0. Preliminaries

This lecture is based on a collection of books and research articles. A general list of books is

- Landau & Lifschitz, Band VII
- Fetter & Walecka, Theoretical mechanics of particles and continua
- M. Meyers, Dynamic behavior of materials
- Ashcroft & Mermin, Solid state physics
- Deymier (Ed.), Acoustic metamaterials and phononic crystals
- Craster & Guenneau (Eds.), Acoustic metamaterials

The references to the relevant research articles will be given throughout the course.

0.1 Exercise classes

We will have small exercises to deepen our understanding. Moreover, after the first few introductory weeks, we will distribute industry and research-projects where you will have to evaluate the taught material against "real-world" problems.

Moreover, I encourage you to bring a device to class with which you can conduct quick internet searches. I will frequently toss small assignments at you, which you will have to solve in class.

0.2 References for the introduction

1.) Lee, S.H. and Wright, O.B., Phys. Rev. B **93** 024302 (2016)
2.) Coulais, C. et al., Nature **535** 529 (2016)
3.) Bertoldi, K. et al., Adv. Mater. **22** 361 (2010)
4.) Moldovan, M., Nature **503**, 209 (2013)
5.) Martínez-Sala, M. et al., Nature **378** 241 (1995)
6.) Liu, Z. et al., Science **289**, 1734 (2000)
7.) Cummer, S.A. et al., Nature Rev. Mat. **1** 16001 (2016)

1. Bragg scattering vs. local resonances

We start our lecture on mechanical metamaterials with the discussion of metamaterials with a target functionality at finite frequencies. In other words, we want to understand how we can control or manipulate the flow of mechanical energy in the form of vibrations.

To understand how vibrations propagate through materials we will have to introduce the wave equation and familiarize ourselves with the properties of wave-propagation. However, before we do so, we introduce two generic building blocks that are used to achieve the goal of manipulating wave propagation in solids: Bragg scattering and the use of locally resonant structures. Once we know how these two principles work, we will understand most modern metamaterial designs.

1.1. Bragg scattering

For now, all we need to know about waves is that they can be described by a travelling modulation of some property $f(x,t)$

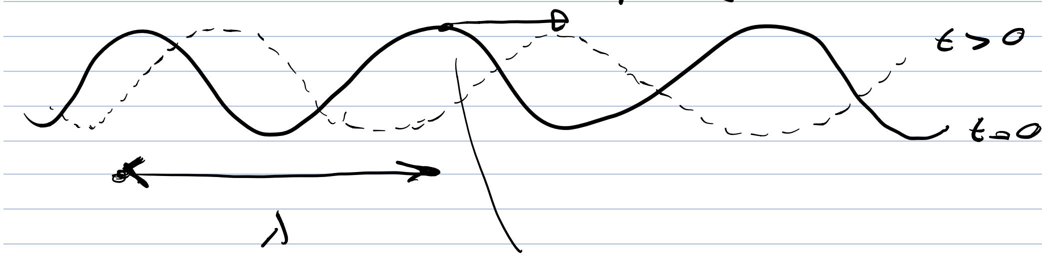
$$f(x,t) = \cos(\omega t + kx + \varphi)$$

where $\omega = \frac{2\pi}{T}$ is the angular frequency related to the period T

$k = \frac{2\pi}{\lambda}$ is the wave number related to the wave-length λ

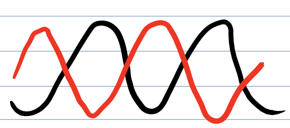
and φ is a "phase".

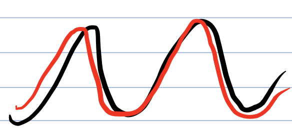
$$v = \frac{\lambda}{T} = \frac{\omega}{k}$$



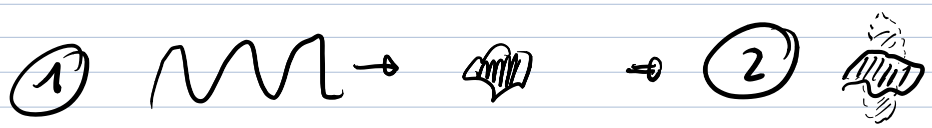
$$\omega t + kx = 0 \Rightarrow x = -\frac{\omega}{k} t$$

Two waves can interfere. In particular:

 $+ \Rightarrow$ destructive interference for $\Delta\varphi = \pi$

 $+ \Rightarrow$ constructive interference for $\Delta\varphi = 0$

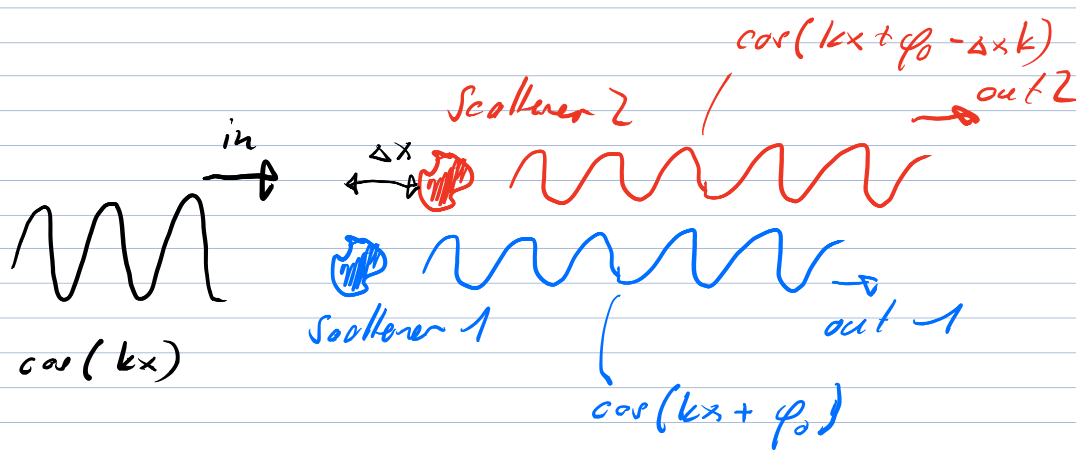
Let us now consider a wave incident on a scatterer:



scatterer is excited

scatterer radiates wave

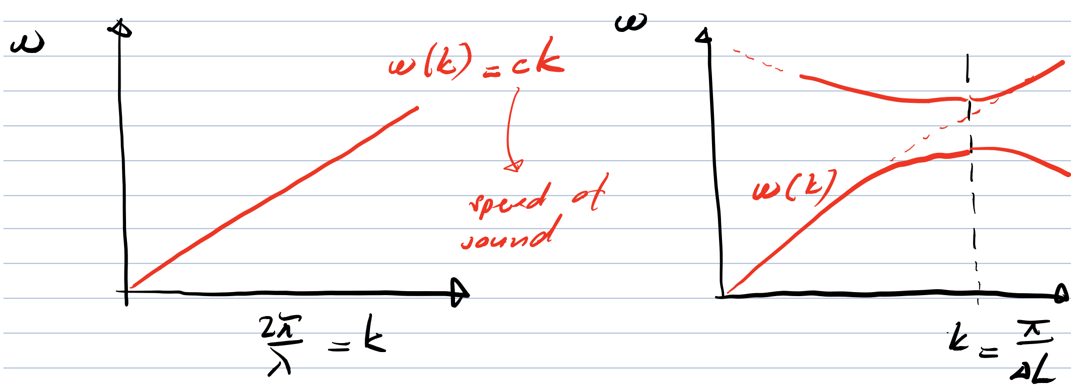
The "time lag", i.e. $\Delta\varphi$, between the incident and outcoming wave is given by the details of the interaction between the wave and the scatterer. However, consider the effect of two scatterers at a distance Δx:



$\Rightarrow \quad \Delta x \cdot k = \pi$: destructive interference

$\Rightarrow \quad \Delta x = \frac{\lambda}{2}$ leads to a strong destructive effect on the propagation of waves.

This phenomena is called "Bragg scattering" if we deal with a periodic array of scatterers. Bragg scattering is at the heart of phononic crystals. It is not hard to imagine, that periodic arrays with a typical length-scale ΔL can have dramatic effects on the wave propagation at frequencies where the Bragg condition is fulfilled.



$\Rightarrow$ $\boxed{\text{If I want to change the propagation of waves at frequency } \nu = \frac{\omega}{2\pi} \text{ we have to design structures with dimension } \Delta L \approx \frac{c}{\nu}}$

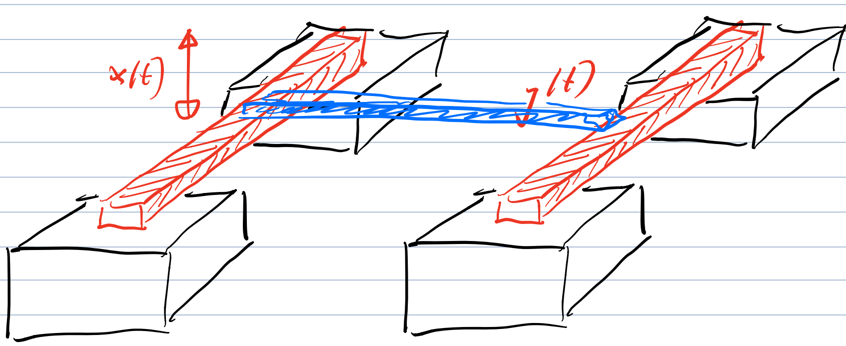
Question: I live in California and want to protect my house from earthquakes with a phononic crystal. How big is it going to be?

1.2 Local resonances

We have seen that periodic structures can have an influence on the propagation of waves. Here we want to prevent a building block to achieve similar effects where we are not limited by the size of the structure.

In order to understand the concept of local resonances, we need to understand the coupling of <u>two</u> oscillators.

Let us assume the following system:



The oscillator $x(t)$ is described by

$$\ddot{x}(t) = -\omega_0^2 x(t) + \gamma^2 y(t) \quad (1) \qquad \text{with } \gamma^2 \ll \omega_0^2$$

$$\ddot{y}(t) = -\omega_0^2 y(t) + \gamma^2 x(t) \quad (2)$$

For $\gamma = 0$ we know that

$$x(t) = x_0 e^{i\omega_0 t} \quad \text{and} \quad y(t) = y_0 e^{i\omega_0 t}$$

solve the equations (1) & (2). If $\gamma \neq 0$, we need to find combined solutions:

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} e^{i\omega t} \Rightarrow$$

$$-\omega^2 \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \cancel{e^{i\omega t}} = \begin{pmatrix} -\omega_0^2 & \gamma^2 \\ \gamma^2 & -\omega_0^2 \end{pmatrix} \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \cancel{e^{i\omega t}}$$

$\Rightarrow$ we are dealing with an eigenvalue problem for ω

Route 1 $\quad \det \begin{pmatrix} \lambda - \omega_0^2 & \gamma^2 \\ \gamma^2 & \lambda - \omega_0^2 \end{pmatrix} = (\lambda - \omega_0^2)^2 - \gamma^4 = 0$

$$\Rightarrow \lambda = \omega_0^2 \pm \gamma^2 \Rightarrow \omega = \sqrt{\omega_0^2 \pm \gamma^2}$$

Route 2:

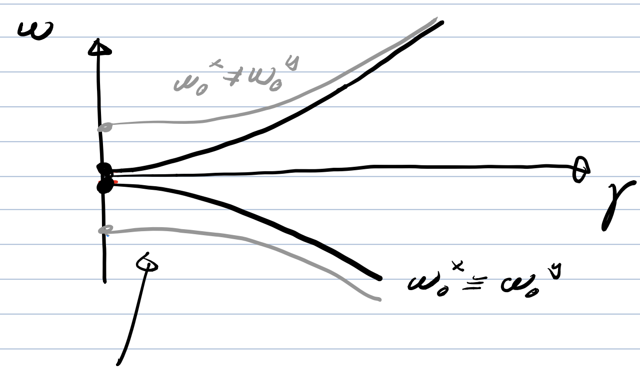
$$\begin{pmatrix} -\omega_0^2 & \gamma^2 \\ \gamma^2 & -\omega_0^2 \end{pmatrix} = -\omega_0^2 \,\mathbb{1} + \gamma^2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= -\omega_0^2 \,\mathbb{1} + \sum_{i=1}^{3} d_i \sigma_i \qquad \text{with}$$

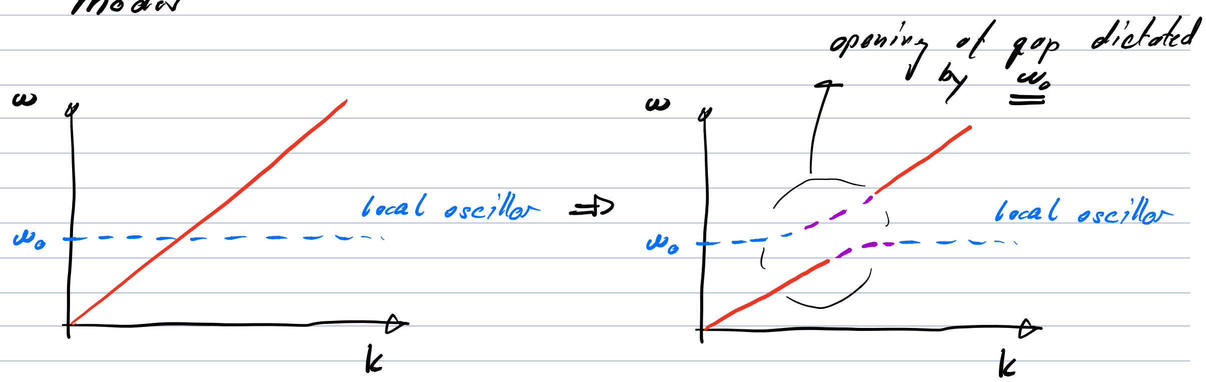
$$d_i = (\gamma^2, 0, 0) \quad \text{and} \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow \quad \lambda = -\omega_0^2 \pm |\vec{d}|$$



influence of coupling strongest for degenerate modes



We can modify the propagation of waves in the vicinity of a frequency ν by coupling them to local resonances $\nu_0 \approx \nu$.