Molecular BEC and BEC-BCS crossover   Cheng Chin

(First session: characterization of pair wavefunction)
  Dissociation of molecules (K-40, JILA, 2003)
    Greiner, Regal, Jin, Nature 426 537 (2003)
  Observation of pairing gap (Li-6, Innsbruck, 2004)
    Chin et al., Science 305 1128 (2004)
    Schunck et al., Nature 454 739 (2008)
  Photo-emission spectroscopy (K-40, JILA, 2009)
    Stewart et al., Nature 454 744 (2008)

(Second session: Quantum nature of Feshbach molecules)

Question 1: How much do we know about the BEC-BCS crossover from experiments?

Question 2: What is different from atomic BECs?
Atom-molecule coexistence, pairing and unitarity limit

Universal conjecture: since $a \to \infty$, only energy scale is E_F, length scale k_F^{-1}

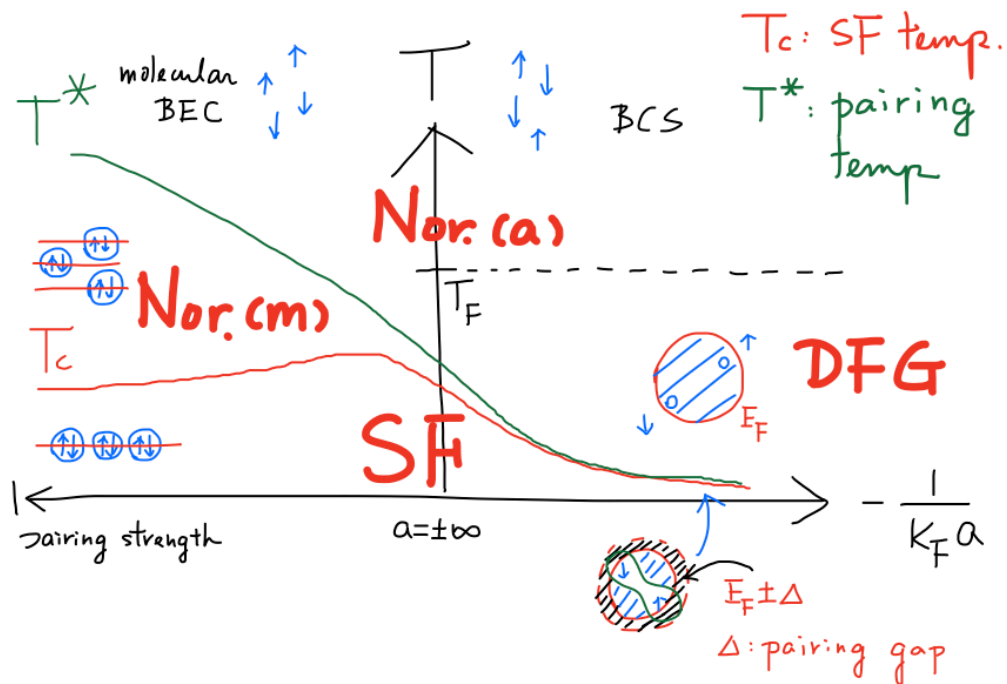All energy scales are either $\sim E_F$ or 0 or ∞
    length scales are either k_F^{-1} or 0 or ∞.   T.L.Ho (04)

This is a restatement of continuous scale invariance.

Q1: How much do we know about the crossover from experiments?
Thermodynamics (2004 JILA, Innsbruck, MIT, ENS, 2005 Rice, Duke)
Collective excitations and damping (2004 Innsbruck, Duke)
Pairing (2004 Innsbruck, 2005 JILA, Rice, 2006 ETH, MIT)
Resonance location (2004 Innsbruck)
Fermi gas in optical lattices (2005 ETH, 2006 MIT, MPQ)
Superfluidity (2004 Duke, 2005 MIT)
Spin imbalance (2006 MIT, Rice)
Sound speed (2007 Duke, MIT)
Other topics: reduced dimensions, equation of state...
False alarm: ferromagnetism, heavy soliton?

Does everything we saw agree with the theoretical phase diagram?

Atom-pair-condensed pair coexistence

Phase diagram does not guarantee that equilibrium can be reached.
1. We need dynamical process to create new species.
Starting from a pure atomic sample, how are molecules created?

$\Rightarrow$ We need 3-body recombination $A+A+A \rightarrow A_2 + A$
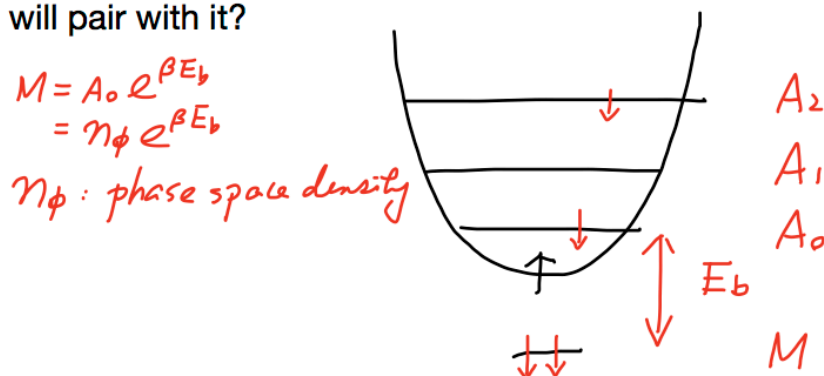
How about starting from a pure molecular sample? Can atoms be
created if we slowly heat up the sample?

$\Rightarrow$ We need collision dissociation $A_2 + A_2 \rightarrow A_2 + A + A$ (unconfirmed)

2. These reactions should not generate much heat.

Equilibrium of weakly interacting atoms and molecules
Assuming we have a spin down atom, how many spin up atoms
will pair with it?

$$M = A_0 \, e^{\beta E_b}$$
$$= n_\phi \, e^{\beta E_b}$$
n_ϕ : phase space density

A_2
A_1
A_0

E_b

M

So in a gas with N spin downs, N spin ups, the total molecule number is
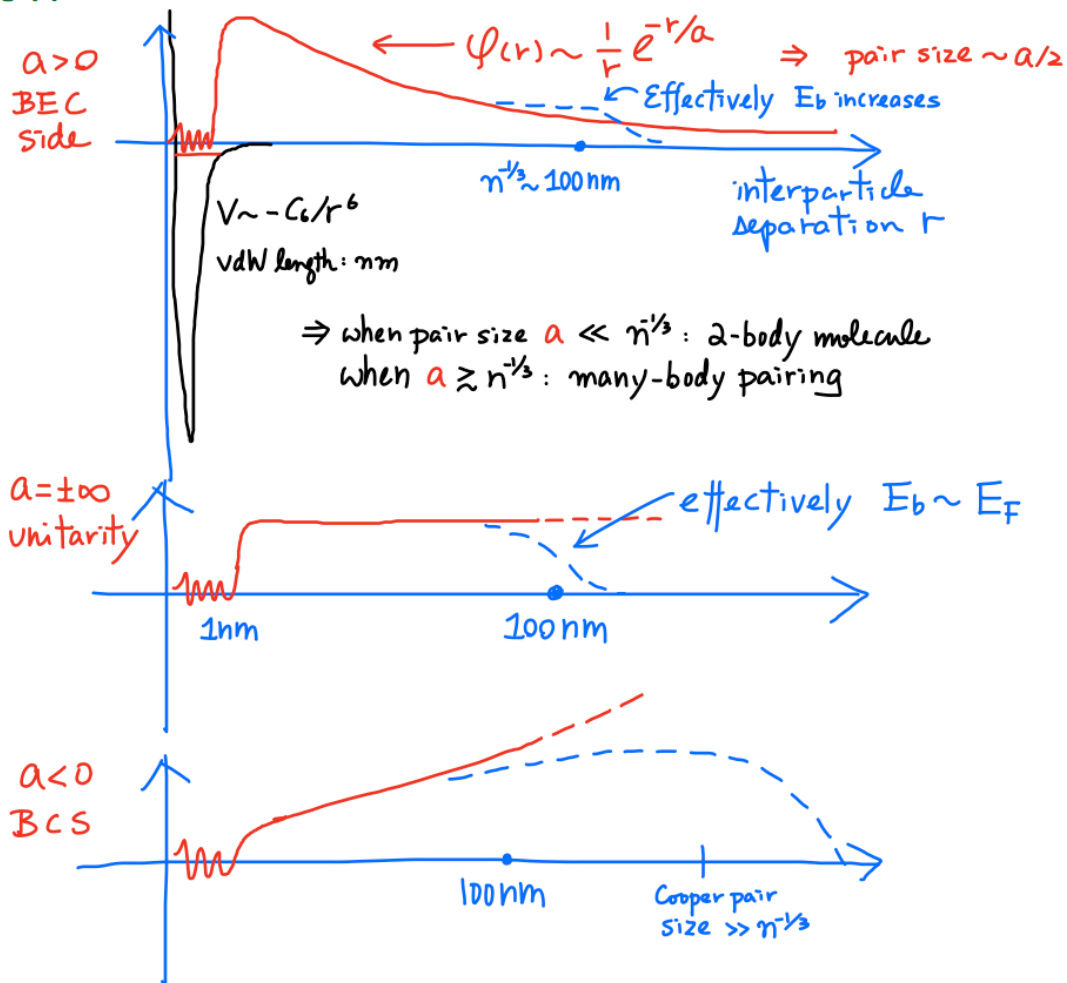
$$M = N \, n_\phi \, e^{\beta E_b}$$

HW1: Show that the characteristic temperature T* to see macroscopic
amount of weakly interacting molecules in a 3D trap satisfies

$$T^* = 12^{-1/3} \, T_F \, e^{(2 T_F / 3 T^*)(k_F a)^{-2}}$$

Pairing:
1. Chemistry molecules: bound by potential, size ~ angstrom ~Eb^0
2. Van der Waals molecules: bound by vdW pot, size ~ Eb^(-1/6)
3. Halo molecules: bound by uncertainty principle, size ~ Eb^(-1/2)
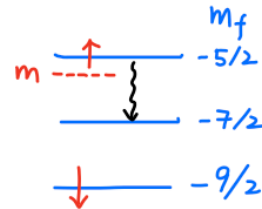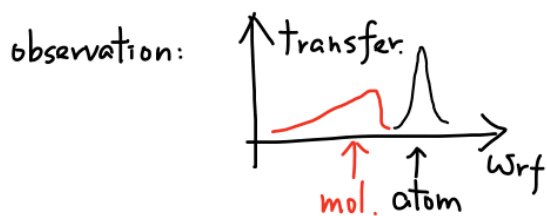4. Cooper pairs: bound by Fermi statistics, size ~ Eb^??

HW2: Justify or disprove the above scaling with your estimation. What is ??

$a > 0$
BEC side

$\psi(r) \sim \frac{1}{r} e^{-r/a}$ $\Rightarrow$ pair size $\sim a/2$

Effectively E_b increases

$n^{-1/3} \sim 100\,nm$ interparticle separation r

$V \sim -C_6/r^6$
vdW length: nm

$\Rightarrow$ when pair size $a \ll n^{-1/3}$: 2-body molecule
when $a \gtrsim n^{-1/3}$: many-body pairing

$a = \pm \infty$
unitarity

effectively $E_b \sim E_F$

1nm   100 nm

$a < 0$
BCS

100nm   Cooper pair size $\gg n^{-1/3}$

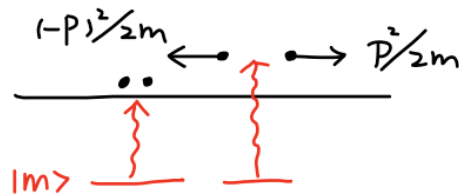How do we probe pair wavefunction?

Radiofreq spectroscopy

2003 JILA "photodissociation of molecules K_2"
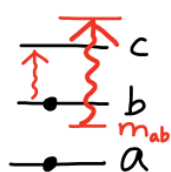
observation:

Interpretation:
1. a finite offset in freq comes from molecular binding energy.
2. Asymmetric lineshape is due to the kinetic energy of the dissociated molecules. A higher energy is required to dissociate the molecules into 2 atoms with high kinetic energy.

This interpretation is absolutely correct and it reveals the pair wavefunction!!

2004 RF spectroscopy on Li fermi gas in the BEC-BCS crossover regime

thermal gas

free-free transition: δ function
bound-free (mol. → atom): asymmetric lineshape
bound-bound (mol. → mol.): molecular spectroscopy.

Theory for bound free transition: (PRA 71 012713 2005)
Initial molecular state ψ(r) at rest
Final state: two atoms with momenta p and - p, total kinetic energy K

Energy conservation:
 RF photon energy = hyperfine energy + binding energy + kinetic energy

Transition rate $T = \frac{2\pi}{\hbar} |< K | \frac{\hbar}{2}\Omega |m>|^2 = \frac{\hbar}{2}\Omega^2 \bar{F}_K$

$$F_K = |<K|m>|^2 : \text{Franck-Condon Factor}$$
$$= \text{probability that the molecule is dissociated}$$
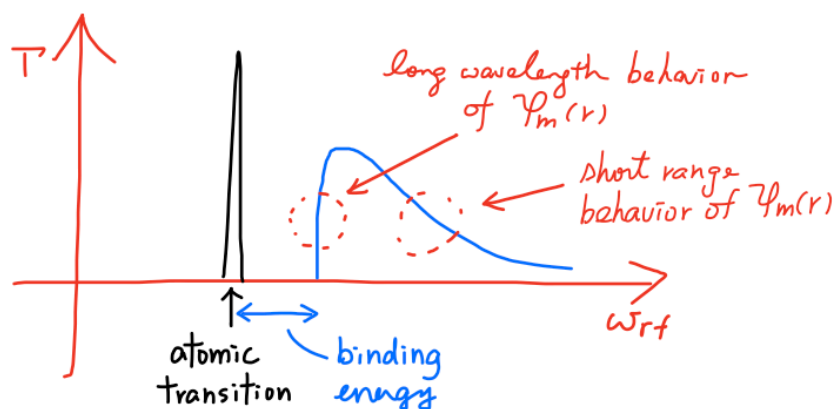$$\text{into 2 atoms with total kinetic energy } K.$$

For example: starting from a halo molecule

$$\psi_m = \left(\frac{2}{a}\right)^{1/2} e^{-r/a}$$
$$\psi_K = A \sin(Kr + \delta')$$
$$A = \left(\frac{2\mu}{\pi\hbar^2 K}\right)^{1/2} \text{ for an energy normalized plane wave}$$
$$\delta': \text{scattering phase shift in the final state. } K\cot\delta' = -1/a'$$

long wavelength behavior of $\psi_m^0(r)$

short range behavior of $\psi_m(r)$
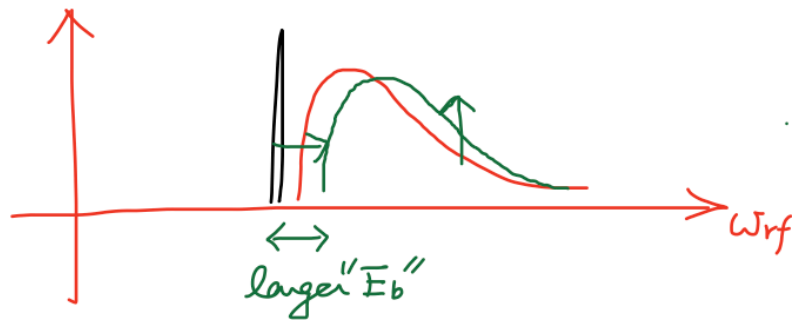
ω_{rf}

atomic transition

binding energy

HW3: Determine the limiting behavior of the rf lineshape.

Signatures of many body effect: $\hat{V} = a^{\dagger}_{kc}\, a_{k\uparrow}$
Deviations from the halo molecule prediction:
1. Larger pair energy in the BEC regime
2. Finite pairing gaps in the unitary and BCS limits.
3. Dependence on temperature and Fermi energy
4. Modified lineshape determined by many body physics.

$\longleftrightarrow$
larger "E_b"

ω_{rf}

Issues with the early rf experiments:
1. Spatial inhomogeneity of the trapped sample
2. Final state effect?
Recent rf spectroscopy experiments:
1. ETH: optical lattices: confinement induced pairing.
2. MIT: final state effects
3. JILA: momentum resolved rf spectroscopy.
Observe the momentum of the spin flipped fermion.

$h\nu = \varphi + E_k - \varepsilon_k \Rightarrow \varepsilon_k = E_k - (h\nu - \varphi)$

BCS theory: $\varepsilon_k = \mu - \sqrt{\Delta^2 + (\mu - E_k)^2}$

HW4: Show that:

In the BCS regime
the rf threshold $\sim \Delta^2/E_F$
In the BEC regime,
we have $\varepsilon_k \sim -E_b - E_k$.