Efimov physics   Cheng Chin

(First session: Efimov physics and Efimov potential)
  Efimov's original paper: Efimov, Phys. Lett. B 33, 563 (1970)
  Special features of Efimov trimers
  Efforts before cold atoms
  Recombination in cold atoms: Esry, Greene & Burke, PRL 83 1751 (1999)
  First observation of Efimov states in Cs133 (Innsbruck, 2006)
  Li6 (2008 Heiderberg), (2009 Penn State)
  Li7 (2009 Rice), (2009 Bar-Ilan)
  K39 (2009 LENS)
  K41Rb87 (2009 LENS)
  Rb85 (2012 JILA)
  Rb87K40 (2013 JILA)
  Li6Cs133 (2014 Chicago)

(Second session: extension of the Efimov scenario)
  Three component Fermions
  Mixed species system
  Three-body parameter and Universality
  Efimov physics beyond three-body
  Efimov physics beyond 3D

Question 1: What is so special about Efimov molecules?

Question 2: What new physics did we find?

Question 3: Why should we care?

Efimov physics (Vitaly Efimov, 1970)

Three identical bosons with resonantly pair-wise interactions.

$$H\varphi = E\varphi \ . \ \varphi = (\vec{x}_1, \vec{x}_2, \vec{x}_3) \ . \ H = \sum_i P_i^2/2m + \sum_{i<j} V(|x_i - x_j|)$$

V(r) is assumed to be identical for any pairs of particles, zero range, attractive and the scattering length is tuned to infinity.

Introducing hyper spherical coordinate, we have

$$R^2 = (r_1 - r_2)^2 + (r_3 - r_3)^2 + (r_3 - r_1)^2$$

and 5 hyperangles Λ_i which depend on the orientation of the three body system. Why 5?

In the low scattering energy limit, we assume all hyper angular momenta are suppressed. The radial equation is reduced to

$$H\varphi(R) = P_R^2/2m - \frac{\hbar^2 (s_0^2 + 1/4)}{2mR^2} \qquad \text{Effective long range potential}$$

Efimov potential

and $s_0 = |i\nu|$, $\nu \cos \nu \frac{\pi}{2} - \frac{8}{\sqrt{3}} \sin \nu \frac{\pi}{6} = 0 \Rightarrow s_0 = 1.0064$
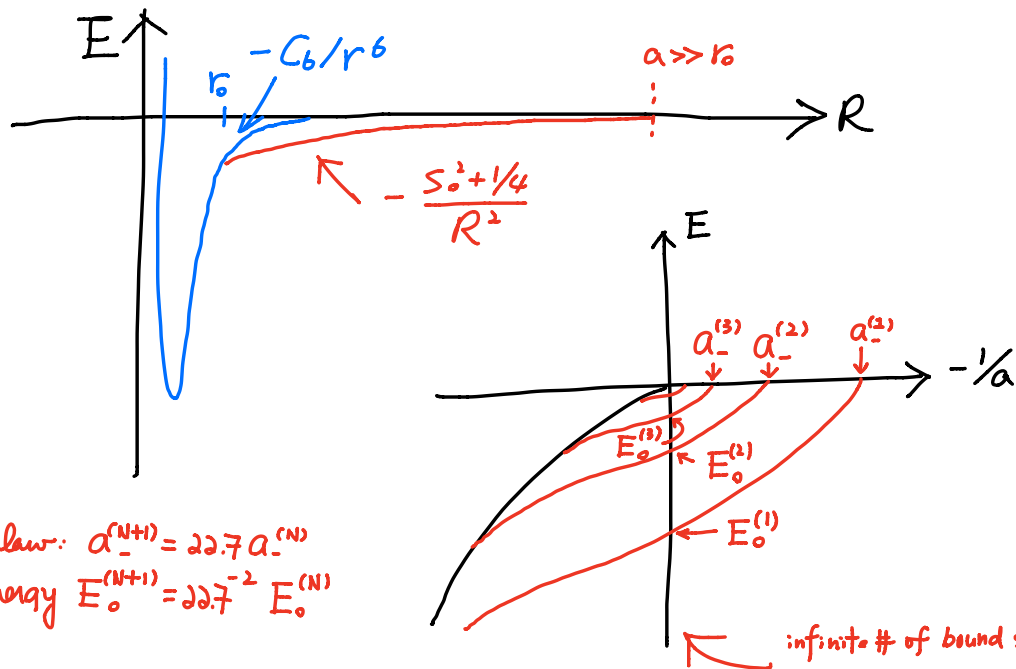
Scaling law: $a_-^{(N+1)} = 22.7 \, a_-^{(N)}$

binding energy $E_0^{(N+1)} = 22.7^{-2} E_0^{(N)}$

HW1: solve the radial Schroedinger's eqn with a central force potential with zero scattering energy E=0.

$$H\Psi(R) = \left[-\frac{\hbar^2}{2m}\nabla_R^2 + V(R)\right]\Psi(R) = E\Psi = 0$$

$$\Psi(R > r_0) = -\frac{\alpha\hbar^2}{2mR^2}$$
$$\Psi(R < r_0) = +\infty$$

From the solution, show that the potential indeed supports infinite number of bound states with a logarithmic-periodic behavior.
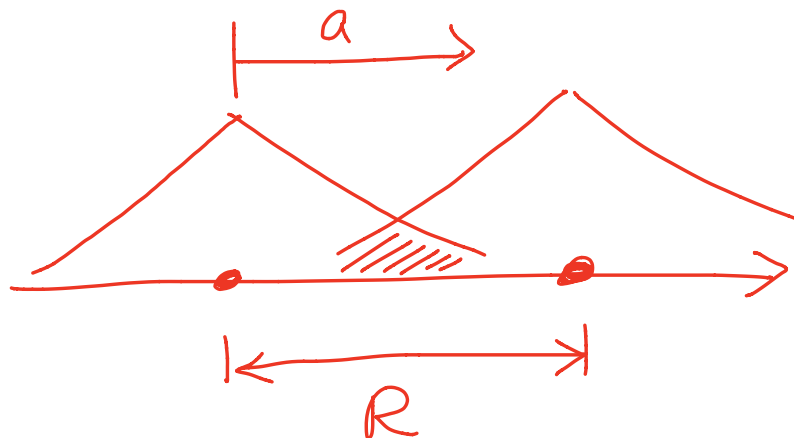
Using WKB, we get $\oint p\,dg = n\hbar = \int \sqrt{2m(-V(r))}\,dr$, assume $V = -\frac{S_0^2\hbar^2}{2mr^2}$

$$\Rightarrow \int (2S_0\hbar/r)\,dr = n\hbar$$
$$\Rightarrow 2S_0 \ln(r_n/r_0) = 2\pi n \Rightarrow \frac{r_{n+1}}{r_n} = e^{\pi/S_0} = 22.7\ldots$$

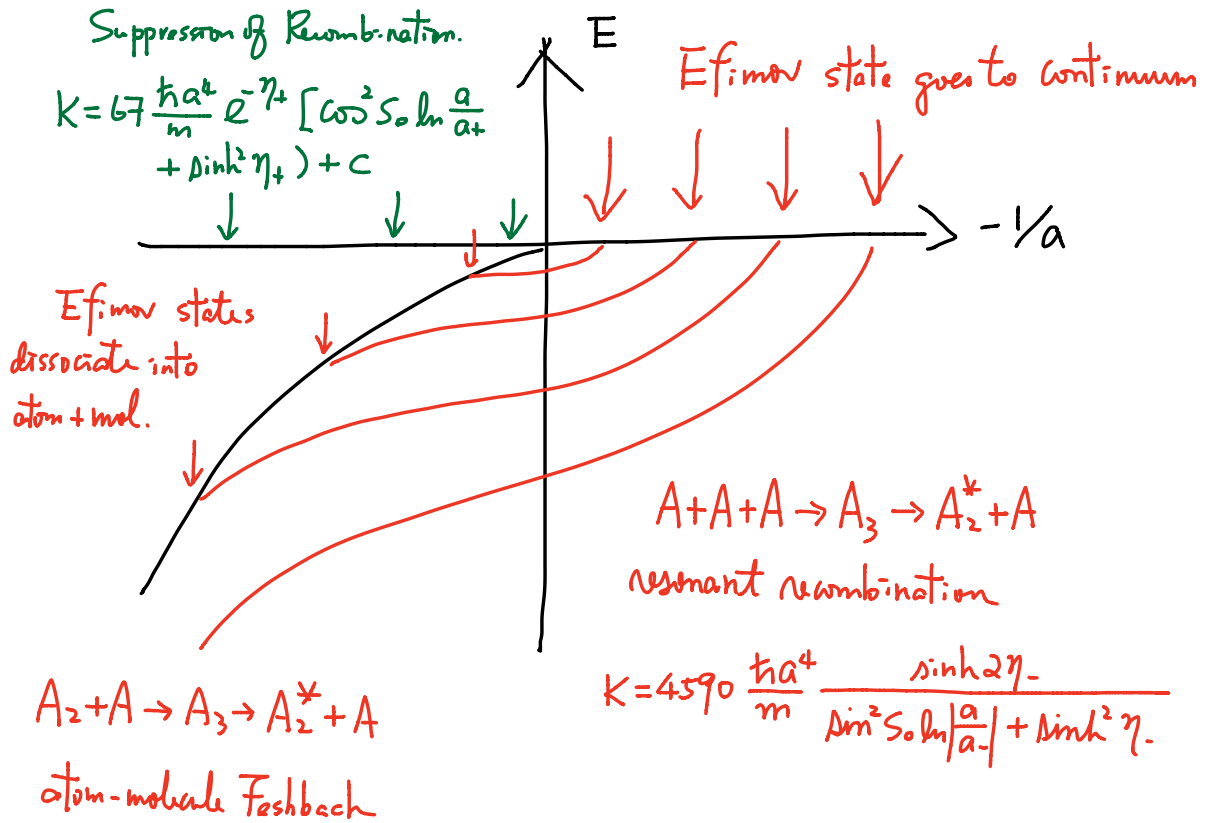Physics interpretation? Why is there a $1/R^2$ potential?
Consider two particles are pinned to the x-axis, what is the wavefunction of the third one?

Enhancement of the wavefunction due to symmetrization of wave function is given by $-\frac{\hbar^2}{2m}\Psi_{ex}'' + V_{eff}\Psi_{ex} = E\Psi = 0 \Rightarrow V_{eff} \sim \frac{-\hbar^2}{2mR^2}$

$$\Rightarrow V_{eff} = 0 . R > a$$

How to observe Efimov states in cold

Suppression of Recombination.

$$K = 67 \frac{\hbar a^4}{m} e^{-\eta_+} [\cos^2 S_0 \ln \frac{a}{a_+} + \sinh^2 \eta_+) + C$$

Efimov states dissociate into atom + mol.

$$A_2 + A \rightarrow A_3 \rightarrow A_2^* + A$$

atom-molecule Feshbach

E

$-1/a$

Efimov state goes to continuum

$$A + A + A \rightarrow A_3 \rightarrow A_2^* + A$$

resonant recombination

$$K = 4590 \frac{\hbar a^4}{m} \frac{\sinh 2\eta_-}{\sin^2 S_0 \ln \left|\frac{a}{a_-}\right| + \sinh^2 \eta_-}$$

Summary: enhancement of 3-body recomb. for $a < 0$
suppression of 3-body recomb. for $a > 0$

Hw2: recombination loss coefficient K is defined here $\frac{dn}{dt} = -3Kn^3$

A. If a is the only scale, show that $K = \frac{\hbar^2 a^4}{m} C(a)$, where the Efimov factor C(a) is dimensionless.

B. What if a diverges? Argue that at finite temperature $\frac{dn}{dt} \propto -\frac{n^3}{T^2}$ How about at zero temperature T=0?

Experimental progress

- First observation of Efimov states (Innsbruck, 2006)
Both enhancement and suppression are observed as well as

- 3 component fermi gas Li6 (2008 Heiderberg), (2009 Penn State) $k \to a^4$
Bosons are not absolutely necessary.

- Many features Li7 (2009 Rice, correction on 2013)
Some secondary features and four-body? Interpretation?

- Universal Efimov in diff spin states (2009 Bar-Ilan)

- Observation of many features in K39 (2009 LENS, correction on 2013)
Some secondary features and four-body? Interpretation

- Mixture system K41Rb87 (2009 LENS)

- New universality across species and resonances (2011 Innsbruck)

Theory (2011, 2012...)

- Single Efimov in Rb85, test of universality (2012 JILA)

- Rb87K40, no recom resonance but atom-dimmer resonance (2013 JILA)

- Discrete scaling invariance Li6Cs133 (2014 Chicago)

Other progress:
- Four and five body recombination (Innsbruck)
- Atom-dimer universality?
- Effective range correction?
- Radio-freq association of Efimov trimers
- Three-body contact?