A dynamical magnetic field accompanying the motion of ferroelectric domain walls: Supplementary Material

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Time evolution of Néel- and Bloch-type ferroelectric domain walls. In the following we review the derivation of the domain wall motion based on Refs. [17, 18], rewriting it in the notation used in this work. The Lagrangian for a ferroelectric domain wall separating domains of different orientation of polarization lying in the xy plane can be written as

$$\begin{aligned}\mathcal{L} = {}& \frac{1}{2}\mathcal{M}_x(\partial_t U_x)^2 - \frac{1}{2}S_x(\partial_r U_x)^2 - \frac{1}{2}A_x U_x^2 - \frac{1}{4}B_x U_x^4 \\ & + \frac{1}{2}\mathcal{M}_y(\partial_t U_y)^2 - \frac{1}{2}S_y(\partial_r U_y)^2 - \frac{1}{2}A_y U_y^2 - \frac{1}{4}B_y U_y^4 \\ & - C_{xy}U_x^2 U_y^2, \end{aligned} \tag{S1}$$

where $U_{x/y}$ is the amplitude of the ferroelectric displacement along direction x/y, $\mathcal{M}_{x/y}$ is the effective mass of the ferroelectric distortion mode, $S_{x/y}$ are the gradient energies and $A_{x/y}$, $B_{x/y}$ and C_{xy} are the coefficients of the harmonic, quartic anharmonic, and coupling terms. The coordinate r denotes the position perpendicular to the domain wall in the xy plane of polarization. For a Bloch-type domain wall, we would require components perpendicular to the plane of polarization, $z \perp x, y$; for a Néel-type domain wall, we can express the change of ferroelectric displacement as simple rotation in the xy plane:

$$\mathbf{U} = \begin{pmatrix} U_x(t) \\ U_y(t) \end{pmatrix} = U_0 \begin{pmatrix} \cos(\phi(r,t)) \\ \sin(\phi(r,t)) \end{pmatrix}, \tag{S2}$$

where U_0 is the amplitude of the bulk ferroelectric displacement. ($\mathbf{U} = U_0\mathbf{Q}$ in Eq. (5) in the main text with $n = 1$.) Inserting this into the Lagrangian (S1), together with $\mathcal{M}_x = \mathcal{M}_y = \mathcal{M}$, $S_x = S_y \equiv S$, $A_x = A_y \equiv A$, $B_x = B_y \equiv B$, $B_x = B_y \equiv B$, and $C_{xy} \equiv C$ we obtain

$$\mathcal{L} = \frac{1}{2}\mathcal{M}U_0^2(\partial_t\phi)^2 - \frac{1}{2}SU_0^2(\partial_r\phi)^2 - \frac{1}{2}AU_0^2 - \frac{1}{4}BU_0^4 - CU_0^4\frac{1}{8}(1-\cos(4\phi)). \tag{S3}$$

The Euler-Lagrange equations for the Lagrangian (S3) yield after some rearrangements

$$\partial_t^2\phi - c_0^2\partial_r\phi + \kappa^2\sin(4\phi) = 0, \tag{S4}$$

where $\kappa^2 = CU_0^2/(2\mathcal{M})$ and $c_0 = \sqrt{S/\mathcal{M}}$ is the characteristic velocity. A substitution $4\phi \to \theta$ and a transformation to a moving frame $r \to \xi = r - vt$, where v is the constant domain wall velocity yields

$$\partial_\xi^2\theta - \alpha^2\sin(\theta) = 0, \tag{S5}$$

where $\alpha^2 = \kappa^2/(c_0^2(1 - v^2/c_0^2))$. The solution to this equation is known for a 360° rotation of θ (corresponding to a 90° rotation of ϕ), see for example Ref. [18]:

$$\theta(\xi) = 4\arctan\left(\mathrm{e}^{\xi\alpha}\right) \tag{S6}$$

$$\Rightarrow \phi(r,t) = \frac{1}{4}\theta(\xi(r,t)) = \arctan\left(\exp\left(\frac{1}{w}\frac{r - vt}{\sqrt{1-\frac{v^2}{c_0^2}}}\right)\right), \tag{S7}$$

where $w = c_0^2/\kappa^2 = S/(2CU_0^2)$ and $2w$ is the width of the domain wall. Eq. (S7) is the expression (4) given in the main text. $n\phi$ with $n = 1, 2, 4$ then corresponds to 90°, 180°, and 360° rotations of the ferroelectric polarization.

Stray field estimate. The vertical magnetic stray field B_z^{s} appearing at height h above a thin ($2w/\beta \ll h$), extended domain wall located (see Fig. 3a) is given by

$$B_z^{\mathrm{s}}(r) = \frac{\mu_0 M_{2\mathrm{d}} h}{2\pi(r^2 + h^2)} \ , \tag{S8}$$

where r is the relative position of the domain wall with respect to the sensor, $M_{2\mathrm{d}}$ is the two-dimensional moment density of the domain wall, and where the material is assumed to be thick ($d \gg 2w/\beta, h$). The magnetic moment density is given by

$$M_{2\mathrm{d}} = \frac{1}{V}\int_{-\infty}^{\infty} \mathrm{d}r m_z(r) = \frac{1}{V}\int_{-\infty}^{\infty} \mathrm{d}t v \gamma n \dot{\phi}(t) = \frac{\gamma}{V} v n \pi. \tag{S9}$$

To detect the magnetic stray field, we repeatedly move the domain wall back and forth to create an oscillatory field $B_z^{\mathrm{s}}(t)$ at the location of the NV spin sensor. When tuned to the spin resonance frequency f_0, the oscillatory field induces a Rabi rotation of the spin providing an experimentally detectable signal. The Rabi field B_1 is given by the Fourier component of $B_z^{\mathrm{s}}(t)$ at frequency f_0,

$$B_1 = \frac{4}{T}\int_{-T/4}^{T/4} \mathrm{d}t \cos(2\pi t/T) B_z^{\mathrm{s}}(t), \tag{S10}$$

where $T = 1/f_0$ is the Larmor period of the spin. Because the Larmor period ($\sim 0.1 - 1$ ns) is typically much longer than the time taken for the domain wall to pass (~ 1 ps), $B_z^{\mathrm{s}}(t)$ is non-zero only for a very short period of time around $t = 0$, and the integral can be approximated by

$$\begin{aligned} B_1 &\approx \frac{4}{T}\int_{-\infty}^{\infty} \mathrm{d}t B_z^{\mathrm{s}}(t) = \frac{4}{T}\frac{\mu_0 m h}{2\pi}\int_{-\infty}^{\infty} \frac{\mathrm{d}t}{v^2 t^2 + h^2} \\ &= \frac{\gamma}{V} n 2\pi \mu_0 f_0, \end{aligned} \tag{S11}$$

which is the expression used in the main text. Note that the expected signal is independent of domain wall width w, speed v and sensor height h as long as the geometric assumptions about the setup (a thick sample and thin moving domain wall) are satisfied and $vT \gg h$.

TABLE S1. Calculated diagonal Born effective charges $Z^*_{i,aa}$ in units of the elementary charge (a denoting spatial coordinates x, y, and z), and ferroelectric displacements $u_{i,x}$ in picometers for polarization along the x direction.

Atom	$Z^*_{i,xx}$	$Z^*_{i,yy}$	$Z^*_{i,zz}$	$u_{i,x}$
Ba	2.8	2.7	2.7	4.2
Ti	6.6	7.6	7.6	8.9
O(1)	-5.3	-2.1	-2.1	-3.4
O(2)	-2.0	-6.0	-2.1	-0.8
O(3)	-2.0	-2.1	-6.0	-0.8