

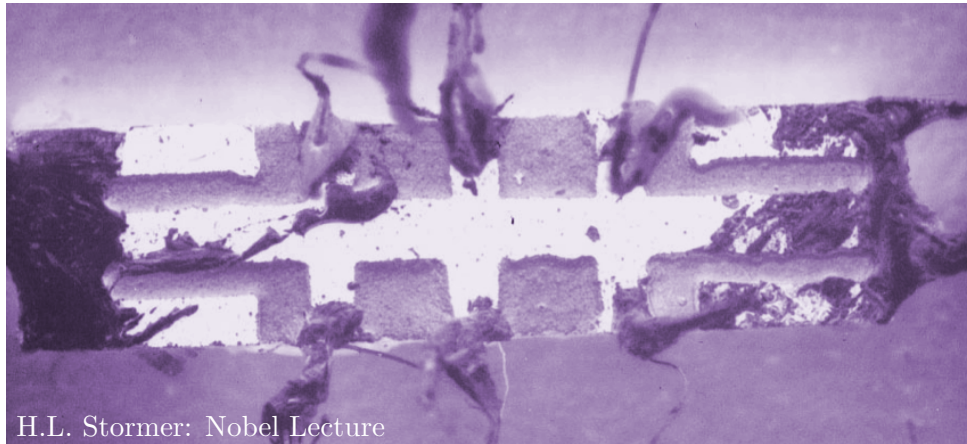
**Week 1**  
**Lecture Notes:**  
**Topological Condensed Matter Physics**

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# Chapter 1

## The integer quantum Hall effect I



### Learning goals

- We know the basic phenomenology of the quantum Hall effect (QHE)
  - We know the structure of the lowest Landau level (LLL)
  - We understand the role of disorder for the QHE.
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- K. von Klitzing, G. Dorda, and M. Pepper, *Phys. Rev. Lett.* **45**, 494 (1980)

In large parts of this chapter we follow reference [2].

### 1.1 Preliminaries

The Lorentz force acting on charged particles moving in a two-dimensional plane leads to a build-up of charges perpendicular to the direction of motion. This is the classical Hall effect first discussed by Edwin Hall in 1879 [3]. To understand this, let us consider a two-dimensional system which is translationally invariant. We move to a frame moving with  $-\mathbf{v}$  where we therefore see electrons moving with velocity  $\mathbf{v}$  and carrying a current

$$\mathbf{J} = -nev, \quad (1.1.1)$$

where  $n$  is the particle density and  $e$  is the electron charge. In the laboratory frame we have  $\mathbf{E} = \mathbf{0}$  and  $\mathbf{B} = B\hat{z}$ . Hence, in the moving frame we obtain

$$\mathbf{E} = -\mathbf{v} \wedge \mathbf{B} \quad \text{and} \quad \mathbf{B} = B\hat{z}. \quad (1.1.2)$$

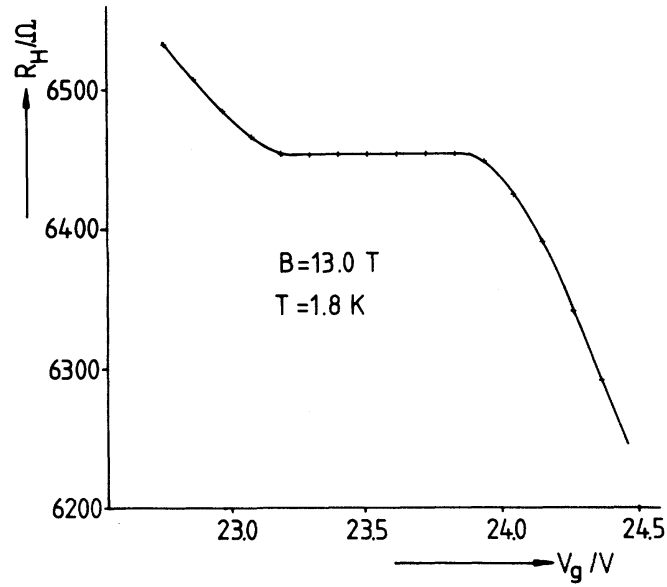


Figure 1.1: Measured Hall resistivity as a function of an applied back-gate which leads to a change in the particle density  $n$ . The pronounced plateau is the hallmark of the quantum Hall effect. Figure taken from Ref [1].

We can express the electric field as

$$\mathbf{E} = \frac{B}{ne} \mathbf{J} \wedge \hat{\mathbf{z}}. \quad (1.1.3)$$

The resistivity  $\rho$  is defined as the relation between the current and the electric field  $E^\mu = \rho_{\mu\nu} J^\nu$ . We thus find

$$\rho = \frac{B}{ne} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \Rightarrow \quad \sigma = \frac{ne}{B} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (1.1.4)$$

We see that owing to the non-zero  $\sigma_{xy}$  the longitudinal resistivity  $\rho_{xx} = \sigma_{xx} = 0$  is equal to the longitudinal conductivity. Moreover, the Hall resistivity is proportional to the magnetic field

$$\rho_{xy} = \frac{B}{ne}. \quad (1.1.5)$$

This is in striking contrast to the seminal discovery of von Klitzing and his co-workers in 1980 [1], see Fig. 1.1. The only ingredient in our theoretical model so far, however, was translational symmetry. In the following, we first take steps towards a quantum mechanical understanding of electrons in a magnetic field before we come back to the issue of translational symmetry breaking via disorder.

## 1.2 Classical Lagrangian

To motivate how the magnetic field enters our quantum mechanical description, we recall that the classical equations of motions are reproduced by the following Lagrangian  $\mathcal{L} = \frac{m}{2} \dot{x}^\mu \dot{x}^\mu - e \dot{x}^\mu A^\mu$ .

$$-\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} + \frac{\partial \mathcal{L}}{\partial x^\mu} = 0 \quad \Rightarrow \quad m \ddot{x} = -e B \dot{y} \quad \text{and} \quad m \ddot{y} = e B \dot{x}. \quad (1.2.1)$$

The canonical momentum is given by  $p^\mu = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu} = m \dot{x}^\mu - e A^\mu$  and therefore the Hamiltonian reads

$$H(x^\mu, p^\mu) = \dot{x}^\mu p^\mu - \mathcal{L}(x^\mu, \dot{x}^\mu) = \frac{1}{2m} (p^\mu + e A^\mu) (p^\mu + e A^\mu). \quad (1.2.2)$$

With this small detour into classical mechanics we are now in the position to tackle the quantum mechanical problem of a particle in a magnetic field.

### 1.3 Landau levels

We have to solve for the eigenstates of the following Hamiltonian

$$H = \frac{1}{2m}(\mathbf{p} + e\mathbf{A})^2. \quad (1.3.1)$$

As only the vector potential  $\mathbf{A}$  enters the Hamiltonian we have to choose an appropriate gauge. For now we choose the Landau gauge where  $\mathbf{A} = xB\hat{y}$ . We check that  $\nabla \wedge \mathbf{A} \equiv \mathbf{B} = (\partial_x A_y - \partial_y A_x)\hat{z} = B\hat{z}$ . Inserted into the above Hamiltonian we obtain

$$H = \frac{1}{2m} \left[ p_x^2 + (p_y + eBx)^2 \right]. \quad (1.3.2)$$

We immediately observe that this Hamiltonian has a translational symmetry in  $y$  direction. We therefore choose the following ansatz for the wave function  $\psi(x, y) = e^{iky} f_k(x)$ . With this ansatz we obtain a family of one-dimensional problems (one per momentum  $k$  in  $y$ -direction)

$$h_k = -\frac{\hbar^2 \partial_x^2}{2m} + \frac{1}{2} m \omega_c^2 (x + kl^2)^2 \quad \text{with} \quad \omega_c = \frac{eB}{m} \quad \text{and} \quad l = \sqrt{\frac{\hbar}{eB}}. \quad (1.3.3)$$

We see that we are dealing with a (displaced) one-dimensional harmonic oscillator. The characteristic frequency is known as the **the cyclotron frequency  $\omega_c$** . The displacement is proportional to the  $y$ -momentum and measured in the natural length scale, the **magnetic length  $l$** . Solving the harmonic oscillator we find that

1.  $\epsilon_k = \hbar\omega_c \left( s + \frac{1}{2} \right)$  with  $s \in \mathbb{N}$ .

2. For  $s = 0$ , i.e., the LLL the wave function is a Gaussian centered at  $X_k = -kl^2$

$$\psi(x, y) = \frac{1}{\sqrt{\pi^{1/2} L_y l}} e^{iky} e^{-\frac{1}{2l^2}(x+kl^2)^2} = \frac{1}{\sqrt{\pi^{1/2} L_y l}} e^{iky} e^{-\frac{1}{2l^2}(x-X_k)^2}, \quad (1.3.4)$$

where  $L_y$  is the extent in  $y$ -direction as shown in Fig. 1.2.

3. We have a vastly degenerate system. The number of degenerate states in each LL is given by

$$N = \frac{L_y}{2\pi} \int_0^{L_x/l^2} dk = \frac{L_x L_y}{2\pi l^2} = \frac{L_x L_y B}{\Phi_0}, \quad (1.3.5)$$

where  $\Phi_0 = h/e$  is the magnetic flux quantum. In other words, per magnetic flux quantum that penetrates the sample we have one state per Landau level.

Before we continue we should remind ourselves that in the case of huge degeneracies any perturbation might have dramatic effect. Moreover, the choice of basis can facilitate the description of these effects. For the case of a magnetic field, the choice of gauge determined the shape of the basis wave-functions. We will come back to this point later.

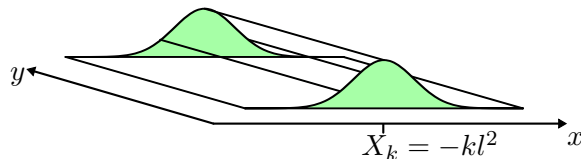


Figure 1.2: Eigenfunctions of the LLL in the Landau gauge.

## 1.4 Currents

We set out to understand the Hall conductivity. To make further progress, we need to calculate currents. We evaluate the current operator in  $y$  direction,  $J_y = -\frac{e}{m}(p_y + eA_y)$ , in the LLL eigenfunctions

$$\langle \psi | J_y | \psi \rangle = -\frac{e}{m\pi^{1/2}l} \int dx e^{-\frac{(x-X_k)^2}{2l^2}} (\hbar k + eBx) e^{-\frac{(x-X_k)^2}{2l^2}} \quad (1.4.1)$$

$$= -\frac{e\omega_c}{\pi^{1/2}l} \int dx e^{-\frac{(x-X_k)^2}{l^2}} (x + kl^2) = -\frac{e\omega_c}{\pi^{1/2}l} \int d\alpha e^{-\frac{\alpha^2}{l^2}} \alpha = 0. \quad (1.4.2)$$

The last equality holds as the integrand is odd under  $\alpha \rightarrow -\alpha$ . In other words, no net current is flowing as shown in Fig. 1.3.

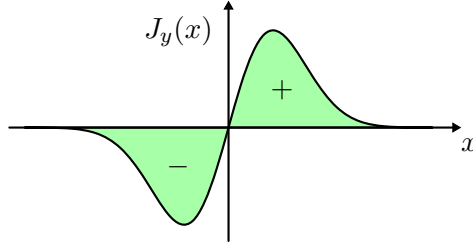


Figure 1.3: Current distribution in the lowest Landau level.

For a current to flow, we need to add an electric field in  $x$ -direction  $V(x) = eEx$ . We still are translationally invariant in  $y$ -direction and the one-dimensional problem is changed to

$$h_k = -\frac{\hbar^2 \partial_x^2}{2m} + \frac{1}{2}m\omega_c^2 (x + kl^2)^2 + eEx \quad (1.4.3)$$

$$= -\frac{\hbar^2 \partial_x^2}{2m} + \frac{1}{2}m\omega_c^2 \left( x + kl^2 + \frac{eE}{m\omega_c^2} \right)^2 + eEX'_k + \frac{1}{2}m\bar{v}^2, \quad (1.4.4)$$

where the center of the Gaussians is shifted

$$X'_k = -kl^2 - eE/m\omega_c^2 \quad (1.4.5)$$

and an additional energy  $\frac{1}{2}m\bar{v}^2$  with  $\bar{v} = -E/B$  arises from the drift of the electrons. The immediate conclusion is that the new energy depends on  $k$ , i.e., the huge degeneracy is lifted

$$\epsilon_k = \frac{1}{2}\hbar\omega_c + eEX'_k + \frac{1}{2}m\bar{v}^2. \quad (1.4.6)$$

With an energy that depends on  $k$  we can also calculate a non-zero group velocity

$$v_{\text{group}} = \frac{1}{\hbar} \frac{\partial \epsilon_k}{\partial k} = \frac{eE}{\hbar} \frac{\partial X'_k}{\partial k} = -\frac{eE}{\hbar} l^2 = -\frac{E}{B} = \bar{v}. \quad (1.4.7)$$

With this we reach the classical result

$$\langle J_y \rangle = -e\bar{v} \quad \Rightarrow \quad \sigma_{xy} = -\frac{ne}{B}. \quad (1.4.8)$$

We reached the same conclusion as with the classical manipulations based entirely on translational symmetry in the beginning of this chapter. In order to make further progress we should take a closer look at the finite extent of a realistic sample as well as on disorder effects to understand the quantization of  $\sigma_{xy}$ .

## 1.5 Edge states

We try to build an understanding of the influence of the edges of a two dimensional sample by considering a strip which is finite in  $x$ -direction and infinite (or periodic) in  $y$ -direction. The basis wave functions of the LLL, or equivalently the gauge choice for  $\mathbf{A}$ , which we used above is optimally tailored to this geometry. Remember that the wave functions are localized in  $x$ -direction with a typical extent  $l$ . If we now consider a potential  $V(x)$

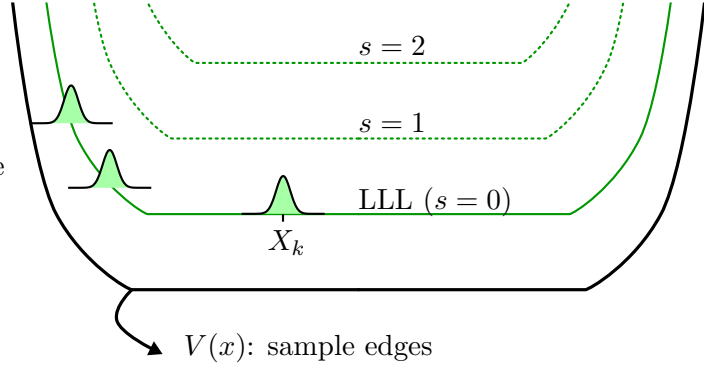


Figure 1.4: Edge states from curved Landau levels.

that confines the electrons to a finite region which is smooth over the length-scale  $l$ , we can expect the wave function to remain approximately Gaussian. However, the wave functions centered in the vicinity of the edges will be lifted in energy. As the position of the wave function is linked to the momentum  $k$  in  $y$ -direction we obtain dispersive edge channels.

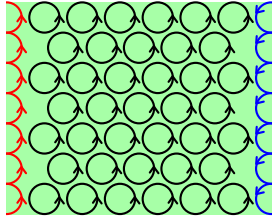


Figure 1.5: Classical skipping orbits.

In order to determine how the current is distributed we again calculate the group velocity.

$$v_{\text{group}} = \frac{1}{\hbar} \frac{\partial \epsilon_k}{\partial k} = \frac{1}{\hbar} \frac{\partial \epsilon_k}{\partial X_k} \frac{\partial X_k}{\partial k} = -\frac{l^2}{\hbar} \frac{\partial \epsilon_k}{\partial X_k} = \begin{cases} < 0 & \text{right edge} \\ > 0 & \text{left edge} \end{cases}. \quad (1.5.1)$$

These simple manipulations reveal that the two opposite edges carry opposite current. This can also be understood from the classical “skipping orbits” picture as shown on the left.

In order to calculate  $\sigma_{xy}$  we now apply a voltage  $V_H$  between the two edges (in  $x$ -direction) and calculate the resulting current along the sample (in  $y$ -direction). Moreover, we assume that the Fermi energy  $E_F$  lies in between two Landau levels.

To obtain the total current  $I_y$  we sum over the contribution  $ev_k$  of all occupied states

$$I_y = -e \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{1}{\hbar} \frac{\partial \epsilon_k}{\partial k} n_k, \quad (1.5.2)$$

where  $n_k$  is the occupation probability of the  $k$ 'th mode. Under the assumption that we only fill the LLL and that we are at zero temperature the occupation numbers only take the values  $n_k = \{1, 0\}$ . Under these assumptions we arrive at

$$I_y = -\frac{e}{h} \int_{\mu_L}^{\mu_R} d\epsilon = -\frac{e}{h} (\mu_R - \mu_L), \quad (1.5.3)$$

where  $\mu_{R/L}$  are the respective chemical potentials on the two sides. As we can write  $eV_H = \mu_R - \mu_L$  we arrive at

$$I_y = -\frac{e^2}{h} V_H \quad \Rightarrow \quad \sigma_{xy} = -\frac{e^2}{h}. \quad (1.5.4)$$

Let us move now the Fermi energy in between any two LL and we immediately conclude that

$$\sigma_{xy} = -\nu \frac{e^2}{h}, \quad (1.5.5)$$

where the integer  $\nu$  counts the number of filled LL's.

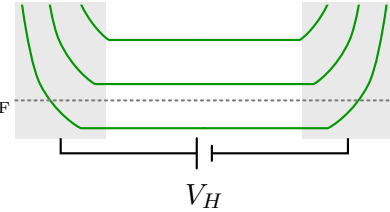


Figure 1.6: Voltage bias.

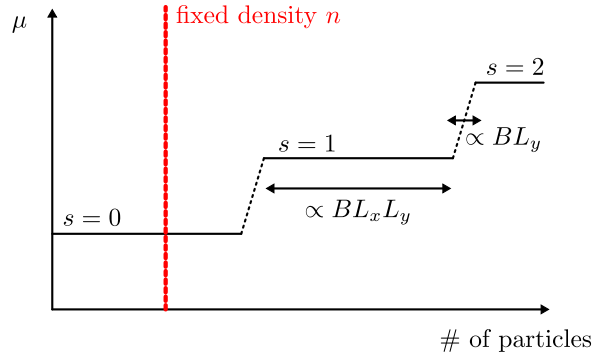


Figure 1.7: Chemical potential stuck to Landau levels.

### 1.5.1 The effect of disorder\*

The above result is strongly suggestive that one dimensional edge channels are responsible for the transport in the quantum Hall effect. Generically the current carried by a one-dimensional channel is given by

$$I = \frac{e^2}{h} |T|^2, \quad (1.5.6)$$

where  $|T|^2$  denotes the probability for an electron to be transmitted through a disordered region. However, our edge channels are *chiral* where the electrons have no way to be back-scattered and therefore  $|T|^2 = 1$ . These arguments explain why even in the case of a disordered sample  $\sigma_{xy}$  can be quantized. However, we did not yet reconcile a quantized  $\sigma_{xy}$  with the general result  $\sigma_{xy} = ne/B$  for a clean system.

We assumed the Fermi energy to lie between two Landau levels. Let us see under which conditions this can be the case. We assume the sample to be  $L_x$  wide and the edge region which is curved up to extend over the length  $W \ll L_x$ . From the finite size (or periodic) quantization in  $y$ -direction we know that the momentum can take the values  $k_i = \frac{2\pi}{L_y} i$  with  $i \in \mathbb{Z}$ . Hence, we find for the centers of the Localized wave functions  $X_i = \frac{2\pi}{L_y} l^2 i$ . We now count how many wave functions fit into the bulk and how many into the edge:

$$\text{edge} \quad : \quad \frac{W}{X_i - X_{i-1}} \propto L_y, \quad \text{bulk} \quad : \quad \frac{L_x}{X_i - X_{i-1}} \propto L_x L_y. \quad (1.5.7)$$

We see that there are extensively many bulk states but only a sub-extensive number of edge states as shown in Fig. 1.7.

Translated to a fixed density but varying magnetic field  $B$  we find that for almost all values of  $B$  the Fermi energy will lie in the bulk, not the edge! Meaning, our assumption that the we have a completely filled LL and the relevant physics is happening only on the edge was not justified. Hence we need to get a better understanding of disorder effects. We do so in App. A.

Let us here briefly summarize some important effects of disorder. First, it spoils translational symmetry that led to the classical result in contrast with the experimental observation. Second, it lifts the degeneracy of the Landau levels. Disorder broadens the Landau levels in the bulk and provides a reservoir of (localized) states which allow the chemical potential to vary smoothly with density. Since they are localized, these states will not contribute to transport and to the Hall conductance.

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1. V. Klitzing, K., Dorda, G. & Pepper, M. “New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance”. *Phys. Rev. Lett.* **45**, 494. <http://link.aps.org/doi/10.1103/PhysRevLett.45.494> (1980).
2. *Topological aspects of low dimensional systems* (eds Comtet, A., Jolicœur, T., Ouvry, S. & David, F.) (Springer-Verlag, Berlin, 1999).
3. Hall, E. “On a New Action of the Magnet on Electric Currents”. *Amer. J. Math.* **2**, 287. <https://www.jstor.org/stable/2369245> (1879).



# Appendix A

## Scaling theory of localization

### Learning goals

- We appreciate the role of the dimensions for the localization of electrons.
  - We can reproduce the gang-of-four scaling plot.
- 
- M. A. Paalanen G. A. Thomas, *Helv. Phys. Acta* **56**, 27 (1983)

### A.1 Conductance versus conductivity

We want to study the influence of disorder on the electrical resistance  $R$  relating the applied voltage  $U$  to the electrical current  $I$

$$U = RI. \tag{A.1.1}$$

$R$  connects two macroscopic observables and therefore characterizes a macroscopic sample. The conductance  $g$  is defined as the inverse of the resistance

$$g = \frac{1}{R}. \tag{A.1.2}$$

These quantities have to be contrasted with the microscopic quantities such as the conductivity  $\sigma$

$$j = \sigma E, \tag{A.1.3}$$

where  $E$  is the electric field and  $j$  the microscopic current density. In this chapter we want to understand if there is a simple bridge between the microscopic quantity  $\sigma$  (which we might be able to calculate from first principles in simple model situations) and the macroscopic conductance  $g$ . We try to do so by starting from a relatively small system where we are in principle up to the task of calculating  $g$  exactly. We then want to successively increase the system size and see what we can deduce.

### A.2 One parameter scaling

The key step in the program of successively increasing the system size dates back to the very influential paper by what we now call the the “gang of four”: Abrahams, Anderson, Licciardello, and Ramakrishnan [1]. Their key insight was that the conductance  $g(2L)$  of a block of size  $2L$  only depends on one parameter, namely the conductance  $g(L)$  of the block of size  $L$  out of which the larger was formed, cf. Fig. A.1. In other words

$$g(2L) = f[g(L)] \quad \text{and not} \quad g(2L) = h[g(L), L, \dots]. \tag{A.2.1}$$

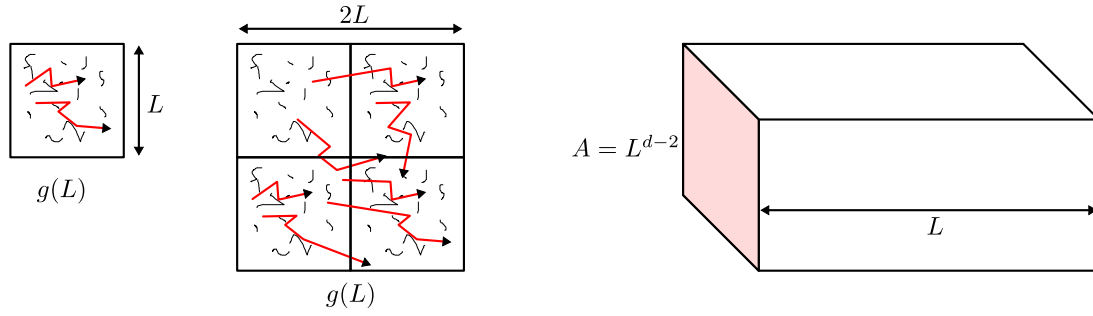


Figure A.1: Setup for the renormalization of the conductance.

This statement is not easy to motivate in a systematic way. Instead of attempting to legitimate (A.2.1), we want to analyze its consequences in the following. To make further progress we write (A.2.1) in a form that contains no scales

$$\frac{L}{g} \frac{dg(L)}{dL} = \frac{d \log(g)}{d \log(L)} = \beta(g). \quad (\text{A.2.2})$$

Let us have a look at simple limiting cases.

For a good conductor  $g \gg 1$  we know that the “one parameter scaling” holds in the form of Ohm’s law

$$R = \rho \frac{L}{A} = \rho \frac{L}{L^{d-1}} \quad \Rightarrow \quad g = \sigma_0 L^{2-d}. \quad (\text{A.2.3})$$

From this we immediately obtain

$$\frac{d \log(g)}{d \log(L)} = d - 2 \quad \Rightarrow \quad \lim_{g \rightarrow \infty} \beta(g) = d - 2. \quad (\text{A.2.4})$$

In the other limit of very strong disorder, all wave-function will be exponentially localized. Therefore, we expect the conductance to behave as

$$g(L) \propto e^{-L/\xi} \quad \Rightarrow \quad \frac{d \log(g)}{d \log(L)} = -\frac{L}{\xi} = \log(g). \quad (\text{A.2.5})$$

Hence in the limit of vanishingly small conductance, the  $\beta$  function reads

$$\lim_{g \rightarrow 0} \beta(g) = \log(g). \quad (\text{A.2.6})$$

We summarize these results in Fig. A.2. Due to the dependence of the  $\beta$ -function on the dimension  $d$ , disorder seems to have very different effects depending on the spatial dimension. Let us discuss the consequences of Fig. A.2 for one, two, and three dimensions separately.

### A.2.1 One dimension

In one dimension,  $\beta(g) < 0$  is always negative. In other words, by increasing the system size, the conductance  $g$  always flows to zero, irrespective of the conductance of a short section of the wire.

Let us define a localization length  $\xi$ , where  $g(L = \xi) = 1$ . We find

$$\frac{d \log(g)}{d \log(L)} = -1 \quad \Rightarrow \quad g(L) = \frac{g_0}{L} \quad \Rightarrow \quad \xi \sim g_0, \quad (\text{A.2.7})$$

where  $g_0$  is the conductance calculated for a small segment.

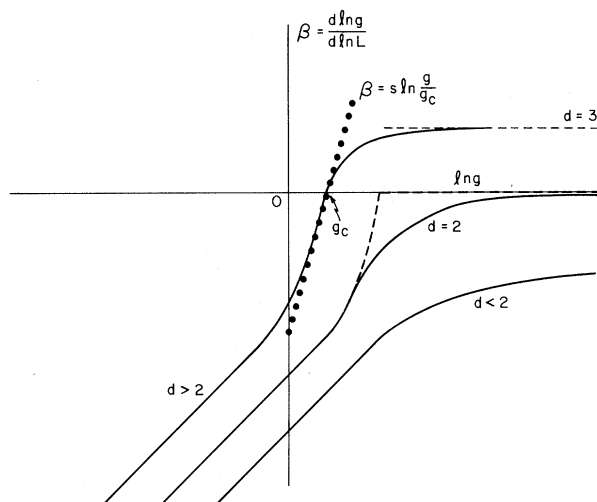


Figure A.2: Plot of  $\beta(g)$  as a function of  $\log(g)$  for various dimensions. Figure taken from Ref. [1] (Copyright (1979) by The American Physical Society).

### A.2.2 Two dimensions

In two dimensions we encounter a somewhat more intriguing situation. For large values of  $g$  the  $\beta$ -function is zero. In other words, to first order in  $1/g$ , the conductance does not change under a change in  $L$ . Such a situation is called marginal. As we have identified the limit  $g \gg 1$  as the classical regime where Ohm's law holds, this means quantum corrections will play a crucial role in how  $\beta(g)$  behaves away from  $g \gg 1$ . These quantum corrections are called “weak (anti-) localization”. Their detailed calculation is beyond the scope of this course. However, we can estimate them using a simple trick. Let us just calculate the probability for a particle to return to the point where we inject it into the system

$$P = |\langle \psi^\dagger(x) \psi(x) \rangle|^2. \quad (\text{A.2.8})$$

When we calculate  $\langle \psi^\dagger(x) \psi(x) \rangle$ , we have to sum over all paths the particle can take from  $x$ , back to the same point  $x$ . In quantum mechanics, each path is associated with an amplitude and a phase. Due to the disorder (which we try to study, after all), all paths sum up incoherently. If we have a time-reversal invariant system, however, there are paths whose amplitude and phase are correlated as shown in Fig. A.3. Owing to the time-reversal symmetry the blue and the red path have a well defined phase relation. If we now calculate  $P$  the sum contains the following contributions shown in Fig. A.4.

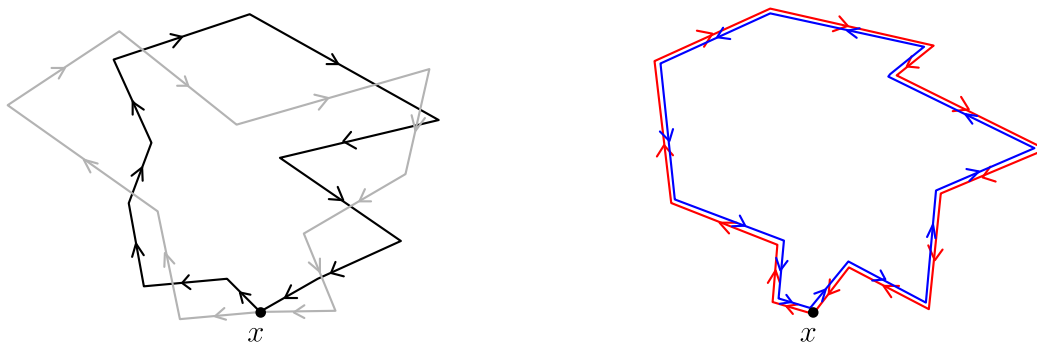
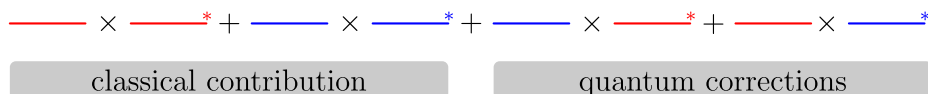


Figure A.3: Return probability.



The endpoints of any segment are related to some state  $|\phi\rangle$ . In order to invert the arrow of time we use the time reversal operator  $\mathcal{T}$  on these states:

Figure A.4: Interference in the return probability.

From this we conclude that we can have two distinctly different situations

1.  $\mathcal{T}^2 = -\mathbb{1}$   $\Rightarrow$  the return probability  $P$  is reduced, hence quantum mechanical effects lead to more extended states and we deal with weak *anti-localization*.
2.  $\mathcal{T}^2 = \mathbb{1}$   $\Rightarrow$  the return probability  $P$  is enhanced, i.e., the states are more localization: weak *localization*.

However, a word of caution is in order here! When applying this argument for spin-1/2 fermions we generically have  $\mathcal{T}^2 = -\mathbb{1}$ . But if the Hamiltonian does not mix the spin degrees of freedom, we can go to the individual spin sectors and describe the physics as two spin-0 problems. In this case however,  $\mathcal{T}^2 = \mathbb{1}$ . The situation changes if we deal with *spin-orbit* coupling. In this case we have to stick with the spin-1/2 description and therefore we generically expect *anti-localization* in this case.

Let us now analyze the case of no spin-orbit interactions, i.e., weak localization. We solve the equation

$$\frac{d \log(g)}{d \log(L)} = -\frac{C}{g} \quad \text{or} \quad \frac{dg}{d \log(L)} = -C < 0. \quad (\text{A.2.9})$$

We find

$$g = g_0 - C \log(L/l), \quad (\text{A.2.10})$$

where  $l$  is the small length at which we managed to solve the problem exactly and found  $g(l) = g_0$ . We can now again determine the localization length by equating  $g(\xi) = 1$  to find

$$\xi \sim l e^{g_0/C}. \quad (\text{A.2.11})$$

Indeed, all states are localized. However,  $g \gg g_0$  and the localization length is astronomical.

### A.2.3 Three dimensions

Three dimensions (or two with spin-orbit) are the most interesting cases. Depending on the initial value  $g_0$ ,  $\beta(g)$  is either positive or negative, i.e., a macroscopic sample can either be conducting or insulating. In other words, there is a metal-insulator transition as a function of  $g_0$ . For the time being three dimensions are not in the scope of the course and we will come back to it (and spin-orbit in two dimensions) later.

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