

Problem Set 5

1. Hertzian dipole

A Hertzian dipole is a time-dependent point dipole $p(t)$ with

$$\rho(x, t) = -p(t) \cdot \nabla \delta(x), \quad \mathbf{j}(x, t) = \dot{p}(t) \delta(x).$$

- (i) Verify the continuity equation.
- (ii) Compute the retarded electromagnetic potentials φ and A in the Lorenz gauge. Furthermore, compute the electromagnetic fields

$$E(x, t) = -\nabla \varphi(x, t) - \frac{1}{c} \frac{\partial A}{\partial t}(x, t), \quad B(x, t) = \nabla \times A(x, t)$$

thereof. Arrange the contributions in terms of powers of r^{-1} .

- (iii) Let the direction of p be constant. What are the directions of E and B ?
- (iv) In the time-harmonic case (wavelength λ), discuss which terms dominate for $r \gg \lambda$ and $r \ll \lambda$ respectively. What is the relative phase of E and B in both limiting cases?
- (v) Compute the Poynting vector for $r \gg \lambda$, as well as the dependence of the power on the angle. What is the amount of the total power emitted?

2. Thomson scattering

A point particle (charge e , mass m) is moving under the influence of a monochromatic, plane electromagnetic wave. At the same time, it is radiating itself. Compute the ratio of the emitted power P and the incident intensity I_0 ,

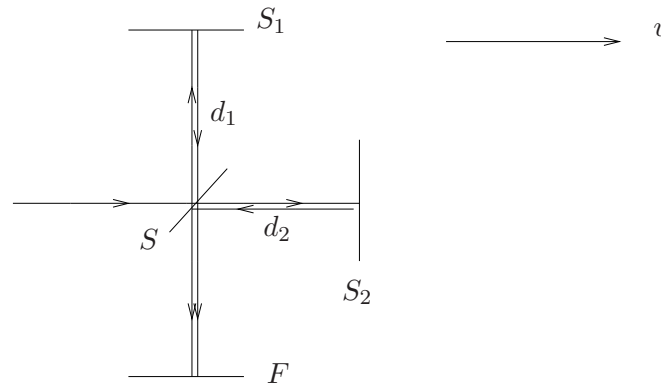
$$\sigma = \frac{P}{I_0},$$

in the case of a small I_0 . The scattering cross section σ has the physical dimension of an area.

Hint: Discuss the oscillation of the particle around $x = 0$ to first order in the wave amplitude. As long as I_0 is small (with respect to what?), it is enough to consider the part of P corresponding to the electric dipole radiation.

3. The Michelson-Morley experiment

In the 19th century, light was considered to be an excitation of an aether. Its velocity of propagation is isotropic w.r.t. the aether. However, as long as space and time coordinates are subject to Galilei transformations, the former is not isotropic w.r.t. a reference frame which is moving w.r.t. the aether. In the course of a year, the Earth can not permanently be at rest with respect to the aether because of its motion around the Sun. Michelson and Morley did not find such an anisotropy in 1886. They were using the apparatus (interferometer) in the picture, where v is the velocity of the laboratory w.r.t. the aether.



A light ray strikes a semipermeable mirror S which divides it into two perpendicular partial beams. These beams are reaching the mirrors S_i after travelling the distances d_i ($i = 1, 2$), and subsequently get back to S respectively. Afterwards, a part of each of them is reaching an observer telescope F , where an interference pattern of stripes is visible. The conditions for this are on one hand a difference $|d_2 - d_1|$ which is small compared to the coherence length of the light; and on the other hand an arrangement of the mirrors S_1 and S_2 which is not exactly perpendicular, such that we have variable path differences. Displacements of the pattern can be measured within fractions of one wavelength.

- (i) Find the elapsed times t_1 and t_2 of the light along the two paths SS_iS , and thereby $\Delta t = t_2 - t_1$ up to relative errors $O((v/c)^4)$. Then compute $\Delta t'$ for an arrangement rotated by 90° . The difference $\Delta t' - \Delta t$ determines the displacement of the pattern caused by the rotation.
- (ii) Numerical example: $v = 3 \cdot 10^4 \text{ m/s}$, $d_1 + d_2 = 3 \text{ m}$, wavelength of the light $\lambda \approx 3 \cdot 10^{-7} \text{ m}$. By which part of the distance between the stripes is the pattern displaced?

Hint: The aether and the laboratory system should be assumed to be connected via a Galilei transformation, since the experiment is testing the classical notion of spacetime.

Due: 13.04.2016.