Problem 1. ***Monte Carlo simulation of the classical 2D Ising model with local updates***

Write a classical Monte Carlo simulation of the Ising model on an $L \times L$ square lattice, defined by the Hamiltonian

$$H = -J \sum_{\langle i,j \rangle} s_i s_j, \tag{1}$$

where the sum runs over all pairs of nearest neighbour sites i, j and the Ising spins can take the values $s_i \in \{\pm 1\}$. We assume periodic boundary conditions. Use local updates, i.e. each simulation step proposes to flip a random spin, which is accepted with Metropolis acceptance probability

$$P(x \to y) = \min\left\{1, e^{-\beta(H(y)-H(x))}\right\}. \tag{2}$$

Measure the absolute value of the magnetization $\langle |m| \rangle = \frac{1}{L^2} \left| \sum_i s_i \right|$ and its square $\langle m^2 \rangle$ for different values of the inverse temperature β. Compute error bars and make sure these are not reduced by autocorrelation effects.

Problem 2. ***Monte Carlo simulation of the classical 2D Ising model with cluster update***

Local updates become inefficient at low temperatures and close to phase transitions. Therefore in this part you improve your simulation by implementing Wolff cluster updates. Verify your implementation by comparing results with the simple updates of problem 1 and observe that autocorrelation effects are a lot smaller.