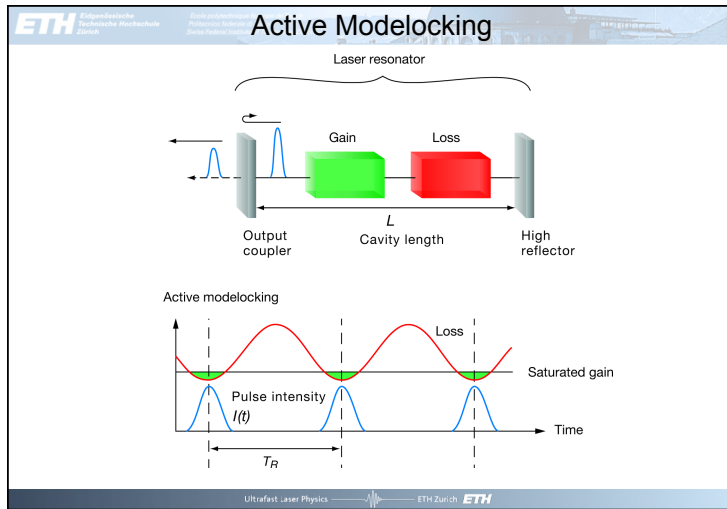
Legend:
 cw: continuous wave
 τ_p : pulse duration
 T_R : roundtrip time
 f_{rep} : pulse repetition frequency

Q-switching: single axial mode $\tau_p \gg T_R$, and $f_{rep} \ll \frac{1}{T_R}$

(fundamental) modelocking: one pulse per cavity roundtrip, multi axial modes, phase-locked
 $\tau_p \ll T_R$, and $f_{rep} = \frac{1}{T_R}$

harmonic modelocking: $f_{rep} = n \frac{1}{T_R}$, $n = 2, 3, \dots$

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Loss modulation for a slow and fast saturable absorber

$$\frac{dq(t)}{dt} = -\frac{q(t) - q_0}{\tau_A} - \frac{q(t)P(t)}{E_{sat,A}}$$

slow saturable absorber: $\tau_p \ll \tau_A$
neglect recovery within pulse duration

fast saturable absorber: $\tau_p \gg \tau_A$
follows "immediately" incoming power

For the slow saturable absorber case, the equation is simplified to:

$$q(t) = q_0 \exp\left[-\frac{E_p}{E_{sat,A}} \int_0^t f(t') dt'\right]$$

For the fast saturable absorber case, the equation is simplified to:

$$q(t) = \frac{q_0}{1 + I_A(t)/I_{sat,A}}$$

ETH Loss modulation for a slow saturable absorber

$$\frac{dq(t)}{dt} = -\frac{q(t) - q_0}{\tau_A} - \frac{q(t)P(t)}{E_{sat,A}}$$

slow saturable absorber: $\tau_p \ll \tau_A$
neglect recovery within pulse duration

$$\frac{dq(t)}{dt} \approx -\frac{q(t)P(t)}{E_{sat,A}}$$

$P(t) = E_p f(t)$, where $\int_0^{\tau_p} f(t) dt = 1$

$$q(t) = q_0 \exp\left[-\frac{E_p}{E_{sat,A}} \int_0^t f(t') dt'\right]$$

loss a pulse experiences through this saturable absorber:

$$q_p(E_p) = \int_0^{\tau_p} q(t) f(t) dt = q_0 \frac{F_{sat,A} A_A}{E_p} \left[1 - \exp\left(-\frac{E_p}{F_{sat,A} A_A}\right)\right]$$

does not depend on pulse shape because: $\tau_p \ll \tau_A$
absorbed energy:
 $E_{abs} = 2q_p(E_p) E_p$

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ETH Loss modulation for a fast saturable absorber

$$\frac{dq(t)}{dt} = -\frac{q(t) - q_0}{\tau_A} - \frac{q(t)P(t)}{E_{sat,A}}$$

fast saturable absorber: $\tau_p \gg \tau_A$
follows "immediately" incoming power

$$0 = -\frac{q(t) - q_0}{\tau_A} - \frac{q(t)P(t)}{E_{sat,A}}$$

$P_{sat,A} = \frac{E_{sat,A}}{\tau_A} \Rightarrow \frac{P(t)}{P_{sat,A}} = \frac{I_A(t)}{I_{sat,A}}$

$$q(t) = \frac{q_0}{1 + I_A(t)/I_{sat,A}}$$

loss a pulse experiences through this saturable absorber:
assuming **soliton pulse shape** and **fully saturated ideally fast absorber**

$$I(t) = I_p \text{sech}^2(t/\tau)$$

$$q_p \approx \frac{q_0}{3}$$

loss now depends on pulse shape!

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ETH Ideally fast saturable absorber

$$q(t) = \frac{q_0}{1 + I_A(t)/I_{sat,A}} = q_0 \left(1 - \frac{I_A(t)}{I_{sat,A}}\right) = q_0 - \gamma_A I_A(t)$$

fast saturable absorber ideally fast saturable absorber

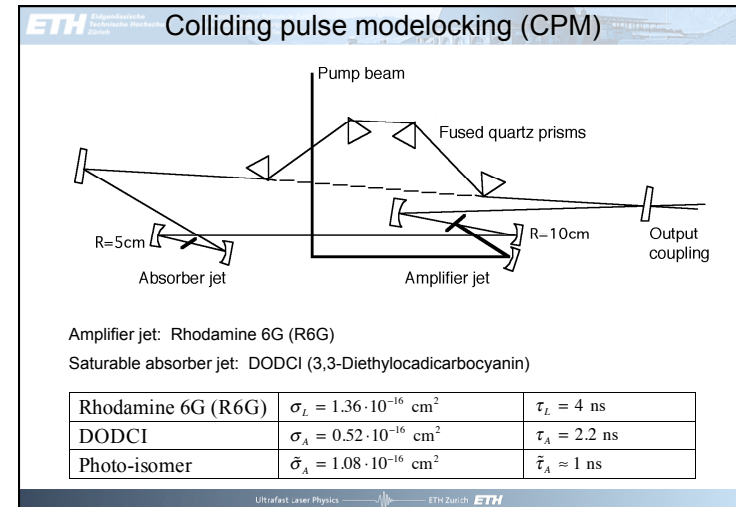
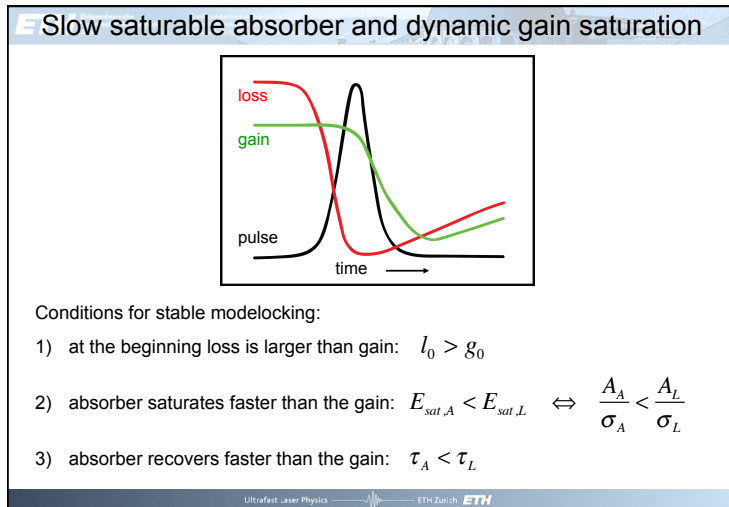
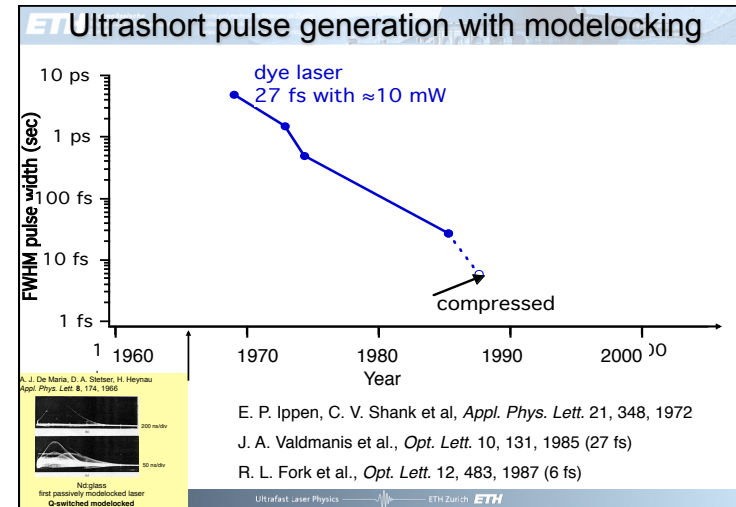
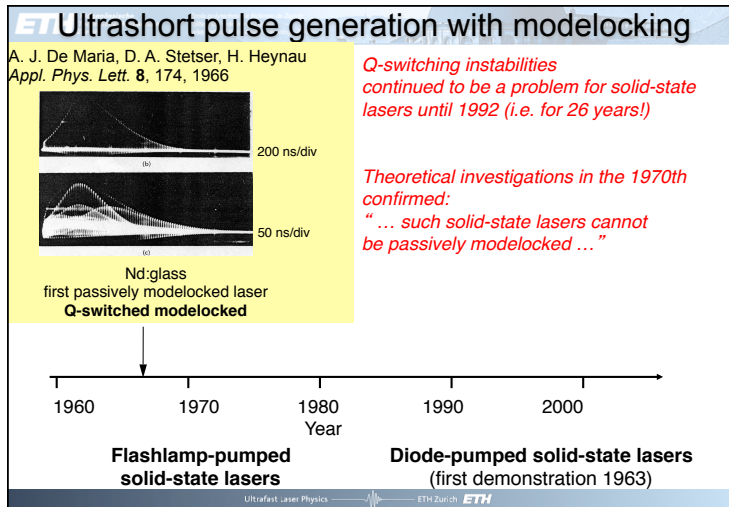
$$\gamma_A \equiv \frac{q_0}{I_{sat,A}}$$

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ETH Pulse-shaping in passive modelocking

U. Keller, Ultrafast solid-state lasers, Landolt-Börnstein, Group VIII/1B1, edited by G. Herziger, H. Weber, R. Proprawe, pp. 33-167, 2007, ISBN 978-3-540-26033-2

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Colliding pulse modelocking (CPM)

CPM Rhodamine 6G-laser	Typical	Best (86Val)
FWHM pulse duration τ_p	100 fs	27 fs
Emission wavelength λ_e	620 nm	620 nm
Average power P_p	2 - 25 mW	2 - 20 mW
Resonator length L	3.3 m	3.3 m
Repetition rate f_r	100 MHz	100 MHz
Pulse peak power in resonator P_{res}	100 kW	190 kW
Output coupling transmission T_{out}	2.5 %	3.5 %
Pump power	5 W at 514 nm	4 W at 514 nm
Amplifier jet thickness (at focus) and pressure	80 μ m 300 μ m nozzle, 40 psi	
Absorber jet thickness (at focus) and pressure	30 - 50 μ m 120 μ m nozzle, 15 psi	
Prisms for GDD-compensation		Quartz glass

Slow saturable absorber and dynamic gain saturation

$$q(t) = \sigma_A N_A d_A e^{-E_p(t)/E_{sat,A}}$$

$$g(t) = \sigma_L N_L d_L e^{-E_p(t)/E_{sat,L}}$$

$$P(t) = E_p f(t)$$

$$E_p(t) = A_L \int_{-\infty}^t |A(t')|^2 dt'$$

Assume slow saturable absorber and slow gain saturation
 net gain window: $g_T(t) = g(t) - q(t) = g_0 e^{-E_p(t)/E_{sat,L}} - q_A e^{-E_p(t)/E_{sat,A}} - l_R$

resonator loss: $l_0 = q_A + l_R$ $q_A = \sigma_A N_A d_A$

s-parameter: $s = \frac{E_{sat,L}}{E_{sat,A}}$

Slow saturable absorber and dynamic gain saturation

$$g_T(t) = g(t) - q(t) = g_0 e^{-E_p(t)/E_{sat,L}} - q_A e^{-E_p(t)/E_{sat,A}} - l_R$$

$$e^x \approx 1 + x + \frac{x^2}{2}$$

$$g_T(E) \approx (g_0 - q_A - l_R) + (q_A - \frac{g_0}{s}) \cdot E + \frac{1}{2} \left(\frac{g_0}{s^2} - q_A \right) \cdot E^2$$

For stable pulse generation we need to have $g_T < 0$ at both beginning and end of the pulse.
 (see later that this does not always need to be the case - see soliton modelocking)

Shorter pulses for larger g_T^{\max}

$$g_T^{\max} = g_T(E_{\max}), \text{ with } \left. \frac{\partial g_T}{\partial E} \right|_{E=E_{\max}} = 0$$

Slow saturable absorber and dynamic gain saturation

$$g_T(E) \approx (g_0 - q_A - l_R) + (q_A - \frac{g_0}{s}) \cdot E + \frac{1}{2} \left(\frac{g_0}{s^2} - q_A \right) \cdot E^2$$

For stable pulse generation we assume to have $g_T < 0$ at both beginning and end of the pulse.

Shorter pulses for larger g_T^{\max}

$$g_T^{\max} = g_T(E_{\max}), \text{ with } \left. \frac{\partial g_T}{\partial E} \right|_{E=E_{\max}} = 0$$

$$g_T^{\max} = \frac{l_0}{2} \frac{(1 - 1/s^2)^2}{(1 - 1/s^2)}$$

optimize s-parameter: goal in CPM

$$3 < s = \frac{\sigma_A A_L}{\sigma_L A_A} < 12$$

Colliding pulse modelocking (CPM)

Pump beam
 Fused quartz prisms
 R=5cm
 Absorber jet jet thickness 30-50 μm
 R=10cm
 Amplifier jet jet thickness ≈80 μm
 Output coupling

Amplifier jet: Rhodamine 6G (R6G)
 Saturable absorber jet: DODCI (3,3-Diethylocadicarboocyanin)

$$3 < s = \frac{\sigma_A A_L}{\sigma_L A_A} < 12$$

Optimization of s-parameter:
 • mode size: factor of 4
 • colliding pulse in absorber (bi-directional pulse propagation in ring laser): factor of 2 - 3
 need thin absorber < spatial extend of pulses (100 fs pulses)
 $d_A < \frac{c}{n} \tau_p \approx 20 \mu\text{m}$
 $s \approx 12$

Haus master equation

pulse envelope $A(t)$
 electric field $E(t)$

$$T_R \frac{\partial A(T, t)}{\partial T} = \sum_i \Delta A_i = 0$$

Haus master equation
 No change in pulse envelope per cavity roundtrip

$$E(t) = A(t) e^{i\omega_0 t}$$

Monochromatic plane wave

$$E(z, t) = E_0 e^{i(\omega t - k_n z)}$$

Vacuum	Dispersive material
Frequency: ν	ν
Period: $T = 1/\nu$	$T = 1/\nu$
Phase velocity: $v_p = c$	$v_p = c_n = c/n$
Wave number: $k = \frac{\omega}{c}$	$k_n = \frac{\omega}{v_p} = \frac{\omega}{c} n = kn$
$k = \frac{2\pi}{\lambda}$	$k_n = \frac{2\pi}{\lambda_n} = kn$
Wavelength: λ	$\lambda_n = \frac{\lambda}{n}$

Laser pulse

Pulse envelope $A(t)$
 Electric Field $E(t)$

$$E(t) = \frac{1}{2\pi} \int \tilde{E}(\omega) e^{i\omega t} d\omega$$

$$E(t) = A(t) e^{i\omega_0 t} \text{ where } A(t) = \frac{1}{2\pi} \int \tilde{A}(\Delta\omega) e^{i\Delta\omega t} d\Delta\omega$$

$$E(t) = \frac{1}{2\pi} \int \tilde{E}(\omega_0 + \Delta\omega) e^{i(\omega_0 + \Delta\omega)t} d\Delta\omega = \frac{1}{2\pi} e^{i\omega_0 t} \int \tilde{A}(\Delta\omega) e^{i\Delta\omega t} d\Delta\omega$$

$$\tilde{A}(\Delta\omega) = \tilde{E}(\omega_0 + \Delta\omega)$$

Example: Gaussian pulse

pulse envelope $A(t)$

electric field $E(t)$

$$E(t) = e^{-\Gamma t^2} e^{i\omega_0 t}$$

$$A(t) = e^{-\Gamma t^2}, \quad \Gamma \equiv \Gamma_1 - i\Gamma_2$$

$\tau_p = \sqrt{\frac{2 \ln 2}{\Gamma_1}}$

$\omega(t) \equiv \frac{d\phi_{tot}(t)}{dt} = \omega_0 + 2\Gamma_2 t$

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Chirped Gaussian Pulse

$$E(t) = A(t) \exp(i\omega_0 t) = \exp(-\Gamma t^2) \exp(i\omega_0 t)$$

$\Gamma \equiv \Gamma_1 - i\Gamma_2$
 $\Gamma_2 = 0$

$\Gamma \equiv \Gamma_1 - i\Gamma_2$
 $\Gamma_2 \neq 0$

$\phi_{tot}(t) \equiv \omega_0 t + \Gamma_2 t^2$

$\omega(t) \equiv \frac{d\phi_{tot}(t)}{dt} = \omega_0 + 2\Gamma_2 t$

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Time-Bandwidth Products

$I(t)$ ($x \equiv t/\tau$)	τ_p/τ	$\Delta\nu_p \cdot \tau_p$
1. Gaussian $I(t) = e^{-t^2}$	$2\sqrt{\ln 2}$	0.4413
2. Hyperbolic secant (soliton pulse) $I(t) = \text{sech}^2 x$	1.7627	0.3148
3. Rectangle $I(t) = \begin{cases} 1, & t \leq \tau/2 \\ 0, & t > \tau/2 \end{cases}$	1	0.8859
4. Parabolic $I(t) = \begin{cases} 1-x^2, & t \leq \tau/2 \\ 0, & t > \tau/2 \end{cases}$	1	0.7276
5. Lorentzian $I(t) = \frac{1}{1+t^2}$	2	0.2206
6. Symmetric two-sided exponent $I(t) = e^{- t }$	$\ln 2$	0.1420

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Frequency comb

Pulse train: Spectrum is a frequency comb
Single pulse: Spectrum is continuous

$$E_{train}(t) = E(t) * \sum_{m=-\infty}^{\infty} \delta(t - mT_R)$$

$$f_{rep} = 1/T_R$$

$$\tilde{E}_{train}(f) = \tilde{E}(f) \sum_{m=-\infty}^{\infty} \delta(f - mf_{rep})$$

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Carrier-Envelope Offset (CEO) Phase

H.R. Telle, G. Steinmeyer, A. E. Dunlop, J. Stenger, D. H. Sutter, U. Keller
Appl. Phys. B **69**, 327 (1999)
 F. W. Helbing, G. Steinmeyer, U. Keller
IEEE J. of Sel. Top. In Quantum Electron. **9**, 1030, 2003

Mode-locked pulse train

Pulse envelope $A(t)$

CEO phase $\Delta\varphi_0$

$$f_{CEO} = \frac{\Delta\varphi_0}{2\pi T_R}$$

Electric field: $\lambda/c = 2.7$ fs @ 800 nm

$$E(t) = A(t)\exp(i\omega_c t + i\varphi_0(t))$$

CEO phase controlled in laser oscillator

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Frequency comb

Intensity vs Frequency

Frequency comb:

$$f_n = f_{CEO} + n f_{rep}$$

f_{rep} : pulse repetition rate frequency
 f_{CEO} : carrier envelope offset frequency

$$\tilde{E}_{train}(f) = \tilde{E}(f) \sum_{m=-\infty}^{\infty} \delta(f - m f_{rep} - f_{CEO})$$

Shift theorem of Fourier transformation

$$\tilde{f}(\omega) \equiv F\{f(t)\} \Leftrightarrow F\{e^{i\omega_0 t} f(t)\} = \tilde{f}(\omega - \omega_0)$$

$$E_{train}(t) = A(t)\exp(2\pi i f_c t - 2\pi i f_{ceo} t) * \sum_{m=-\infty}^{\infty} \delta(t - m T_R)$$

H.R. Telle, G. Steinmeyer, A. E. Dunlop, J. Stenger, D. H. Sutter, U. Keller, *Appl. Phys. B* **69**, 327 (1999) and U. Keller, *Appl. Phys. B* **100**, 15-28, 2010

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Carrier-Envelope Offset (CEO) Phase

$$E_{train}(t) = A(t)\exp(2\pi i f_c t - 2\pi i f_{ceo} t) * \sum_{m=-\infty}^{\infty} \delta(t - m T_R)$$

Mode-locked pulse train

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Electric field: $\lambda/c = 2.7$ fs @ 800 nm

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CEO phase controlled in laser oscillator

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Optical Dispersion

Absorption coefficient vs Frequency ν

Absorption coefficient vs Wavelength λ (nm)

Refractive index n vs Frequency ν

Refractive index n vs Wavelength λ (nm)

Positive dispersion: $\frac{\partial^2 n}{\partial \omega^2} > 0$

Negative dispersion: $\frac{\partial^2 n}{\partial \omega^2} < 0$

Input pulse vs Dispersive medium vs Output pulse

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Dispersive pulse broadening

Dispersive medium

$\phi(\omega) = k \cdot n(\omega) \cdot L$

$\tau_p(0)$ $\tau_p(z)$

$n(\omega)$

L

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First and second order dispersion

Taylor expansion around the center frequency ω_0 : $\Delta\omega = \omega - \omega_0$

$$k_n(\omega) \cong k_n(\omega_0) + k'_n \Delta\omega + \frac{1}{2} k''_n \Delta\omega^2 + \dots$$

First order dispersion: $k'_n = dk_n/d\omega$

Second order dispersion: $k''_n = d^2k_n/d\omega^2$

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Phase and group velocity

$$E(z,t) \propto \exp\left[i\omega_0\left(t - \frac{z}{v_p(\omega_0)}\right)\right] \times \exp\left[-\Gamma(L) \times \left(t - \frac{z}{v_g(\omega_0)}\right)^2\right]$$

$$v_p(\omega_0) \equiv c_n = \frac{\omega}{k_n}|_{\omega=\omega_0}$$

$$v_g(\omega_0) \equiv \frac{1}{k'_n(\omega_0)} = \frac{1}{\left(\frac{dk_n}{d\omega}\right)_{\omega=\omega_0}} = \left(\frac{d\omega}{dk_n}\right)_{\omega=\omega_0}$$

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Dispersive pulse broadening

Dispersive medium

$\phi(\omega) = k \cdot n(\omega) \cdot L$

$\tau_p(0)$ $\tau_p(z)$

$n(\omega)$

L

Gaussian pulse: (initially unchirped pulse) $\frac{\tau_p(z)}{\tau_p(0)} = \sqrt{1 + \left(\frac{4 \ln 2}{\tau_p^2(0)} \frac{d^2\phi/d\omega^2}{\tau_p^2(0)}\right)^2}$

Approximation for (strong pulse broadening) $\frac{d^2\phi}{d\omega^2} \gg \tau_p^2(0)$ $\tau_p(z) \approx \frac{d^2\phi}{d\omega^2} \Delta\omega_p$

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Example: fused quartz

$n(0.8 \mu\text{m}) = 1.45332$	
$\left. \frac{\partial n}{\partial \lambda} \right _{800 \text{ nm}} = -0.017 \frac{1}{\mu\text{m}}$	$\left. \frac{\partial k_n}{\partial \omega} \right _{800 \text{ nm}} = 4.84 \frac{\text{ns}}{\text{m}}$
$\left. \frac{\partial^2 n}{\partial \lambda^2} \right _{800 \text{ nm}} = 0.04 \frac{1}{\mu\text{m}^2}$	$\left. \frac{\partial^2 k_n}{\partial \omega^2} \right _{800 \text{ nm}} = 36.1 \frac{\text{fs}^2}{\text{mm}}$

Fused quartz @ 800 nm

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Phase Velocity v_p	$\frac{\omega}{k_n}$	$\frac{c}{n}$
Group Velocity v_g	$\frac{d\omega}{dk_n}$	$\frac{c}{n} \frac{1}{1 - \frac{dn}{d\lambda} \frac{\lambda}{n}}$
Group Delay T_g	$T_g = \frac{z}{v_g} = \frac{d\phi}{d\omega}, \phi \equiv k_n z$	$\frac{nz}{c} \left(1 - \frac{dn}{d\lambda} \frac{\lambda}{n} \right)$
Dispersion 1. Order	$\frac{d\phi}{d\omega}$	$\frac{nz}{c} \left(1 - \frac{dn}{d\lambda} \frac{\lambda}{n} \right)$
Dispersion 2. Order	$\frac{d^2\phi}{d\omega^2}$	$\frac{\lambda^3 z}{2\pi c^2} \frac{d^2 n}{d\lambda^2}$
Dispersion 3. Order	$\frac{d^3\phi}{d\omega^3}$	$\frac{-\lambda^4 z}{4\pi^2 c^3} \left(3 \frac{d^2 n}{d\lambda^2} + \lambda \frac{d^3 n}{d\lambda^3} \right)$

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Haus master equation: active modelocking

$$T_R \frac{\partial A(T,t)}{\partial T} = \sum_i \Delta A_i = 0$$

Gain: $\Delta A_1 = g \left(1 + \frac{1}{\Omega_g^2} \frac{d^2}{dt^2} \right) A(T,t)$

Loss modulator: $\Delta A_2 \approx -M(1 - \cos \omega_m t) A(T,t)$

Constant loss: $\Delta A_3 \approx -IA(T,t)$

parabolic approximation: $M(1 - \cos \omega_m t) \approx M \frac{\omega_m^2 t^2}{2}$

Schrödinger equation for harmonic oscillator

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g \left(1 + \frac{1}{\Omega_g^2} \frac{d^2}{dt^2} \right) - I - M(1 - \cos \omega_m t) \right] A(T,t)$$

H. A. Haus, "Short pulse generation," in *Compact Sources of Ultrashort Pulses*, I. I. N. Duling, Eds. (Cambridge University Press 1995, New York, 1995) pp. 1-56.

Linearized operators: Gain

Gain dispersion:

$$\Omega_g \equiv \frac{\Delta \omega_g}{2}$$

$$g(\omega) = \frac{g(z)L_g}{1 + \left[2(\omega - \omega_0) / \Delta \omega_g \right]^2} = \frac{g}{1 + \left[(\omega - \omega_0) / \Omega_g \right]^2} = g \left(1 - \frac{(\omega - \omega_0)^2}{\Omega_g^2} \right)$$

$\exp[g(\omega)] \tilde{A}(\omega)$

$\Delta \omega = \omega - \omega_0$

$$\exp[g(\omega)] \tilde{A}(\omega) \approx \left[1 + g \left(1 - \frac{\Delta \omega^2}{\Omega_g^2} \right) \right] \tilde{A}(\omega) = \left[1 + g - \frac{g}{\Omega_g^2} \Delta \omega^2 \right] \tilde{A}(\omega)$$

$$\Delta A_{\text{Gain}} = g \left(1 + \frac{1}{\Omega_g^2} \frac{d^2}{dt^2} \right) A(T,t)$$

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Zürich **Linearized operators: Modulator**

$$A_{out}(t) = \exp[-M(1 - \cos\omega_m t)] A_{in}(t)$$

\downarrow
 $e^x \approx 1 + x$

$$A_{out}(t) \approx [1 - M(1 - \cos\omega_m t)] A_{in}(t)$$

$$\Rightarrow \Delta A_{Mod} = A_{out}(t) - A_{in}(t) \approx -M(1 - \cos\omega_m t) A_{in}(t)$$

$$\Delta A_{Mod} \approx -M(1 - \cos\omega_m t) A(T, t)$$

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Zürich **Linearized operators: Loss**

$$A_{out}(t) = e^{-l} A_{in}(t)$$

\downarrow
 $e^x \approx 1 + x$

$$A_{out}(t) = e^{-l} A_{in}(t) \approx (1 - l) A_{in}(t)$$

$$\Rightarrow \Delta A_{Loss} = A_{out}(t) - A_{in}(t) \approx -l A_{in}(T, t)$$

$$\Delta A_{Loss} \approx -l A(T, t)$$

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Zürich **Slow saturable absorber and dynamic gain saturation**

Master equation: $T_R \frac{\partial A(T, t)}{\partial T} = \left(g(t) - q(t) + \frac{g_0}{\Omega_s^2} \frac{d^2}{dt^2} + t_D \frac{d}{dt} \right) A(T, t) = 0$

SAM (self-amplitude modulation)

$$A_{out}(t) = \exp[-q(t)] A_{in}(t)$$

$$\Delta A_{SAM} \approx -q(t) A(T, t)$$

$$q(t) = q_A \exp\left(-\frac{A_L \sigma_A}{A_A h\nu} \int_{-\infty}^t |A(t')|^2 dt'\right)$$

q_A : unsaturated loss of the absorber

Solution: $A(t) = A_0 \operatorname{sech}\left(\frac{t}{\tau}\right)$

time shift of pulse

$$\Delta A_s = t_D \frac{d}{dt} A(T, t)$$

Rhodamin 6G:
 $\Delta v_s \approx 4 \cdot 10^{13}$ Hz
 $\tau_p = 1.76 \cdot \tau \leq 56$ fs

With SPM: factor of 2 shorter (Martinez numerically)

H. A. Haus, "Theory of modelocking with a slow saturable absorber," *IEEE J. Quantum Electron.* **11**, 736, 1975

Ultrafast Laser Physics **ETH** Zürich

ETH Universität
Technische Hochschule
Zürich **Passive mode locking with an ideally fast saturable absorbers**

loss

gain

pulse

time →

Semiconductor and dye lasers:
Dynamic gain saturation
G.H.C. New,
Opt. Com. **6**, 188, 1974

loss

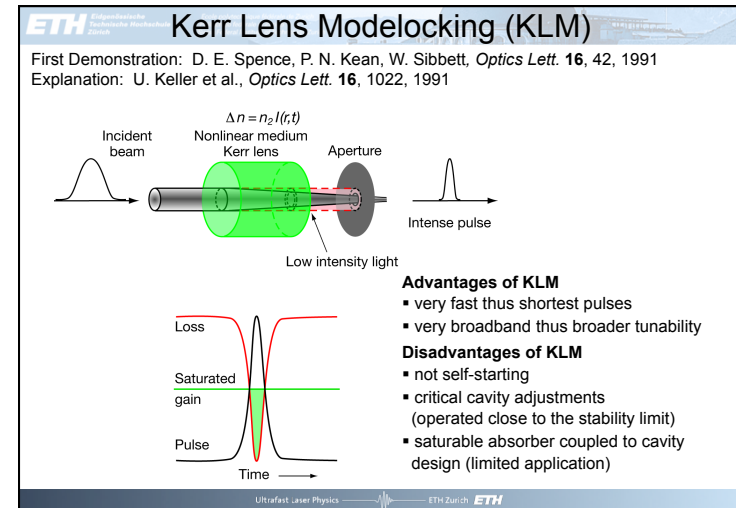
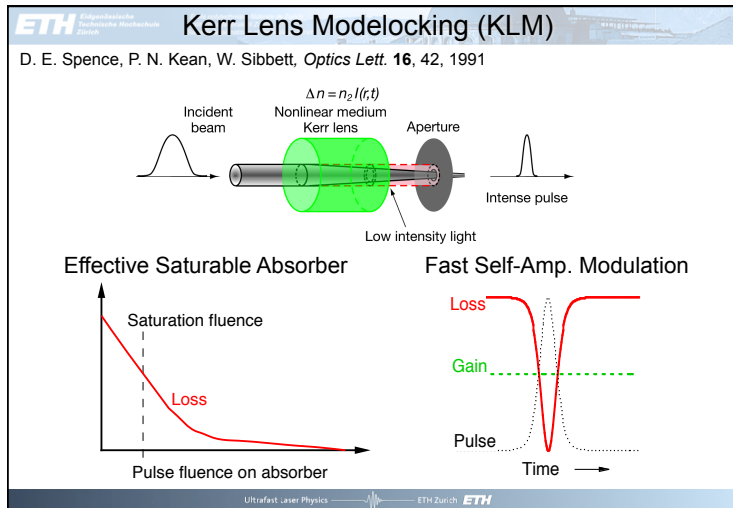
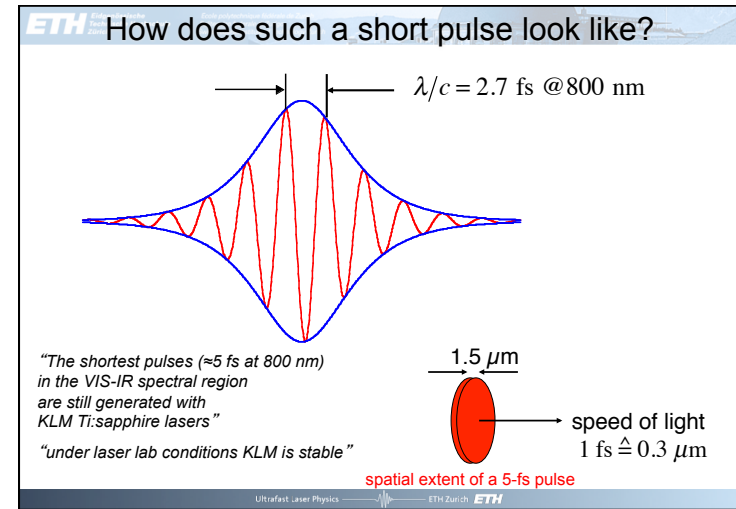
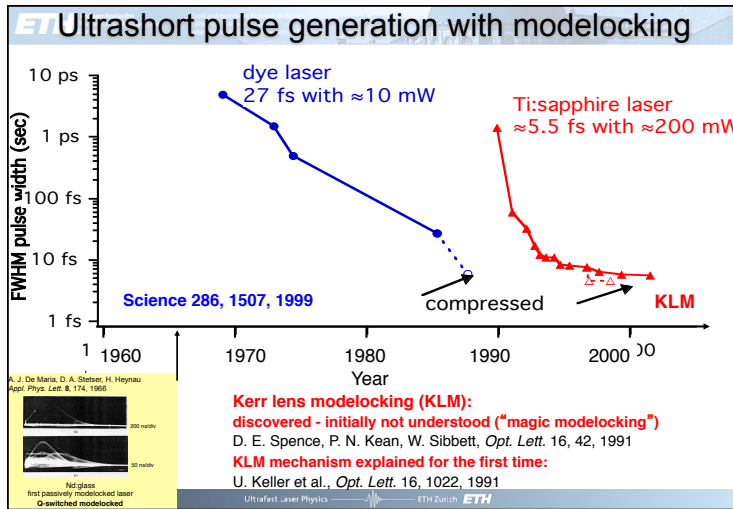
gain

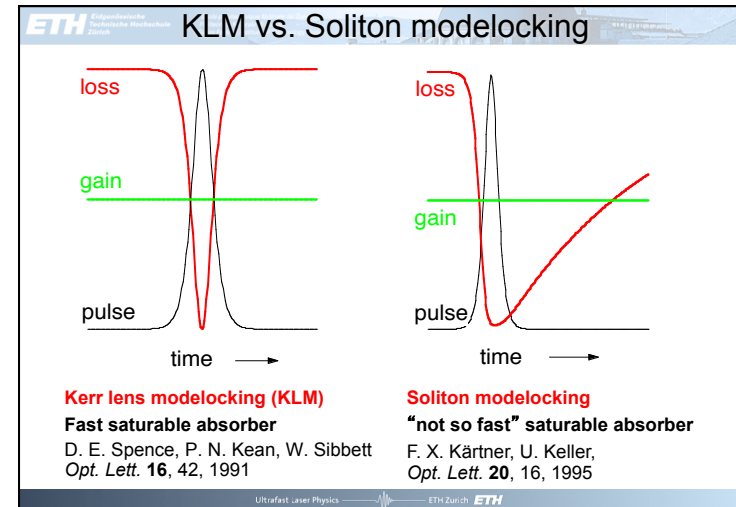
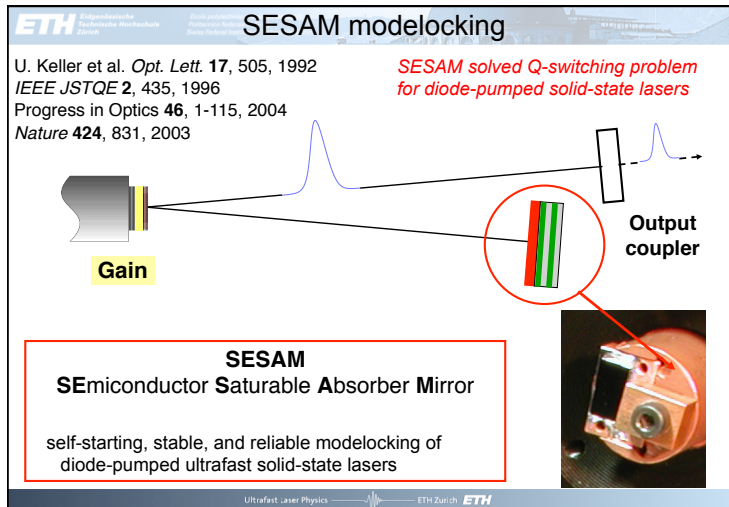
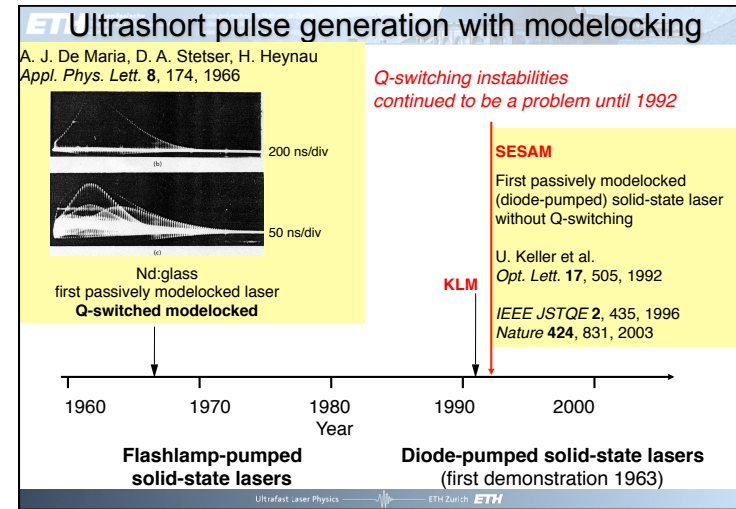
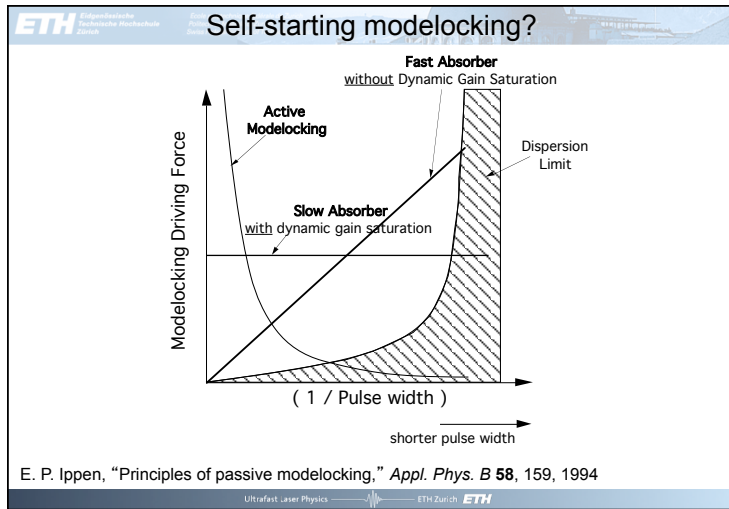
pulse

time →

Solid-state lasers (e.g. Ti:sapphire)
Kerr lens modelocking (KLM)
Ideally fast saturable absorber
Opt. Lett. **16**, 42, 1991

Ultrafast Laser Physics **ETH** Zürich





ETH Passive mode locking with slow saturable absorbers

loss
gain
pulse
time →

Semiconductor lasers:
Dynamic gain saturation
G.H.C. New,
Opt. Com. **6**, 188, 1974

Ion-doped solid-state lasers:
Constant gain saturation:
soliton modelocking
F. X. Kärtner, U. Keller,
Opt. Lett. **20**, 16, 1995

Ultrafast Laser Physics ETH Zurich ETH

ETH Pulse-shaping

loss
gain
pulse
Time →

VECSEL

KLM

Solid-state and SESAM

Gain window can be up to 20 times longer than the pulse before mode locking becomes unstable

- fast/broadband sat. abs.
- critical cavity adjustment: KLM better at cavity stability limit
- typically not self-starting
- "not so fast" sat. abs.
- absorber independent of cavity design
- self-starting

For stable pulse generation it was initially assumed that we need $g_T < 0$ at both beginning and end of the pulse. **This is however not the case!**
See SESAM modelocking with slow saturable absorber and soliton modelocking

Ultrafast Laser Physics ETH Zurich ETH

ETH Slow saturable absorber modelocking

R. Paschotta, U. Keller, *Appl. Phys. B* **73**, 653, 2001

loss
absorber delays pulse
Fully saturated absorber: negligible loss for trailing edge of pulse

leading edge of pulse has significant loss from the saturable absorber

time →

Dominant stabilization process:
Picosecond domain: absorber delays pulse
The pulse is constantly moving backward and can swallow any noise growing behind itself
Femtosecond domain: dispersion in soliton modelocking

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ETH Soliton modelocking: GDD negative, $n_2 > 0$

Group Delay Dispersion | Self-Phase Modulation | Gain | Slow Absorber | Output-coupler

Master equation:

$$T_R \frac{\partial}{\partial T} A(T, t) = \left(iD \frac{\partial^2}{\partial t^2} - i\delta |A(T, t)|^2 \right) A(T, t) + \left(g - l + D_s \frac{\partial^2}{\partial t^2} - q(t) \right) A(T, t) = 0$$

$A_{out}(T, t) = e^{-at(t)} A_{in}(T, t)$
 $A_{out}(T, t) = [1 - q(t)] A_{in}(T, t)$
 $\Rightarrow \Delta A(T, t) = -q(t) A(T, t)$

Ultrafast Laser Physics ETH Zurich ETH

ETH Kerr effect and self-phase modulation (SPM)

$n(I) = n + n_2 I$ $n_2 \left[\frac{\text{cm}^2}{\text{W}} \right] = 4.19 \times 10^{-13} \frac{n_2 \text{ [esu]}}{n}$

Material	Refractive index n	n_2 [esu]	n_2 [cm ² / W]
Sapphire (Al ₂ O ₃)	1.76 @ 850 nm	1.25×10^{-13} [89Ada]	3×10^{-16}
Fused quartz	1.45 @ 1.06 μm	0.85×10^{-13} [89Ada]	2.46×10^{-16}
Glass (Schott LG-760)	1.5 @ 1.06 μm	1.04×10^{-13} [93Aza]	2.9×10^{-16}
YAG (Y ₃ Al ₅ O ₁₂)	1.82 @ 1.064 μm	3.47×10^{-13} [93Aza]	6.2×10^{-16}
YLF (LiYF ₄)	$n_o = 1.47$ @ 1.047 μm		1.72×10^{-16} [93Aza]

Typical order of magnitude for the nonlinear index coefficient: $n_2 \approx 10^{-16} \text{ cm}^2/\text{W}$

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ETH Linearized operators: self-phase modulation (SPM)

$n_2 > 0$

leading edge SPM: red trailing edge SPM: blue

Gaussian Pulse $I(t)$

Spectral broadening $\omega_2(t)$

$\phi_2(t) = -kn_2 I(t) L_K = -kn_2 L_K |A(t)|^2 \equiv -\delta |A(t)|^2$

$\delta \equiv kn_2 L_K$

$\omega_2(t) = \frac{d\phi_2(t)}{dt} = -\delta \frac{dI(t)}{dt}$

$E(L_K, t) = A(0, t) \exp[i\omega_0 t + i\phi(t)] = A(0, t) \exp[i\omega_0 t - ik_n(\omega_0) L_K - i\delta |A(t)|^2]$

$A(L_K, t) = e^{-i\delta |A|^2} A(0, t) e^{-ik_n(\omega_0) L_K} \xrightarrow{\delta |A|^2 \ll 1} \approx (1 - i\delta |A(t)|^2) A(0, t) e^{-ik_n(\omega_0) L_K}$

$\Delta A_{SPM} \approx -i\delta |A(T, t)|^2$

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ETH Linearized operators: group delay dispersion (GDD)

Please derive this on your own

Ultrafast Laser Physics ETH Zurich ETH

ETH Soliton modelocking: GDD negative, $n_2 > 0$

Group Delay Dispersion Self-Phase Modulation Gain Slow Absorber Output-coupler

$A_{out}(T, t) = e^{-q(t)} A_m(T, t)$

$A_{out}(T, t) = [1 - q(t)] A_m(T, t)$

$\Rightarrow \Delta A(T, t) = -q(t) A(T, t)$

Master equation:

$T_R \frac{\partial}{\partial T} A(T, t) = \left(iD \frac{\partial^2}{\partial t^2} - i\delta |A(T, t)|^2 \right) A(T, t) + \left(g - l + D_s \frac{\partial^2}{\partial t^2} - q(t) \right) A(T, t) = 0$

$A(T, t) \approx \left(A_0 \operatorname{sech} \left(\frac{t}{\tau} \right) \right) \exp \left[i\phi_0 \frac{T}{T_R} \right] + \text{continuum}$ $\tau = \frac{4|D|}{\delta \cdot F_{p,L}}$

Ultrafast Laser Physics ETH Zurich ETH

ETH Soliton modelocking: GDD negative, $n_2 > 0$
 F. X. Kärtner, U. Keller, *Optics Lett.* 20, 16, 1995
 Invited Paper: F. X. Kärtner, I. D. Jung, U. Keller, *IEEE JSTQE*, 2, 540, 1996

Soliton Perturbation Theory:

$$A(T,t) = \underbrace{\left(A \operatorname{sech} \left(\frac{t}{\tau} \right) \right)}_{\text{soliton}} \exp \left[i \Phi_0 \frac{T}{T_R} \right] + \underbrace{\text{small perturbations}}_{\text{"continuum" only GDD \& SAM (no SPM)}}$$

Stabilization: Dispersion spreads continuum out where it sees more loss

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E Experimental confirmation: example Ti:sapphire laser

SESAM: LT-GaAs
 Impulse response measured with 300 fs pulses
 clearly a slow saturable absorber

No KLM: cavity operated in the middle of cavity stability regime (and use etalon for bandwidth limitation)

I. D. Jung, F. X. Kärtner, L. R. Brovelli, M. Kamp, U. Keller, "Experimental verification of soliton modelocking using only a slow saturable absorber," *Opt. Lett.*, vol. 20, pp. 1892-1894, 1995

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E Experimental confirmation: example Ti:sapphire laser

Solution: soliton pulse $A_s(T,t) = \sqrt{\frac{F_{p,L}}{2\tau}} \operatorname{sech} \left(\frac{t}{\tau} \right) e^{i\phi_0 \frac{T}{T_R}}$

soliton phase per resonator roundtrip $\phi_0 = \frac{|D|}{\tau^2} = \frac{\delta \cdot F_{p,L}}{4\tau}$

Stability (soliton perturbation theory): continuum loss l_c is larger than soliton loss l_s

Normalized abs. recovery $w = \frac{\tau_A}{\tau} \sqrt{\frac{q_0}{\phi_0}} = \tau_A \sqrt{\frac{q_0}{|D|}} \propto \frac{1}{\sqrt{|D|}}$

I. D. Jung, F. X. Kärtner, L. R. Brovelli, M. Kamp, U. Keller, "Experimental verification of soliton modelocking using only a slow saturable absorber," *Opt. Lett.*, vol. 20, pp. 1892-1894, 1995

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E Experimental confirmation: example Ti:sapphire laser

$\tau_{FWHM} = 1.76 \cdot \tau = 1.76 \cdot \frac{4|D|}{\delta \cdot F_{p,L}}$

I. D. Jung, F. X. Kärtner, L. R. Brovelli, M. Kamp, U. Keller, "Experimental verification of soliton modelocking using only a slow saturable absorber," *Opt. Lett.*, vol. 20, pp. 1892-1894, 1995

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