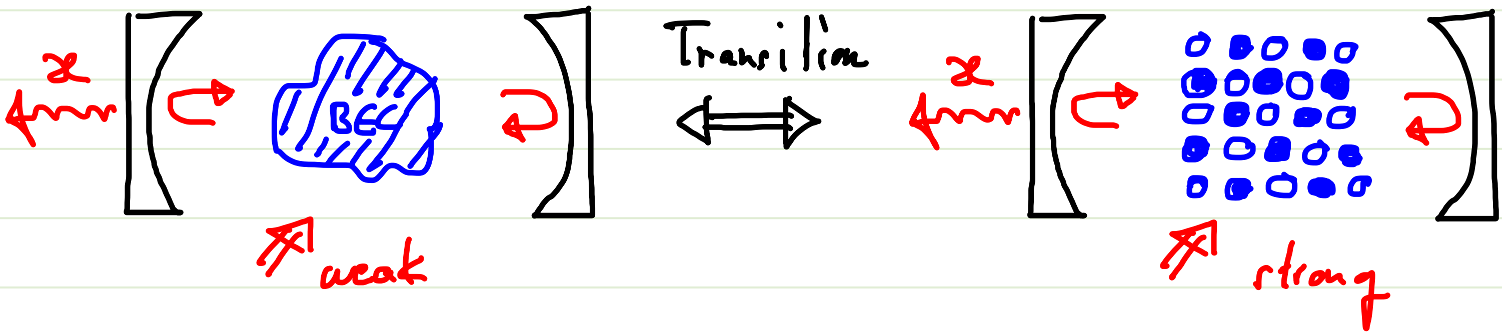
Topic: Non-equilibrium phase transitions

- We know several examples of intrinsically non-equilibrium systems.
- We can define non-equilibrium

## 1. Examples, Motivation

A.) Atoms in a cavity: Dicke transition

Science 336 1570
PNAS 110 29

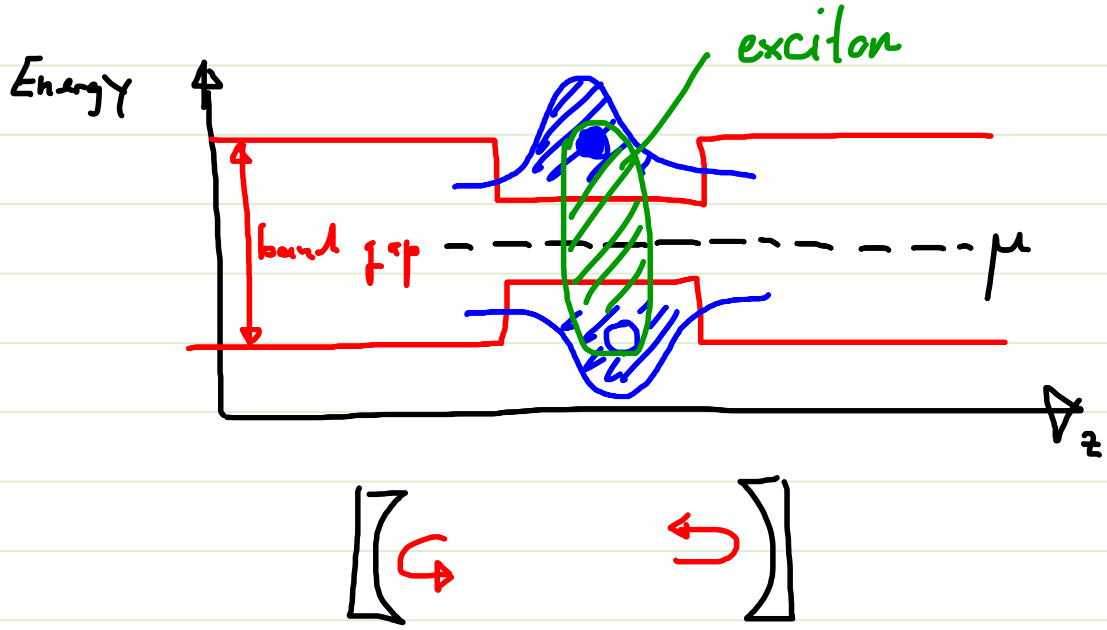



The transition (and the steady state) would be well captured by an equilibrium theory if there were no (only small) cavity losses. ⇒ However: if $\kappa$ is not negligible were are dealing with a non-equilibrium phase transition.

B.) Exciton-polariton condensation

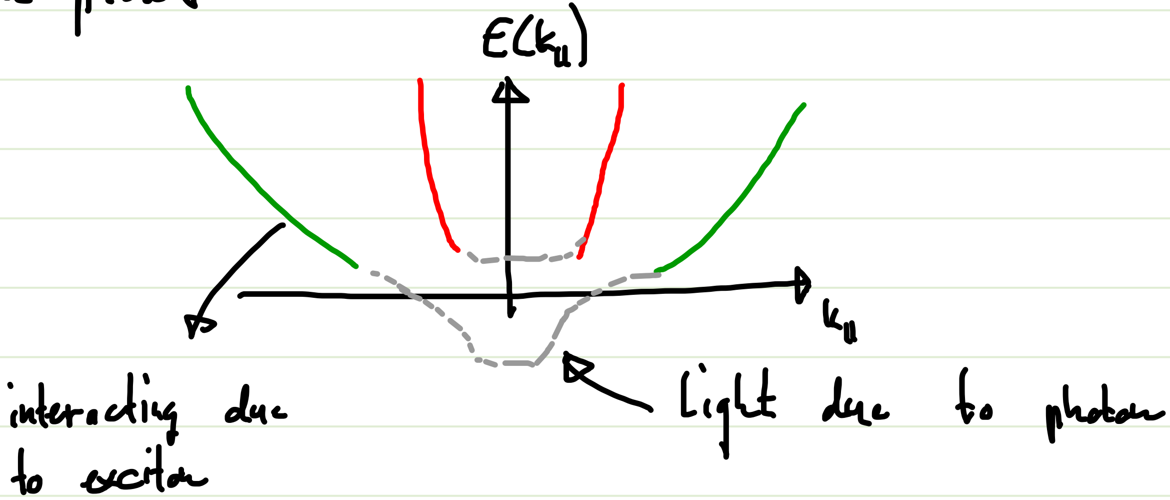
The critical temperature for BEC scales as $T \propto \frac{1}{m}$.
For Rubidium, which is relatively heavy, this required

extremely low temperatures, which can be achieved owing to the extrem isolation / control we have on dilute quantum gases. Another "bosonic" degree of freedom are excitons:



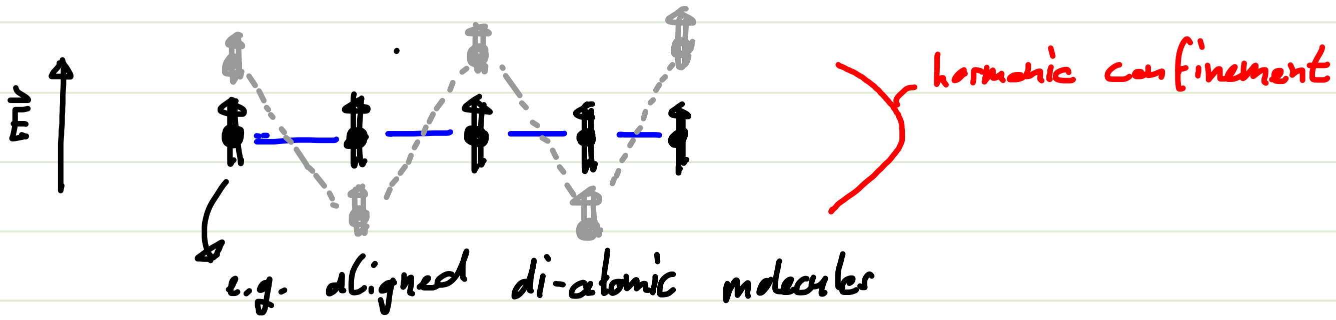

Nature 443 409 (2006)

Excitons, however are too heavy to condense at temperatures reasonably achievable in solid states. ⇒ mix with light photons:




But: Excitons recombine & mirrors are leaky ⇒ need to drive the system constantly ⇒ strong non-equilibrium situation.

## C:) Dipoles in a linear chain

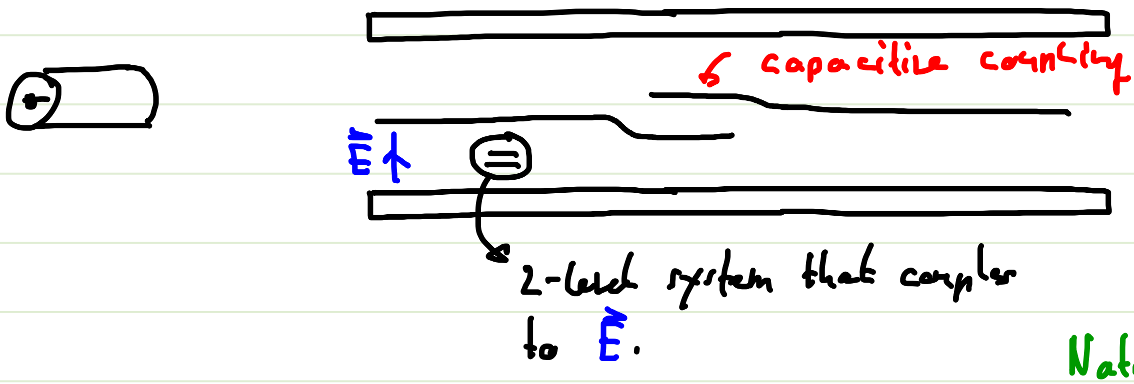



If the repulsion exceeds a certain value: zig-zag transition to trade interaction with potential energy. However: gates that make up the $\vec{E}$-field suffer from $1/f$ noise.

Nature phys. <u>6</u> 806 (2010); PRB <u>85</u> 125121 (2012)

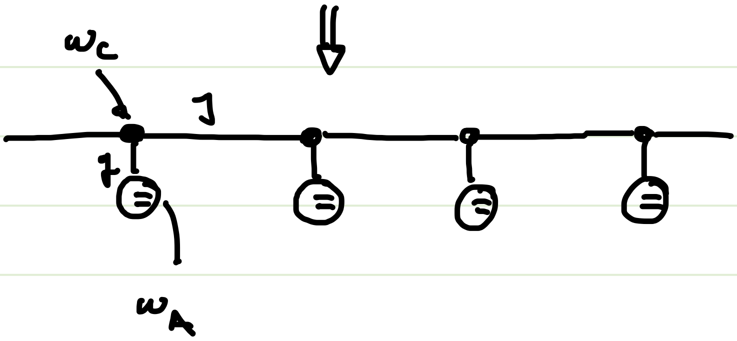
## D:) Microwave cavities

In another type of engineered system, the photon plays the central role.




Nature <u>431</u> 162 (2004)

$\Downarrow$




$\Rightarrow$ interacting photons $\Rightarrow$ Mott transition.

Nature phys <u>8</u> 292 (2012)

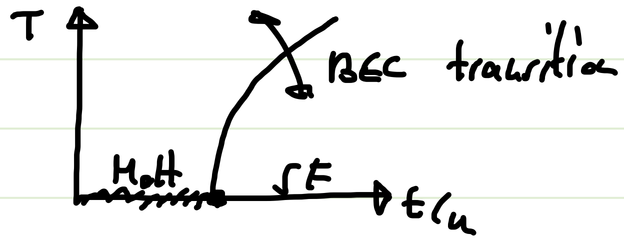
## 2. What are the problems?

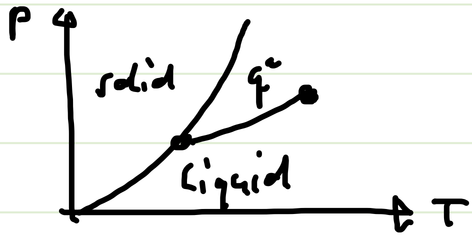
In equilibrium we know, in principle, how to calculate measurable quantities for a given microscopic model

$$A = \langle \hat{A} \rangle = \frac{\mathrm{Tr}(\hat{A}\hat{\rho})}{\mathrm{Tr}(\hat{\rho})} \quad \text{with } \hat{\rho} = e^{-\beta \hat{H}} .$$

Using $e^{-\beta \hat{H}} = \sum_{n,m} |n\rangle\langle n| e^{-\beta \hat{H}} |m\rangle\langle m| = \sum_n e^{-\beta E_n} |n\rangle\langle n|$

we can obtain e.g.:




What is now non-equilibrium? $A = \mathrm{tr}(\hat{A}\hat{\rho})$ with $\hat{\rho} = ??$

The best way to define <u>non</u>-equilibrium is by stating what it is <u>not</u>: (Altland & Simons)

- An equilibrium system is characterized by a unique set of extensive and intensive variables which do not change in time.
- After isolation from the environment, all variables remain unchanged.

The second point is important to distinguish it from a non-thermal steady state like the exciton-polariton or the Dicke system described above.

Therefore, to make further progress, we need to develop tools to deal with the problem of finding $\hat{\rho}$. Or, more directly, a way to calculate $\langle \hat{A} \rangle$.

We will discuss two ways to attack this problem:

(i) Derive an equation of motion for $\hat{\rho}$, solve it, and calculate properties of interest. This approach goes under the label Master equation.

Pros:
- easy to derive
- transparent machinery
- "simple" to solve

Cons:
- Not good for a continuum of modes
- Hard to distill universal features.

(ii) Set up a protocol to evaluate a path integral for the calculation of expectation values:

in equilibrium: $\mathrm{Tr}\, \hat{\rho} \hat{A}$: $\quad |\vec{\alpha}\rangle = e^{\sum \alpha_i b_i^\dagger} |0\rangle$

$$\Rightarrow e^{\beta \hat{H}} = \prod_i e^{\Delta\beta_i \hat{H}}$$

$$\Rightarrow e^{\beta \hat{H}} = \sum_{\{\alpha_i\}} e^{\Delta\beta_0 \hat{H}} |\vec{\alpha}_0\rangle\langle\vec{\alpha}_0| e^{\Delta\beta_1 \hat{H}} |\vec{\alpha}_1\rangle\langle\vec{\alpha}_1| \ldots\ldots$$

$$\Rightarrow \mathrm{Tr}\,\hat{\rho} = \int \{\mathcal{D}\vec{\alpha}\}\, e^{-\int_0^\beta d\tau\, H(\vec{\alpha},\vec{\alpha}^*)}$$

with $\vec{\alpha}(0) = \vec{\alpha}(\beta)$

$\Rightarrow$ we need the same logic but $\hat{\rho} \neq e^{-\beta\hat{H}}$

$\Rightarrow$ Keldysh path integral.

Pros:
- Can handle mode continuum
- We can do RG

Cons:
- Slightly more sophisticated
- Technicalities sometimes subtle.

Program for the remaining course:

- 13.5. : Master equation & Mott transition
- 20.5. : Keldysh P.I. & Driven-dissipative transitions
- 27.5. : Numerical tools for non-equilibrium problems.