Problem 1. ***DMRG study of spin-1 Heisenberg chain***

In this exercise you will study the phase diagram of a spin-1 antiferromagnetic Heisenberg chain:

$$H = \sum_{i=1}^{L} J\left(\mathbf{S}_i \cdot \mathbf{S}_j\right) + D(S_i^z)^2 + \lambda\left(\mathbf{S}_i \cdot \mathbf{S}_j\right)^2 . \tag{1}$$

The phase diagram of such model is very rich. We are interested in a sector where three different phases are realized. One of them is a symmetry breaking antiferromangetic phase. The other two do not break any symmetry and are indistinguishable according to Landau theory of phase transitions. Nonetheless, these two phases are topologically distinct. For $D \to +\infty$, the ground state becomes a product state of spins with $m_z = 0$ and it is topologically trivial. The phase connected to $D = 0$ is the topological Haldane phase. Such phase is realized by the AKLT model seen in the lectures, which is the limit of (1) for $D = 0$ and $\lambda = J/3$.

Use the previously installed library TeNPy. Consider an infinite MPS and run a iDMRG algorithm to find the ground state wavefunction. Measure the staggered magnetization, the entanglement entropy and the entanglement spectrum of the ground state wavefunction. The magnetization allows to detect symmetry broken phases and long range order. The entanglement entropy diverges at phase transition and will help us to identify phases that are not captured by the symmetry-breaking paradigm of Landau. The degeneracy of the entanglement spectrum provides information on the topological nature of the phase under study (Phys. Rev. B, **81**, 064439 (2010)).

Perform these studies for different parameters:

1. $J = 1$, $\lambda = 0$, $D \in [-J, 2J]$. Here you should observe the transition from the antiferromagnetic phase, to the Haldane phase and then to the topologically trivial large D phase.

2. $J = 1$, $D = 0$, $\lambda \in [J/3, 0]$. The absence of divergence in the entanglement entropy and the double degeneracy of the entanglement spectrum proves that the AKLT model realizes the topological Haldane phase.

Hints: Take advantage of the large online documentation available at `tenpy.github.io`. Both the TeNPy Reference and the TeNPy Usuer Guide