

Problem 1. *Path Integral Monte Carlo - Harmonic Oscillator*

Solve the one-particle harmonic oscillator problem at finite temperature with PIMC. The system configuration is comprised of positions x_i for the M time slices between 0 and β of a particle in harmonic potential.

- Implement functions that evaluate the potential and kinetic energy of a configuration $\{x_m\}$ using the proper boundary conditions.
- Implement the Metropolis algorithm of sampling the configurations as described in the section 7.1.4 of the script.
- Implement observables:
 1. Potential Energy
 2. Kinetic Energy
 3. Density operator in position space (optional)
- Estimate the autocorrelation time and errors for your observables.

Hints for the implementation:

- Reasonable values for the simulation are:
 - $\beta = 1$
 - $\tau = 0.1$
 - thermalization sweeps: 20000
 - total sweeps: 500000
- Choose the maximum displacement in each step such that your acceptance probability is neither close to one nor close to zero.
- In case your code takes long time to complete, you might want to do the debugging with less sweeps.